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A METHODOLOGY FOR OPTIMAL DESIGN OF WATER DISTRIBUTION SYSTEMS.(U)

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6 A METHODOLOGY FOR OPTIMAL DESIGN OF WATER

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A METHODOLOGY FOR OPTIMAL DESIGN OF WATER

DISTRIBUTION SYSTEMS

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To my parents who always expected
nothing but my best.

To my wife Kathi whose steadfast
support kept me going when
everything seemed bleak.

To my children, Bryan and Jenni,
who kept asking, "Does Daddy have
to go to school again?"

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W.F.R.

The University of Texas at Austin

December 1979

ABSTRACT

A comprehensive methodology for the design of municipal water distribution systems that explicitly incorporates reliability and performance into the system design is developed. The complex design problem is decomposed within the context of a three-level hierarchically integrated system of models. The first and second level models combine to select the links in the distribution system layout. The third level model accomplishes the detailed system design for the layout from the upper level models. Two alternative first level models, a shortest path tree and a nonlinear programming model, are developed to select the minimum cost tree layout. Two second level, complementary 0-1 integer programming models are developed to select the loop-forming links for the minimum cost tree layout. The third level nonlinear programming model optimizes the detailed distribution system design (link diameters, pump capacities, elevated storage heights, and valve resistance) of the resulting network layout with respect to distribution system performance under expected emergency loading conditions (fire demand,

broken links, pump outage). This detailed design is performed subject to satisfying steady state conditions, minimum performance levels under normal loading conditions, and maximum budget level. The methodology is applied to the design of a real life water distribution system.

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CHAPTER 1

LITERATURE REVIEW

1.1 Introduction to Water Distribution Systems

1.1.1 Major System Components

A water distribution system generally consists of a set of sources, pipes, pumps, and valves that supply water to a set of demand points. In network terms the source and demand points may be represented by nodes and the pipes may be represented by links or arcs connecting the nodes. Source nodes bring flow into the network while demand nodes withdraw flow from the network. A special type of source, the balancing storage reservoir, has a dual function of filling up with water during periods of low demand (night) and releasing water during periods of high demand (late afternoon/early evening).

1.1.2 Conservation of Energy

Flowing water contains both kinetic and potential energy. It possesses kinetic energy due to its motion. It contains two

forms of potential energy, one by virtue of its elevation and the other by virtue of its pressure. The energy per unit weight (E/g') of a fluid is the sum of these three energy components:

$$\frac{E}{g'} = EL + \frac{P}{\gamma} + \frac{V^2}{2g'} \qquad (1\text{-}1)$$

energy/unit weight = due to elevation + due to pressure + Kinetic

where EL is the vertical distance above some datum plane, P is the fluid pressure, γ the specific weight of the fluid, g' the acceleration of gravity, and V the velocity of the liquid [1]. Since the units of energy are force times length and gravity is a force, the dimension of equation (1-1) is length (more correctly energy per pound). Each of the terms is designated as a "head," i.e., EL, is the elevation head, P/γ is the pressure head and $V^2/2g'$ is the velocity head. The sum of $EL + P/\gamma$ is denoted as the piezometric or hydraulic head and the sum $EL + P/\gamma + V^2/2g'$ is the total or stagnation head.

Whenever fluid flow passes a fixed wall or boundary, fluid friction exists. Thus, between any two distinct points in a pipeline there is a frictional head loss ΔHF due to pipe resistance and valve

resistance. The calculation of frictional head loss will be discussed in section 1.1.3.

A pump is associated with a link and adds pressure head to each unit weight of fluid passing through the pump. The pressure head or head lift added by a pump will be denoted by XP.

Figure 1-1 depicts water flowing from point 1 to point 2 in a link with a pump adding head in between the two points. Bernoulli's equation for incompressible fluid flow accounts for the change in energy level that occurs between the two points:

$$EL_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g'} + XP = EL_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g'} + \Delta HF \qquad (1\text{-}2)$$

In pipeline design problems the velocity head is usually negligible compared to the other head components simplifying equation (1-2) to

$$EL_1 + \frac{P_1}{\gamma} + XP = EL_2 + \frac{P_2}{\gamma} + \Delta HF \qquad (1\text{-}3)$$

1.1.3 Frictional Head Loss Equations

There are several equations which may be used to evaluate a link's frictional head loss, i.e., the conversion of energy per unit weight into a nonrecoverable form of energy. These equations are categorized as either empirical or rational equations. The empirical

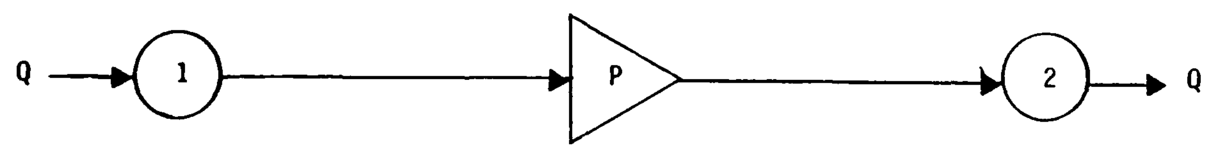

Figure 1-1

PIPE FLOW BETWEEN TWO POINTS

frictional head loss equation for a link has the general form

$$\Delta HF = \frac{K\,Q^n\,L}{D^m} \tag{1-4}$$

where Q is the link flow rate, D its diameter, L its length, K a constant which is determined by the roughness of the pipe and the particular units of measurement, and n and m are positive constants. The most widely used empirical equation is the Hazen-Williams equation [2]

$$\Delta HF = \frac{10.471\,Q^{1.852}\,L}{(HW)^{1.852}\,D^{4.87}} \tag{1-5}$$

where HW is the Hazen-Williams roughness coefficient, flow Q is given in gallons per minute (GPM), link length L is given in feet, and link diameter D is given in inches. Empirical equations were specifically derived for waterworks practice and do not take into account variations in gravity, temperature, or type of liquid.

In contrast the newer rational equations were developed analytically and verified by extensive, systematic laboratory testing. Unlike the empirical equations any consistent units of measurement and liquids of different viscosities and temperatures may be used. The Darcy Weisbach equation is the most widely used

rational equation:

$$\Delta HF = \frac{f' \, L \, V^2}{D \, 2g'} \tag{1-6}$$

where f' is a dimensionless friction factor. The friction factor depends on several factors including the type of flow, i.e., laminar, turbulent, the Reynolds number (Re), and the relative roughness of the pipe wall (e'/D). For water flow in closed conduits the Colebrook-White equation is usually used to calculate f'.

$$\frac{1}{\sqrt{f'}} = 1.14 - 2 \log_{10} \left(\frac{e'}{D} + \frac{9.35}{Re \sqrt{f'}} \right) \tag{1-7}$$

In most cases the rational equations cannot be solved directly because of the requirement to use iterative techniques to solve for f'. Thus, although theoretically more sound the rational equations are somewhat more difficult to use than the older empirical equations.

The general form of the empirical head loss equation (1-4) will be used throughout this paper. All mathematical models and numerical examples presented in this paper use the Hazen-Williams formula (1-5) with units of flow rate in gallons per minute, diameter in inches, and link length and head loss in feet.

1.1.4 Steady State Flow Conditions

To properly design a water distribution system it is necessary to study its behavior under steady state flow conditions, i.e., where flow does not change over time. The laws of conservation of flow and energy characterize steady state conditions.

Conservation of flow requires that the flow rate entering a node must equal the flow rate leaving a node. For each node i this requirement can be expressed mathematically as

$$\sum_{k \varepsilon O_i} Q_k - \sum_{k \varepsilon T_i} Q_k = b_i \qquad (1\text{-}8)$$

$$i = 1, \ldots, \text{NNODE}$$

where Q_k is the flow rate on link k, O_i is the set of links with flows leaving node i, T_i the set of links with flows entering node i, b_i the external flow at node i, and NNODE the number of nodes in the network. External flow b_i is positive if it enters a node (source node) and negative if it leaves a node (demand node). The seven conservation of flow equations for the network in Figure 1-2 are written below.

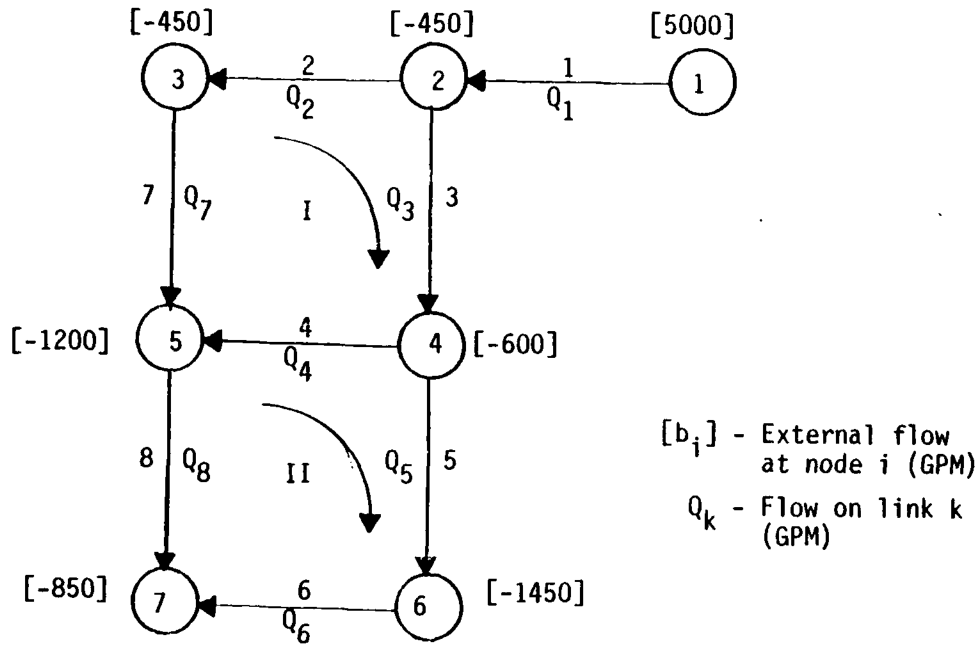

Figure 1-2

TWO LOOP, SINGLE SOURCE DISTRIBUTION SYSTEM

$$\begin{array}{lllllllllll}
Q_1 & & & & & & & & & = & 5000 \\
-Q_1 & + Q_2 & + Q_3 & & & & & & & = & -450 \\
& -Q_2 & & & & & & + Q_7 & & = & -450 \\
& & -Q_3 & + Q_4 & + Q_5 & & & & & = & -600 \\
& & & -Q_4 & & & & -Q_7 & + Q_8 & = & -1200 \\
& & & & -Q_5 & + Q_6 & & & & = & -1450 \\
& & & & & -Q_6 & & & -Q_8 & = & -850
\end{array} \qquad (1\text{-}9)$$

Any one of the equations in the linear system of equations (1-8) may be deleted as redundant leaving NNODE - 1 equations in NLINK unknown link flows.

For an arbitrary network of NLINK links and NNODE nodes there are

$$\text{NLOOP} = \text{NLINK} - \text{NNODE} + 1 \qquad (1\text{-}10)$$

non-overlapping loops in the network [3]. For a tree network NLOOP = 0 and NLINK = NNODE - 1 [3]. Thus, for a tree network the number of independent nodal equations is equal to the number of unknown link flows and the system (1-8) can be solved directly for Q_k.

Conservation of energy requires that the net frictional head losses around any loop equal zero. For a network with NLOOP loops

we have the system of NLOOP equations

$$\sum_{k \varepsilon \text{ LOOP}_i} \pm \Delta HF_k = 0 \qquad i = 1, \ldots, \text{NLOOP} \tag{1-11}$$

where $LOOP_i$ is the set of links in loop i and ΔHF_k is the frictional head loss on link k. Using the general empirical frictional head loss relationship (1-4) results in

$$\sum_{k \varepsilon \text{ LOOP}_i} \pm \frac{K_k \, Q_k^n \, L_k}{D_k^m} = 0 \qquad i = 1, \ldots, \text{NLOOP} \tag{1-12}$$

where Q_k is the flow rate on link k, L_k its length, D_k its diameter, and K_k a constant which depends on the link's roughness coefficient (HW_k for the Hazen-Williams equation) and the particular empirical equation and units of measurement chosen. The sign of each head loss term in (1-12) depends on the direction of flow in the link with respect to the direction (clockwise or counterclockwise) that the loop is traversed in writing the equation. The two loop equations for Figure 1-2 are written below. Both loops are traversed in a clockwise direction. Each link is assumed to have a pipe of a single diameter D_k.

$$\frac{-K_2\, Q_2^n\, L_2}{D_2^m} + \frac{K_3\, Q_3^n\, L_3}{D_3^m} + \frac{K_4\, Q_4^n\, L_4}{D_4^m}$$

LOOP I

$$+ \frac{-K_7\, Q_7^n\, L_7}{D_7^m} = 0$$

(1-13)

$$\frac{-K_4\, Q_4^n\, L_4}{D_4^m} + \frac{K_5\, Q_5^n\, L_5}{D_5^m} + \frac{K_6\, Q_6^n\, L_6}{D_6^m}$$

LOOP II

$$+ \frac{-K_8\, Q_8^n\, L_8}{D_8^m} = 0$$

Combining the set of NNODE - 1 linear equations of (1-8) and the NLOOP = NLINK - NNODE + 1 nonlinear equations of (1-12) results in a system of NLINK equations in as many unknowns. The unique flow solution to this nonlinear system of equations characterizes steady state flow in the network.

1.2 Steady State Network Analysis

Because of the fundamental importance of balancing the network, i.e., finding steady state flow conditions, in any distribution

system analysis or optimization model, a great deal of research has been devoted to finding efficient techniques to solve this problem. The two most widely used methods for network balancing, the Hardy Cross and the Newton-Rhapson methods, will be treated in detail. This section will conclude with a summary of the major features of alternative balancing methods.

1.2.1 Hardy Cross Method

The Hardy Cross method [4] (1936) is the oldest and most widely used method for pipe network analysis. This method is an iterative scheme originally developed for hand computation. With the advent of the digital computer it was used as the basis for numerous programs (Hoag and Weinberg (1957) [5], Graves and Branscome (1958) [6], Adams (1961) [7], Bellamy (1965) [8], and Dillingham (1967) [9]).

To satisfy steady state conditions both the system of nodal conservation of flow equations (1-8) and the system of conservation of energy loop equations (1-12) must be satisfied. By appropriate choice of unknowns, the Hardy Cross method can be applied to solving either nonlinear system of equations, (1-8) or (1-12), where the remaining system is linear and is automatically satisfied at all times. However, before discussing the specific application of the

Hardy Cross method to the nodal or loop equations, we will discuss its use in solving a general system of nonlinear equations.

In general, given a system of N simultaneous nonlinear equations

$$h_i(\hat{x}) = 0 \tag{1-14}$$

where $\hat{x} = (x_1, \ldots, x_N)$ is a vector of unknowns, the Hardy Cross method attempts to solve the system of equations by making corrections to one equation at a time. Let $\hat{x}^k = (x_1^k, \ldots, x_N^k)$ be the value of the unknowns at iteration k. If $h_i(\hat{x}^k) = 0$ for all i, then $\hat{x}^k$ is the solution. Otherwise, we seek corrections to the unknowns, $\Delta\hat{x}^k = (\Delta x_1^k, \ldots, \Delta x_N^k)$ such that $|h_i(\hat{x}^k + \Delta\hat{x}^k)| < |h_i(\hat{x}^k)|$. Using a Taylor series expansion of equation i about the current point $\hat{x}^k$ but only perturbing a single variable x_j, i.e., $\Delta\hat{x}^k = (0, \ldots, \Delta x_j^k, 0, \ldots)$, we obtain

$$\begin{aligned} h_i(\hat{x}^k + \Delta\hat{x}^k) &= h_i(\hat{x}^k) + \Delta x_j^k \frac{\partial h_i(\hat{x}^k)}{\partial x_j} \\ &+ \frac{1}{2!}(\Delta x_j^k)^2 \frac{\partial^2 h_i(\hat{x}^k)}{\partial x_j^2} + \ldots \end{aligned} \tag{1-15}$$

where $\partial^{\ell} h_i(\hat{x}^k) / \partial x_j^{\ell}$ is the ℓth partial derivative of h_i with respect to x_j evaluated at $\hat{x}^k$. Retaining only the first two terms of the expansion (1-15), setting the right hand side equal to zero, and solving for the correction term gives us

$$\Delta x_j^k = \frac{-h_i(\hat{x}^k)}{\dfrac{\partial h_i(\hat{x}^k)}{\partial x_j}} \tag{1-16}$$

The above algorithm continues until the convergence criteria are satisfied, e.g., $|h_i(\hat{x}^k)| < \varepsilon_1$ for $i = i, \ldots, N$ or $|\Delta x_j^k| < \varepsilon_2$ for $j = 1, \ldots, N$, $\varepsilon_1, \varepsilon_2 > 0$.

To solve the nonlinear system of loop equations (1-12) first an initial flow distribution is chosen that satisfies the nodal conservation of flow equations (1-8). For the resulting loop equations we have

$$h_i = \sum_{j \varepsilon \text{ LOOP}_i} \pm \frac{K_j Q_j^n L_j}{D_j^m} = 0 \tag{1-17}$$

$$i = 1, \ldots, \text{NLOOP}$$

The value of h_i at the current flow distribution is the head imbalance on loop i. The correction term is ΔQ_i, the flow change on loop (equation) i. ΔQ_i is applied to every link in the loop, i.e., $j \varepsilon$ $LOOP_i$, according to the link's flow direction. If $\Delta Q_i > 0$, the flow increases by $|\Delta Q_i|$ in those links with plus signs in loop equation i and decreases by $|\Delta Q_i|$ in those links with minus signs. If $\Delta Q_i < 0$, the direction of link flow change is reversed. To compute ΔQ_i we compute

$$\frac{\partial h_i}{\partial \Delta Q_i} = \sum_{j \varepsilon \, LOOP_i} \left| \frac{n K_j Q_j^{n-1} L_j}{D_j^m} \right| \qquad (1\text{-}18)$$

and substitute (1-17) and (1-18) into (1-16) to obtain

$$\Delta Q_i = \frac{- \sum_{j \varepsilon \, LOOP_i} \left(\frac{K_j Q_j^n L_j}{D_j} \right)}{\sum_{j \varepsilon \, LOOP_i} \left| \frac{n K_j Q_j^{n-1} L_j}{D_j^m} \right|} \qquad (1\text{-}19)$$

or

$$\Delta Q_i = \frac{- \sum_{j \varepsilon \, LOOP_i} \Delta HF_j}{\sum_{j \varepsilon \, LOOP_i} \left| \frac{\Delta HF_j}{Q_j} \right|} \qquad (1\text{-}20)$$

It is common in the Hardy Cross method to apply only one iterative correction to each equation before proceeding to the next equation. The algorithm terminates when either $| h_i | < \varepsilon_1$ or $| \Delta Q_i | < \varepsilon_2$ for all loops where $\varepsilon_1, \varepsilon_2 > 0$. A detailed statement of the Hardy Cross loop method and its application to a two-loop network is presented in Appendix A.

Alternatively, the Hardy Cross method may be applied to the nodal conservation of flow equations (1-8). Applying the empirical head loss equation (1-4) to link k and solving for Q_k we have

$$Q_k = \left[\frac{D_k^m \, \Delta \, HF_k}{K_k \, L_k} \right]^{\frac{1}{n}} \tag{1-21}$$

Substituting (1-21) into (1-8) results in the following nonlinear system of equations

$$\sum_{k \varepsilon O_i} \left[\frac{D_k^m \, \Delta \, HF_k}{K_k \, L_k} \right]^{\frac{1}{n}} - \sum_{k \varepsilon T_i} \left[\frac{D_k^m \, \Delta \, HF_k}{K_k \, L_k} \right]^{\frac{1}{n}} - b_i = 0 \tag{1-22}$$

$$i = 1, \ldots, NNODE - 1$$

Heads at all nodes (except fixed head nodes) are arbitrarily initialized thus automatically satisfying the conservation of energy loop equations (1-12). The link head losses ΔHF_k are computed by subtracting the nodal heads at the end of the link. The direction of link flow is from the node with the higher head to the node with the lower head. The magnitude of the flow rate Q_k is computed using equation (1-21). However, now nodal conservation of flow equations (1-8) may be violated. Similar to the loop method, nodal head corrections are applied in such a manner as to satisfy nodal conservation of flow equations using the correction term

$$\Delta H_i = - \frac{\sum_{k \varepsilon O_i} Q_k - \sum_{k \varepsilon T_i} Q_k - b_i}{\sum_{k \varepsilon O_i \cup T_i} \left| \frac{Q_k}{n \, \Delta HF_k} \right|} \qquad (1\text{-}23)$$

$$i = i, \ldots, NNODE - 1$$

where ΔH_i is the head change at node i. Early implementations of the Hardy Cross method used the loop method ([5], [6]) while later work ([7], [8]) tended to use the node method principally because of the relative ease in specifying the input data. For large and complex networks the Hardy Cross method frequently converges very slowly if at all.

1.2.2 Newton-Rhapson Method

The Newton-Rhapson method, also referred to as Newton's method, differs from the Hardy Cross method in that it computes corrections to all unknowns simultaneously rather than individually and therefore uses either the entire system of nodal (1-8) or loop (1-12) equations at once.

Given the system of simultaneous nonlinear equations (1-14) and a current point $\hat{x}_1^k$, each equation is expanded in a Taylor series about $\hat{x}^k$ allowing all unknowns to be perturbed simultaneously. Retaining only first order terms in the expansion and setting each equation to zero results in the linear system of equations at iteration k

$$h_i\,(\hat{x}^k) + \sum_{j=1}^{N} \frac{\partial\, h_j\,(\hat{x}^k)}{\partial\, x_j}\,\Delta\, x_j^k = 0 \qquad (1\text{-}24)$$

$$i = 1, \ldots, N$$

The vector of corrections $\Delta\,\hat{x}^k$ is the solution of the simultaneous system of linear equations

$$JAC^k\,\Delta\,\hat{x}^k = -\,h\,(\hat{x}^k) \qquad (1\text{-}25)$$

where JAC^k is the Jacobian matrix

$$JAC^k = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_N} \\ \vdots & & \vdots \\ \frac{\partial h_N}{\partial x_1} & & \frac{\partial h_N}{\partial x_N} \end{bmatrix} \tag{1-26}$$

evaluated at the current point $\hat{x}^k$ and $h(\hat{x}^k) = (h_1(\hat{x}^k), \ldots, h_N(\hat{x}^k))$. The new values of all the unknowns can be computed immediately

$$x_j^{k+1} = x_j^k + \Delta x_j^k \tag{1-27}$$

$$j = 1, \ldots, N$$

The above algorithm continues until the selected convergence criteria are satisfied.

Martin and Peters [10] in 1963 first applied the Newton-Rhapson method to the network analysis problem. Since then several researchers have refined its application to network analysis and incorporated it as part of optimization models (Shamir [11] (1964), Shamir and Howard [12] (1968), Epp and Fowler [13] (1970), Zarghamee [14] (1971), Lemieux [15] (1972), and Donachie [16] (1973)). In general, the Newton-Rhapson method is superior to the Hardy Cross

method assuming that the necessary matrix storage is available. However, because of the nonconvexity of the system of loop and nodal equations, for a general starting point, the inverse Jacobian may not be positive definite or may not even exist. Thus, a poor initial solution may not yield a direction of descent and the algorithm may not converge (Luenberger [17]).

1.2.3 Alternative Methods

Wood and Charles (1972) [18] developed a linear theory method for solving the network analysis problem. Linear theory transforms the NLOOP nonlinear loop equations into linear equations by approximating the head loss in each link by

$$\Delta\, HF_k = \frac{K_k\, L_k\, (Q_k^0)^{n-1}}{D_k^m}\, Q_k \qquad (1\text{-}28)$$

where Q_k^0 is an initial estimate of the flow rate in each link and Q_k is unknown. The NLOOP linearized equations are then combined with the NNODE - 1 nodal equations to form a linear system of NLINK equations in as many unknowns. The solution of the system of linear equations provides flow estimates for the next iteration. In practice, initial flows are automatically set to 1 flow unit. The authors claim convergence in a relatively small number of iterations.

In a similar manner, Collins and Johnson (1975) [19] applied the finite element method to the network balancing problem. Using one dimensional finite element analysis, a system of linear equations was derived. Iterative solution of the resulting system balances the network.

Kesavan and Chandrashekar (1972) [20] developed a graph-theoretic model for network analysis. Unlike previous approaches which automatically satisfy either conservation of flow (1-8) or conservation of energy equations (1-12), the graph-theoretic model directly utilizes both sets of constraints. The main advantage of this approach is that the formulation procedure is independent of the numerical technique used to solve the resulting set of nonlinear equations.

Collins, Cooper, and Kennington (1976) [21] show that the pipe network analysis problem is mathematically equivalent to a nonlinear optimization model. The nonlinear functions are replaced with piece-wise linear functions. The resulting model is a linear network flow problem for which excellent solution techniques exist. This method makes solution of quite large network analysis problems possible.

1.3 Distribution System Layout Models

The first major task in water distribution system design involves determining the layout of the major links in the network. Although restricted somewhat by the requirement to use public rights-of-way and private easements, there remains considerable flexibility in selecting the links to connect the source nodes to major nodal concentrations of demand [22]. In contrast to recent work in sewer system design and layout (see Mays et al. (1976) [23]) existing methods ([24], [25], [26], [27]) of selecting the network configuration generally make no real attempt to explicitly generate and evaluate alternative network configurations in terms of their ultimate impact on total system cost and on reliability of water service. Existing methods provide little guidance to the design engineer in selecting links other than on the proper use of contour maps, the benefits of looped vs tree-shaped systems, and the importance of proper location of elevated storage reservoirs. Although the cost of pipes account for well over half of the total distribution system cost [28], the water distribution system engineer must rely on an assortment of rules of thumb in selecting the network layout that must serve as the foundation for his detailed design effort.

1.4 Optimization Models for Distribution System Design

A number of water distribution design optimization models have been developed to assist the water engineer. Given a specific set of links in the network layout, the optimization models determine pipe diameters, pump capacities, heights of elevated reservoirs, valve locations and other design parameters subject to satisfying steady state flow conditions and various bounds placed on pipe diameters, flow rates, and nodal heads. The objective function of these models focuses exclusively on monetary cost including acquisition, operation, and maintenance costs. Important capabilities of the models include the type of system analyzed (branched and/or looped), the number of sources allowed (single or multiple), the number of loading (demand) design conditions handled. Solution techniques range from linear programming to sophisticated nonlinear optimization techniques.

The first significant optimization model was developed by Shamir [11] in 1964. The decision variables were pipe diameters. The objective function considered a single loading (demand) condition and was related to the energy loss in flow through all the pipes. The steady state hydraulic solution was obtained by the Newton-Rhapson method with the Jacobian of the solution used to compute the components of the gradient.

Pitchai [29] in 1966 used a random sampling technique to search for the optimal diameters of a pipe network operating under a number of loadings. The objective function contained the initial and operating costs. Constraints on heads were taken into consideration by adding penalties on constraint violation to the objective function to be minimized.

Jacoby [30] in 1968 used a numerical gradient technique to treat the same problem. Diameters were handled as continuous variables and the values obtained in the unconstrained optimization were rounded to the nearest commercially available size. This rounding could cause the selected design to be infeasible. The objective function to be minimized was the combined cost of pumps and pipelines, and penalties for violation of loop and nodal equations.

Karmeli et al. [31] in 1968 handled the design of branching networks. Unlike the looped network, the steady state flow conditions can be computed directly once supply and demand at each node are given. Since the frictional head loss on a pipe and its cost are linear functions of its length, by selecting the pipe lengths as the decision variables, Karmeli et al. formulated a linear programming model. Like previous researchers, the model only considered the initial cost in the objective function.

Lai [32] in 1970 developed a dynamic programming model to handle water distribution system capacity expansion. However, his analysis was limited to tree shaped networks only.

Deb and Sarkar [33] present a method based on the equivalent pipe diameter concept which allows a pipe with a single diameter to replace a set of series or parallel pipes. The diameter of the new pipe can be chosen to provide the equivalent frictional head loss as the set of pipes it replaces. The authors handled only a single source network requiring nodal heads to be specified in advance. Costs of pipe, pumping, and the storage reservoir are included.

Kolhaas and Mattern [34] in 1971 used separable programming to determine not only the optimal diameters but also the pumps and reservoirs for a looped system with all heads known. With heads given the constraints become linear if flows are decision variables. Diameters can be computed directly from the Hazen-Williams equation with heads and flows fixed. The nonlinear objective function contained the cost of pipes, pumps, and reservoirs.

Kally [35] in 1972 extended the method of using pipe lengths as the decision variable to looped networks. To find the network flow solution involved iteratively changing the decision variables, approximating the resulting change in head pressures, and solving

the new linear program until convergence is achieved. The objective function only considered the initial cost of the pipe.

Cembrowicz and Harrington [36] in 1973 minimized the initial pipe cost of a network subject to a single loading. Using graph theory, the problem was decomposed so that the nonconvex total objective function is separated into subsets of convex functions. Each function, which relates to either a pipe or a loop, is minimized separately using the method of feasible directions [37]. Continuous pipe diameters are assumed.

Swamee, Kumar and Khanna [38] in 1973 handle the problem of minimizing the cost of a single source tree distribution system. Using dynamic programming, the authors developed a closed form solution with an objective function covering pipe, pump, and elevated reservoir capital and maintenance cost plus pumping energy costs.

Lam [39] in 1973 developed a discrete gradient optimization technique for a water distribution system consisting only of a single source, pipes, and demands. Pipe diameters were treated as discrete variables. This technique avoids the rounding of a continuous diameter variable to the nearest commercially available size.

Watanatada [40] in 1973 developed an optimization technique for multiple source networks and applied it to real networks of

moderate size. The constrained nonlinear optimization problem was converted to an unconstrained optimization problem by incorporating the constraints into the objective function with appropriate penalty terms. Minimization of the resulting function was performed using the variable metric [41] and conjugate gradient [42] methods.

Shamir [43] in 1974 extended his earlier work by developing a methodology for handling both the optimal design and operation of a water distribution system under one or several loading conditions. Optimization was obtained by a combination of the generalized reduced gradient (GRG) and penalty methods. The objective function included initial cost of the design and cost of operation. The author claims that physical measures of performance and penalties for violating constraints may be incorporated into the objective function but offers little guidance on properly defining these measures of performance.

Delfino [44] in 1975 formulated a nonlinear programming model to minimize the cost of pipe and pumping for a looped network using continuous pipe diameters. He used the generalized reduced gradient (GRG) method to solve the problem.

Deb [45] in 1976 considered a distribution network with the decision variable as the size of pipes, pressure surface over the

network, height and location of the elevated service reservoir, and capacity of the pumping station. A gradient-like technique is used to perform the optimization. The objective function encompassed the initial cost of pipes, pumps, and elevated storage reservoir; operation costs; and maintenance costs.

Alperovits and Shamir [46] in 1977 employed a method called the linear programming gradient (LPG) method in optimizing a distribution system including pipes, pumps, valves, and reservoirs. Decision variables have been expanded to include reservoir elevations and operational parameters such as the pumps to be operated under each of the loading conditions. The objective function included overall capital costs.

Cenedese and Mele [47] in 1978 minimize the capital cost of pipe for looped networks by incorporating the constraints into the objective function with a change of variable and by the addition of a penalty term. The decision variables for the modified objective function are the loop flows. Loop flows and nodal heads are alternately changed using a direct search technique until a local minimum is reached.

Deb [48] in 1978 developed a simple mathematical model for a single source pumping system. Including the cost of pumps, pipes, operation and maintenance, and energy, he formulated an equation

for the total system cost as a function of pipe diameter (all pipes are assumed to have the same diameter). Differentiating the objective function with respect to pipe diameter and setting the expression to zero, a closed form solution for the single optimal diameter is derived for this special case.

Bhave [49] in 1978 developed a manual iterative approach for minimizing the cost of a single source distribution system. The heads at the demand nodes are treated as independent variables and iteratively changed until convergence to an optimal solution occurs. Diameters are continuous rather than discrete variables.

1.5 Reliability/Performance Models

The previous section reflects the great amount of research devoted to minimum cost design of water distribution systems. The emphasis has been placed on designing the system to function under normal loading conditions, e.g., peak hour demand, maximum daily demand, etc. This section reviews the work done on abnormal or emergency loading conditions such as fire demand, pump failure, and broken link loading conditions.

In 1970 de Neufville et al. [50] described their systems analysis on the design of proposed additions to the primary supply network of New York City. The authors examined four primary

measures of water distribution system design: (1) overall performance; (2) fail-safe reliability; (3) distribution of performance; and (4) cost.

These measures were used to evaluate the desirability of manually generated major design alternatives. The authors recognized the shortcomings of available optimization methods and their simplistic cost oriented objective functions, stating that "available optimization methods do not reflect the several criteria whereby distribution networks are usually evaluated." They further concluded that "mathematical techniques do not now consider all the relevant factors of quality, reliability, and distribution of the benefits." Most significant was their effort to quantitatively evaluate water distribution system performance (nodal head values) under realistic emergency loading conditions and to examine the cost/benefit trade-offs associated with designing this performance into the system.

Damelin, Shamir, and Arad [51] in 1972 developed a simulation model to evaluate the reliability of supplying a known demand pattern in a given water supply system in which shortfalls are caused by random pump failures. An economic model is developed that allows the user to evaluate the benefits (additional water obtained) vs the cost of making specific improvements in the reliability of

the system. The researchers strongly emphasize the difficulty of evaluating water distribution system reliability as follows:

> Reliability has an economic value. Perfect reliability is not necessarily the best economic solution as already has been mentioned. To be able to compute the penalty due to imperfect reliability, one has to assign an economic loss function to shortfalls according to their magnitude and the time at which they occur. We consider this assignment of economic loss function to be impossible, at least for the moment, since the actual value of water as a resource used by some production system, say agriculture, has not been defined to everyone's satisfaction.

Rao et al. [52] developed a simulation model to evaluate the performance of an existing water distribution system under a variety of loading conditions including both normal and emergency conditions. The behavior of the system was examined over a 24-48 hour period. Emphasis was placed on the detailed operation and control of the system including the level of the storage reservoirs.

Several researchers have discussed the need for research into developing explicit measures of water distribution reliability and performance under emergency loading conditions. Kolhaas and Mattern [34] claim to handle the requirement for reliability of supply to each demand node in a looped network by simply imposing non-zero lower bounds on minimum pipe diameters. Watanatada [40] discusses the need to explicitly incorporate measures of reliability into an optimization model to predict the way the system will

perform under emergency loading conditions. He identifies the need for future research into a model in which various failure conditions are contained explicitly. Shamir [43] proposes the maximization of weighted nodal heads as a potential measure of system reliability. Delfino [44] formulates a combined minimum cost layout and detailed design problem for a network requiring two alternate paths from the source to each demand node. However, the author only examines possible solution approaches and leaves the problem as a subject for future research. Shamir and Alperovits [46] conclude that there is a need for additional distribution system performance criteria (other than cost) in the objective function and that a more basic definition of reliability of the network should be developed instead of setting arbitrary constraints on minimal pipe diameters.

1.6 Summary

A review of the literature indicates that considerable research has been done and numerous models have been developed and solved in the areas of steady state network analysis and minimum cost optimization for a given network layout. However, there is almost a complete absence of engineering design tools for the critical network layout problem. Likewise, very little work has been

performed on developing basic measures of reliability/performance for water distribution systems under expected emergency loading conditions such as fire demand, link failure, and pump/power outage.

CHAPTER 2

STATEMENT OF THE PROBLEM/SOLUTION APPROACH

2.1 Introduction

A review of the literature revealed two specific areas in the design of water distribution systems that merited further research effort:

1. Optimal network layout.
2. Reliability/performance of the distribution system under emergency loading conditions.

Moreover, there appears to be a need to develop a comprehensive, unified methodology for the total water distribution design process. Such a methodology would be applicable not only to the design of a new system but also provide a framework for the capacity expansion of an existing system.

This chapter presents a verbal statement of the problem, examines the potential solution approaches that were considered during the process of the research, and outlines the three-level hierarchical approach that resulted. Emphasis will be placed on analyzing important conceptual aspects of the problem and its

solution rather than detailed discussion about specific mathematical models and solution algorithms. Our purpose here is to lay a solid conceptual foundation for the detailed description of the solution technique presented in Chapters 3, 4, and 5.

2.2 Verbal Statement of the Problem

The following is a verbal statement of the problem presented in the format of a mathematical programming problem:

GIVEN:

1. Set of source nodes and associated flow capacities.
2. Set of demand nodes.
3. Set of potential links and any unusual (high excavation/ right of way) extra costs for pipe installation.
4. Set of normal loading (demand) conditions.
5. Set of emergency loading conditions.
6. Set of potential pump locations, maximum capacities, and costs.
7. Set of elevated storage reservoirs, maximum elevations, and costs to elevate.
8. Set of commercially available pipe diameters and costs.

9. Minimum performance levels for normal loading conditions.
10. Maximum annual capital and operating budget.

FIND:

1. Layout of network links.
2. Link diameters.
3. Pump capacities.
4. Additional height for elevated storage reservoirs.

IN ORDER TO:

Maximize the distribution system performance under emergency loading conditions.

SUBJECT TO:

1. Satisfying steady state flow conditions.
2. Satisfying minimum performance levels under normal loading conditions.
3. Not exceeding the maximum annual budget.
4. Not exceeding maximum storage heights.
5. Not exceeding maximum pump capacities.

The statement of the problem is intended to reflect the general situation encountered by the water distribution system design engineer during the reconnaissance stage of the design process for a new system, i.e., selection of major system components. The general nature of the problem statement allows it to subsume important special cases such as capacity expansion of or extensive modification to an existing system. Further, it is important to note that this problem involves design of both the network layout and major system components rather than assuming a given layout. Also, by incorporating reliability directly into the objective function, the problem statement explicitly addresses the evaluation of water distribution system performance under emergency loading conditions.

2.3 Water Distribution System Reliability

As revealed by the literature survey, there is no accepted definition or measure of reliability for water distribution systems although researchers often use the term. In the literature of systems analysis reliability is usually defined as the probability that a system performs its mission within specified limits for a given period of time in a specified environment [53]. To analytically compute the mathematical reliability for a large system with many

interactive subsystems requires knowledge of the precise reliabilities of the basic subsystems and the impact on mission accomplishment due to the set of all possible subsystem failures. Except perhaps for the pumping subsystem there is little data available on the mathematical reliability of water distribution subsystems [54]. Thus, in analyzing water distribution systems conventional mathematical reliability measures appear inappropriate.

The mission of a water distribution system is to deliver water to its users in an economical yet reliable manner. Under normal loading conditions (usually defined in terms of peak hourly or maximum daily demands) the emphasis must naturally be on economy. However, under emergency loading conditions, i.e., critical pump failures, high fire demands, and broken links, quantity and quality of service may degrade catastrophically unless the system design adequately considers these conditions. Thus, consistent with de Neufville etal. [50] reliability for a water distribution system will be defined in terms of the system's performance under emergency loading conditions. The specific measure of performance and hence reliability will depend on the specific nature of the emergency loading condition. In general, the quantity of service (flow rate) and/or quality of service (nodal head pressure) will serve as measures of performance.

2.4 Potential Solution Approaches

2.4.1 Single Integrated Mathematical Programming Model

Attempts to formulate a single integrated mathematical programming model to solve the problem revealed the following:

1. The requirement to select the network layout requires integer (0,1) variables.
2. The nonlinear frictional head loss terms result in a nonlinear constraint set.
3. To measure the nodal head pressures and incorporate them as a constraint requires knowledge of a set of links forming a path from a fixed head node to each node of interest. Likewise, for multiple source networks conservation of energy requirements dictate knowledge of a set of links forming a path between each pair of fixed head nodes. If the loop conservation of energy constraints (1-12) are used to enforce steady state conditions, the appropriate set of loop constraints must also be identified. Thus, the formulation of the appropriate steady state and other layout dependent constraints may involve enumerating all possible constraints associated with each potential network layout.

4. Depending on the specific constraint formulation, it may be necessary to introduce additional 0 - 1 variables to insure the network satisfies connectivity requirements.

5. Introducing broken link emergency loading conditions into such a model would be virtually impossible since the network layout is itself a decision variable.

Thus, based on the above observations not only solving but even formulating the problem as a single integrated mathematical programming model is extremely difficult and cumbersome, if not actually impossible. Further, such a model would be almost certain to defy solution even if it were formulated.

2.4.2 Two-Level Hierarchical Integrative Approach

Recognizing the difficulty of solving the problem with a single, large, detailed, integrated model, the problem was initially decomposed into a two-level ([55], Bradley et al.) or two layer (Haimes [56]) hierarchically integrated system. This approach recognizes the need for decomposing the elements of complex problems within the context of a hierarchical system that links higher level (strategic) decisions into lower level (tactical/operational) decisions. The complete decision-making (design) process is partitioned to select adequate models to deal with individual decisions at each

hierarchical level. Linking mechanisms are developed for the transferring of the higher level results to the lower hierarchical levels.

The initial decomposition of the problem elements partitioned the design process into two levels:

1. Strategic - Selection of a set of links forming a spanning tree in the network.
2. Tactical/Operational - Selection o, :he loop forming links and the detailed system design.

Thus, the network layout was split among the two models. Two heuristic models, to be discussed in Chapter 3, were developed to handle the selection of the "primary" links in the "core" tree. The presence of a spanning tree in the network eliminated many of the formulation difficulties of the single integrated model but there still remained the task of developing a solution algorithm for the resulting nonlinear integer programming model (selection of redundant links).

Considerable effort was invested in developing an algorithm to solve this nonlinear integer programming model. A complex heuristic algorithm based on comparing the benefit/cost ratio [57] of adding (deleting) each candidate loop-forming "redundant" link to (from)

the core tree was developed. Although the mechanics of the algorithm worked well, unexpected results on a small, two-looped network for a single normal and emergency (fire demand) condition led to further decomposition of the model. For the fire demand loading condition the benefit/cost ratio of adding a redundant link to the core tree was negative. This result led to the recognition that the real value of redundant links was their ability to provide continuing service in case of failure of the larger primary links. Thus, selection of the redundant links (which is based on satisfying the broken primary link emergency loading conditions) became the task of a separate intermediate level model. The third level of the hierarchy accomplishes the detailed system design using the network layout from the first and second level models and takes into account the remaining emergency loading conditions (fire demand, pump outage).

2.4.3 Three-Level Hierarchical Integrative Approach

The approach chosen to handle the problem involves a hierarchy of three models:

1. Strategic - Selection of the core tree of primary links.
2. Tactical - Selection of the loop forming redundant links.
3. Operational - Detailed design of the system.

For each level it was necessary to develop an appropriate model properly integrating the results of the higher level model(s). The first two models combine to design the system layout while the lowest level model optimizes the detail design of the resulting layout with respect to performance/reliability under the selected non-broken links emergency loading conditions. The resulting decomposition eliminated the requirement to solve a nonlinear integer program but more importantly it represents a logical, comprehensive approach to solution of the problem. The specific description of and rationale for selecting each of the three models is presented in Chapters 3, 4 and 5.

CHAPTER 3

SELECTION OF TREE LAYOUT

3.1 Introduction

Let us consider the problem of connecting a set of demand nodes to a single source node with a set of potential links. The minimum number of links required to satisfy all nodal demands is NNODE - 1 where NNODE is the total number of nodes. This set of NNODE - 1 links forms a spanning tree for the network. For rural water distribution systems where demand nodes are far apart it is not unusual to install a tree shaped distribution system because of the high cost to provide multiple paths to each demand node. Municipal water distribution systems, on the other hand, usually are looped providing at least two paths to each demand node. In this chapter we will consider the problem of selecting the optimal tree layout for the distribution system. After fully characterizing the nature of the optimal tree, we will examine existing techniques for identifying this optimal tree and complete the analytical development of a recently proposed technique [49]. Then, we will present a new technique that remedies the difficulties of existing

techniques. Finally, efficient methods for generating alternative near optimal tree layouts will be discussed.

3.2 Properties of the Core Tree

3.2.1 Definition

The minimum cost spanning tree under the normal loading condition will be termed the core tree and the links in the core tree, the primary links. The links not in the core tree will be referred to as the non-tree links or candidate redundant links. Non-tree links which are eventually selected as part of the full network layout (see Chapter 4) will be called redundant links.

3.2.2 Economy

3.2.2.1 Problem P1

Consider the following problem of minimizing the total costs of designing a looped distribution system subject to satisfying steady state conditions and minimum head levels under the normal loading condition:

PROBLEM P1

$$\text{Minimize} \quad Z = \sum_{k=1}^{NLINK} \ell_1 D_k^{\ell_2} L_k + \sum_{k=1}^{NPUMP} PU\,[XP_k, QP_k] \tag{3-1}$$

$$+ \sum_{k=1}^{NST} STC_k \, XS_k$$

subject to

$$\sum_{k \in O_i} Q_k - \sum_{k \in T_i} Q_k = b_i \tag{3-2}$$

$$i \in DNODE \cup SNODE$$

$$\left(\bar{H}_{k_1} - \bar{H}_{k_2}\right) D_k^m = K_k Q_k |Q_k|^{n-1} L_k \tag{3-3}$$

$$k = 1, \ldots, NLINK$$

$$\bar{H}_i = EL_i + \sum_{k \in PS_i} (XP_k + XS_k) \tag{3-4}$$

$$i \in SNODE$$

$$\bar{H}_i \geq EL_i + HMIN_i \tag{3-5}$$

$$i \in DNODE$$

$$H_i = \bar{H}_i - EL_i \qquad (3\text{-}6)$$

$$i \in \text{DNODE}$$

$$D_k \geq 0 \qquad k = 1, \ldots, \text{NLINK} \qquad (3\text{-}7)$$

$$XP_k \geq 0 \qquad k = 1, \ldots, \text{NPUMP} \qquad (3\text{-}8)$$

$$XS_k \geq 0 \qquad k = 1, \ldots, \text{NST} \qquad (3\text{-}9)$$

where

NLINK--the number of links (primary and non-tree) in the network

ℓ_1, ℓ_2--constant dimensionless link cost parameters

D_k--the diameter of link k in inches

L_k--the length of link k in feet

NPUMP--the number of pumps in the system

XP_k--the head lift provided by pump k in feet

QP_k--the flow rate through pump k in gallons per minute

PU $[XP_k, QP_k]$--the equivalent uniform annual cost in dollars for pump k. The capital cost component of PU is a nonlinear function of head and flow rate.

NST--the number of elevated storage reservoirs in the system

XS_k--the additional height to raise storage reservoir k in feet

STC_k--the equivalent uniform annual cost in dollars per foot for raising storage reservoir k

Q_k--the flow rate on link k in gallons per minute

b_i--the external flow at node i in gallons per minute

K_k--a constant dependent on link k's roughness coefficient

H_i--the pressure head at node i in feet

$\overline{H}_i$--the total head at node i in feet which is the sum of potential head due to elevation (EL_i) and the pressure head (H_i)

EL_i--the elevation above a specified datum plane, e.g., sea level, in feet

k_1, k_2--the two nodes incident to link k

PS_i--the set of pumps and storage reservoirs at source node i

DNODE--the set of demand nodes

SNODE--the set of source nodes

$HMIN_i$--the minimum pressure head at demand node i in feet

The objective function (3-1) composed of link, pump, and storage costs is the total equivalent uniform annual cost of the

distribution system in dollars. The linear system of equations (3-2) insures nodal conservation of flow equation (1-8) is satisfied. Equation (3-3) is the frictional head loss equation for each link.

In this model the total nodal heads ($\overline{H}_i$) are explicitly chosen. Thus, as in the Hardy Cross nodal method (see section 1.2.1) an arbitrary selection of $\overline{H}_i$ automatically satisfies loop conservation of energy requirements (equation 1-11) but may not satisfy nodal conservation of flow. The direction of head loss in equation (3-3) determines the flow direction and sign of Q_k. Equation (3-4) states that the total head at each source node is the sum of the nodal elevation plus the head added by pumps and storage reservoirs located at the node. Inequality (3-5) and equation (3-6) combine to insure that the pressure head (H_i) at each demand node exceeds the minimum required pressure head ($HMIN_i$). Inequalities (3-7), (3-8) and (3-9) are the nonnegative diameter, pump head lift, and storage height decision variables, respectively.

3.2.2.2 Theorem I

The following theorem (Delfino [44]) demonstrates the desirability of identifying and using the core tree as a base for the network layout problem.

THEOREM I

Assuming that Problem PI has a finite optimal solution, there is an optimal solution corresponding to a spanning tree of the looped network.

PROOF: Assume we have a finite optimal solution for Problem PI with optimal values of the decision variables $\bar{H}_i^*$, $i \varepsilon$ DNODE U SNODE; XP_k^*, k = 1, ..., NPUMP; XS_k^*, k = 1, ..., NST; D_k^*, k = 1, ..., NLINK; and Q_k^*, k = 1, ..., NLINK. Therefore the following inequality holds

$$Z\ (D_k^*\ ,\ Q_k^*\ ,\ \bar{H}_i^*,\ XP_k^*\ ,\ XS_k^*) \leqq Z\ (D_k\ ,\ Q_k\ ,\ \bar{H}_i^*,\ XP_k^*\ ,\ XS_k^*) \tag{3-10}$$

for any feasible D_k and Q_k.

Fix $\bar{H}_i$ at $\bar{H}_i^*$, $i \varepsilon$ DNODE U SNODE; XP_k at XP_k^*, k = 1, ..., NPUMP; and XS_k at XS_k^*, k = 1, ..., NST. Thus, using equation (3-3) we can obtain the following expressions:

1. For links k such that $\bar{H}_{k_1}^* - \bar{H}_{k_2}^* \neq 0$ using equation (3-3) we have

$$D_k = \left[\frac{K_k\ |Q_k|^n\ L_k}{|\ \bar{H}_{k_1}^* - \bar{H}_{k_2}^*\ |}\right]^{1/m} \tag{3-11}$$

2. For links k with $\overline{H}^{*}_{k_1} - \overline{H}^{*}_{k_2} = 0$, $D_k = 0$ and $Q_k = 0$.

Let L be the set of links with this property.

Eliminating D_k using equation (3-11) Problem P1 becomes

PROBLEM P2

Minimize $$\sum_{\substack{k=1 \\ k \notin L}}^{NLINK} \overline{K}_k L_k |Q_k|^{\ell_3} \tag{3-12}$$

subject to

$$\sum_{\substack{k \varepsilon O_i \\ k \notin L}} Q_k - \sum_{\substack{k \varepsilon T_i \\ k \notin L}} Q_k = b_i \tag{3-13}$$

$i \varepsilon$ DNODE U SNODE

where

$$\overline{K}_k = \ell_1 \left[\frac{K_k \quad L_k}{|\overline{H}^{*}_{k_1} - \overline{H}^{*}_{k_2}|} \right]^{\ell_2/m} \tag{3-14}$$

$$\ell_3 = \frac{n \ \ell_2}{m} . \tag{3-15}$$

The objective function (3-12) is concave under the condition that

$$\ell_3 = \frac{n\,\ell_2}{m} < 1 \qquad (3.16)$$

For the Hazen-Williams equation $n = 1.852$ and $m = 4.87$. Thus, the expression (3-16) becomes

$$\ell_3 = \frac{1.852\,\ell_2}{4.87} < 1 \qquad (3\text{-}17)$$

or

$$\ell_2 < 2.63 \qquad (3\text{-}18)$$

For 1976 cost data the value of ℓ_1 is 1.01 ℓ_2 is 1.29 [48].

Thus, Problem P2 involves minimizing a concave function over a convex set. Since Problem P1 has a finite optimal solution, Problem P2 also has a finite optimal solution which is given by a spanning forest $\bar{T}$ of the network. If the spanning forest is connected, it is also a spanning tree. Otherwise, $\bar{T}$ plus some links with zero flow, i.e., links with $\bar{H}_{k_1} - \bar{H}_{k_2} = 0$, form a spanning tree T in the network.

Let Q_k^{**} be the link flows associated with the spanning tree T and D_k^{**}, the corresponding diameters computed using

equation (3-11). Thus, we can write

$$Z\ (D_k^{**}, Q_k^{**}, \bar{H}_i^{*}, XP_k^{*}, XS_k^{*}) \leqq Z\ (D_k, Q_k, \bar{H}_i^{*}, XP_k^{*}, XS_k^{*}) \tag{3-19}$$

for any feasible D_k and Q_k.

From (3-10) and (3-19) we must have

$$Z\ (D_k^{**}, Q_k^{**}, \bar{H}_i^{*}, XP_k^{*}, XS_k^{*}) = Z\ (D_k^{*}, Q_k^{*}, \bar{H}_i^{*}, XP_k^{*}, XS_k^{*}) \tag{3-20}$$

Since $(D_k^{*}, Q_k^{*}, \bar{H}_i^{*}, XP_k^{*}, XS_k^{*})$ is an optimal solution the following inequality holds

$$Z\ (D_k^{**}, Q_k^{**}, \bar{H}_i^{*}, XP_k^{*}, XS_k^{*}) \leqq Z\ (K_k, Q_k, \bar{H}_i, XP_k, XS_k) \tag{3-21}$$

for any feasible $(D_k, Q_k, \bar{H}_i, XP_k, XS_k)$.

Hence $(D_k^{**}, Q_k^{**}, \bar{H}_i^{*}, XP_k^{*}, XS_k^{*})$ is also optimal for Problem P1.

Q.E.D.

Consider the two loop, single source distribution system with an elevated storage reservoir at node 1 shown in Figure 3-1. Figure 3-1 also depicts the normal nodal demands, nodal elevations, and link lengths. To illustrate the importance of flow distribution (Q_k) the flow distribution was fixed at a number of points (approximately 1200) and Problem P1 was solved using linear programming [46]. The base flow distribution corresponds to zero flow in both links 7 and 8. Loop flow changes (ΔQ_I and ΔQ_{II}), which preserve nodal conservation of flow, are made to the base flow distribution. The base flow distribution corresponds to $\Delta Q_I = \Delta Q_{II} = 0$. The flow distribution was varied parametrically in 50 GPM increments about $\Delta Q_I = \Delta Q_{II} = 0$. A three-dimensional perspective of the minimum cost (Z) vs. the loop flow changes (ΔQ_I and ΔQ_{II}) is shown in Figure 3-2. The large valleys in the figure correspond to flow distributions with either one or two links at zero flow. This figure also illustrates the low cost of the spanning trees with layouts similar to that of the core tree.

3.3 Identification of Core Tree

Based on the desirable properties of the core tree as a basis for the distribution system layout, it appears worthwhile to have the capability to identify the core tree in an efficient manner.

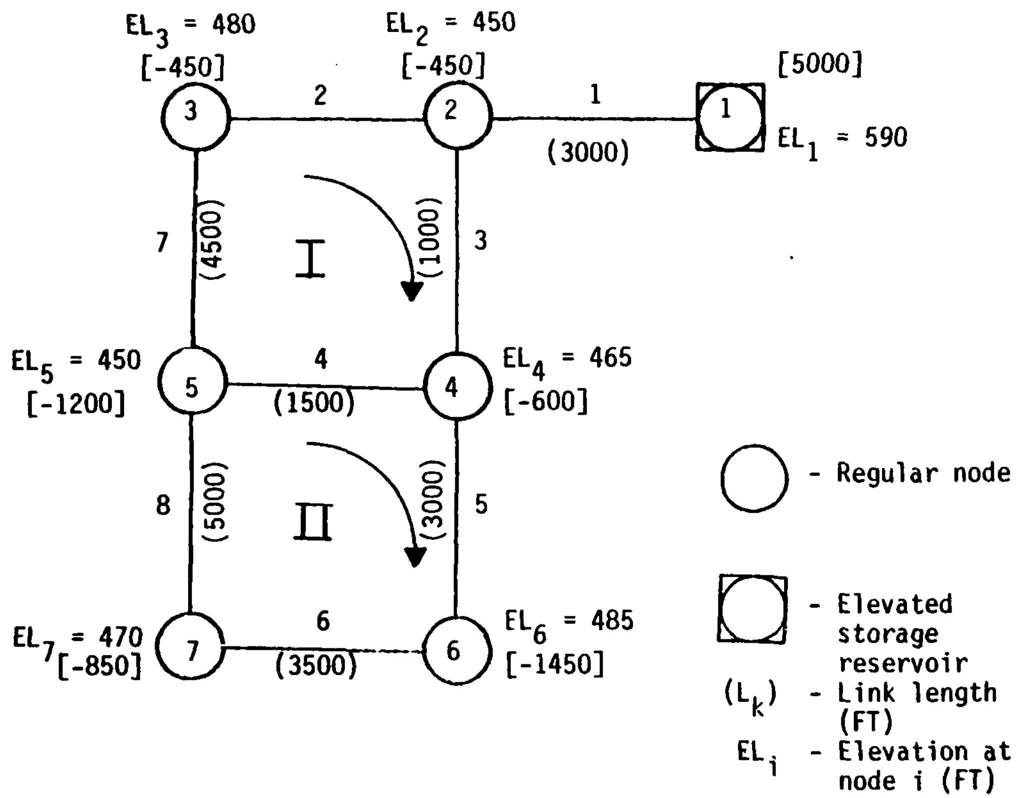

Figure 3-1

TWO LOOP DISTRIBUTION SYSTEM WITH ELEVATED STORAGE RESERVOIR

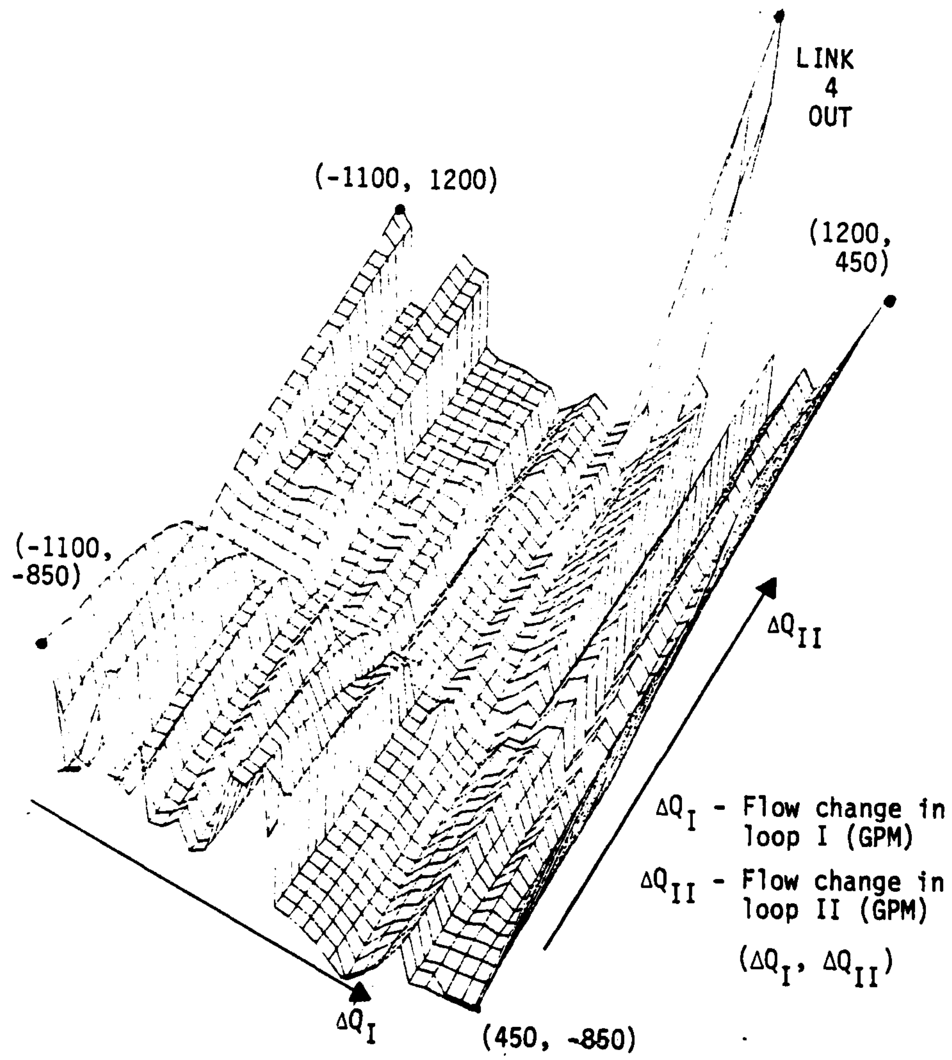

Figure 3-2

MINIMUM COST VS. LOOP FLOW CHANGES

First, we will evaluate three existing techniques for finding the core tree. Next, we will complete the development of a promising technique recently suggested by Bhave [49]. Finally, we will present a new model that overcomes the inadequacies of existing techniques.

3.3.1 Exhaustive Enumeration

One possible way to identify the core tree is to enumerate all spanning trees, optimize each tree with respect to cost, and select the tree with the lowest cost. Graph theory can be used to compute the number of possible spanning trees for an arbitrary set of nodes and potential links.

The fixed nodes of the distribution system and the potential links can be represented by an undirected graph GRAPH = [NODE, LINK] where NODE is the set of all nodes and LINK the set of all potential links in the graph. Let NNODE be the number of nodes in NODE and NLINK be the number of links in LINK. To determine the number of different spanning trees for a specific distribution network requires the Matrix-Tree Theorem for Graphs [58]. Let M′(GRAPH) be an NNODE by NNODE matrix with the diagonal elements of M′, m'_{ii}, equal to the degree of node i. The degree of a node is the number

of links incident to the node. For the off-diagonal elements of M' let $m'_{ij} = -1$ if nodes i and j are adjacent, i.e., connected by a single link and $m'_{j} = 0$ otherwise.

MATRIX TREE THEOREM FOR GRAPHS

For any connected labeled graph GRAPH all cofactors of the matrix M'(GRAPH) are equal and their common value is the number of spanning trees of GRAPH.

Consider the graph GRAPH with four nodes and four links shown in Figure 3-3. The three potential spanning trees are derived by deleting any link except (3, 4) and are also shown in Figure 3-3.

$$M'(\text{GRAPH}) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \end{array}$$

Since all the cofactors are equal, we can take the cofactor of m'_{11}

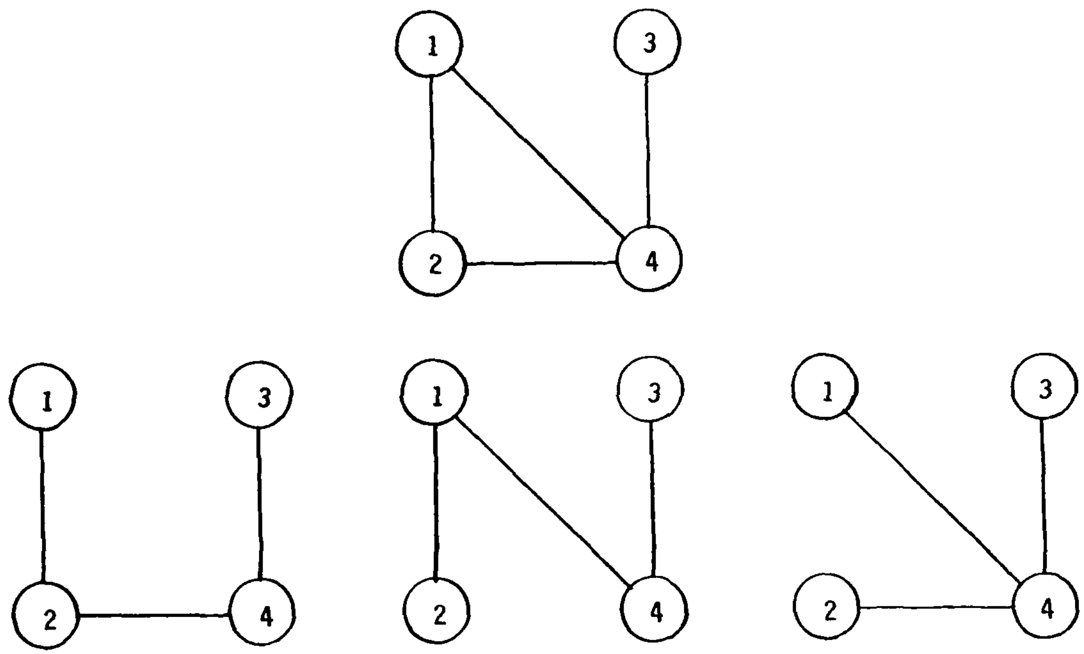

Figure 3-3

ENUMERATION OF SPANNING TREES

$$\begin{vmatrix} 2 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 3 \end{vmatrix} = 2(2) - 0(-1) + (-1)(1) = 3$$

The network of Figure 3-4 [36] with only 10 nodes and 13 links has 208 possible spanning trees. The 20 node, 28 link network of Figure 3-5 [47] has 135,320 possible spanning trees. Thus, for any reasonable size network, exhaustive enumeration and optimization of all spanning trees is infeasible.

3.3.2 Steady State Network Analysis

Barlow and Markland [22] propose using steady state network analysis for finding a "basic" tree in the network which roughly corresponds to our core tree. The procedure involves the following steps:

1. Assign each link in the network the same fixed diameter.
2. Balance the network under the normal loading condition.
3. Select the links in the core tree as those links carrying the larger flows in the network.

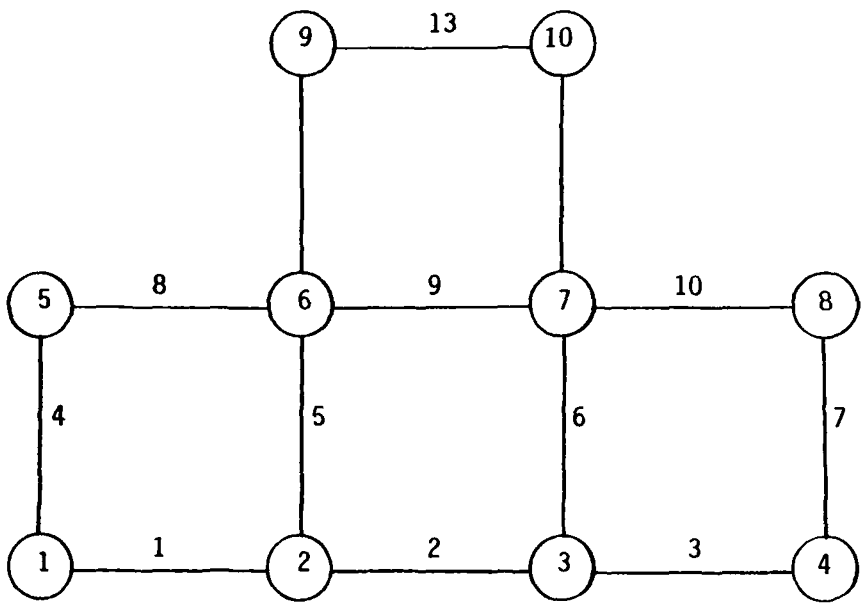

Figure 3-4

NETWORK WITH 10 NODES AND 13 LINKS

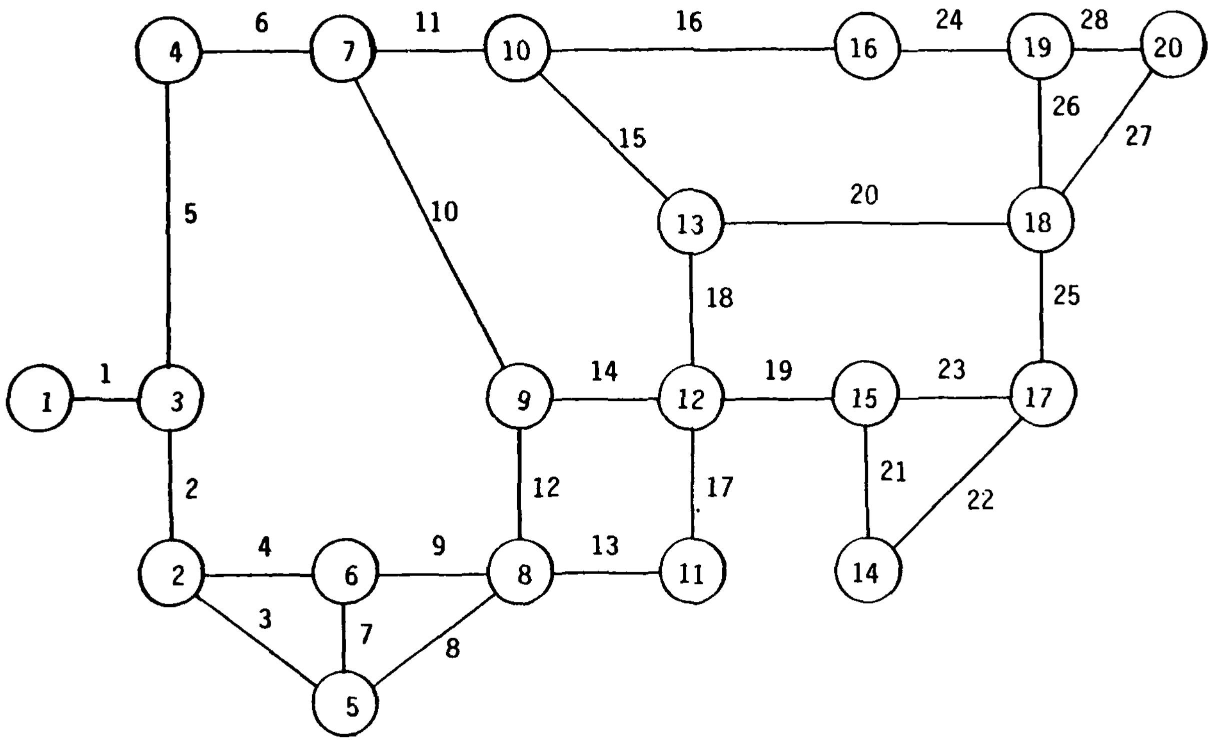

Figure 3-5

NETWORK WITH 20 NODES AND 28 LINKS

This method appears to be based on the observation that water tends to concentrate in the primary links of the system. However, the authors present no justification for this heuristic, provide no examples, and provide no guidance concerning the specific pipe diameter to select or procedure for recognizing flow concentration. Furthermore, this method fails to take into account that the cost of a link varies with its diameter.

3.3.3 Direct Optimization

Alperovits and Shamir [46] state without proof that when a network is designed for a single loading condition that unless a minimum diameter is specified for all links that the minimum cost network will have a branching (tree) configuration. The authors imply that the core tree can be identified using their Linear Programming Gradient (LPG) technique by initially including all potential links in the system and setting very small minimum diameters on all links (1 inch). The minimum cost network is found by solving a sequence of linear programming problems. Between each linear programming iteration, the loop flows are changed using a gradient computed from a combination of the dual variables and the derivatives of the loop equations. Theoretically, the minimum cost solution will have all links not in the core tree at the minimum

diameter and with minimal flow in them. This author's own extensive experience using the LPG method has indicated that the final flow distribution is highly sensitive to the initial flow distribution, i.e., the flow distribution tends to move towards the flow distribution of the nearest tree. This behavior is not surprising because of the nonconvex constraint set that can only guarantee a local optimum solution and because of the general superiority of tree layouts imbedded within a looped network. Furthermore, the computational expense of using several different initial flow distributions in an attempt to find a global optimum and identify the core tree becomes very burdensome even for a moderate size network; Alperovits and Shamir [46] report a cost of $60 for a single LPG run to minimize the cost of a 65-link, 52-node network.

3.3.4 Shortest Path Tree Model

Bhave [49] uses the shortest path tree as part of an algorithm to minimize the cost of a fixed layout single source distribution system. Although the author claims that the shortest path tree is generally the optimal network, he provides no empirical and little analytical support beyond what is necessary to support the use of the shortest path tree in his optimization model. This section analytically derives the shortest path tree model and

3.3.4.1 Analytic Derivation

In a water distribution system external energy is imparted to water by pumps (pressure energy) and elevated storage reservoirs (potential energy). The principal internal energy loss is due to frictional head losses in the pipe. To provide flow to a demand node i at some minimum energy (head) level, $HMIN_i$, involves a tradeoff between the cost of adding external energy and reducing internal energy losses. Assuming a fixed tree layout for a single source network with all links composed of single diameter pipes D_k of length L_k, the head at node i is

$$H_i = EL_s - EL_i + \sum_{k \varepsilon PATH_{si}} XS_k + \sum_{k \varepsilon PATH_{si}} XP_k - \sum_{k \varepsilon PATH_{si}} \frac{K_k Q_k^n L_k}{D_k^m} \qquad (3\text{-}22)$$

Where s is the source node and $PATH_{si}$ is the set of links, pumps, and elevated storage on the path from source s to node i.

Since the precise tradeoff between external energy gains and internal energy losses is part of the final, detailed design model, we will focus on the last term of (3-22) involving internal frictional energy loss. To reduce internal frictional energy loss for a tree layout involves

1. Increasing the link diameters (D_k) on the unique path from the source node to the demand node in the current network layout.
2. Finding an alternate path from the source node to node i that has the lower total head loss.

Since the first alternative involves detailed design, we will consider the second alternative of finding improved paths.

For any link k the quantity

$$J_k = \frac{K_k Q_k^n}{D_k^m} = \frac{\Delta HF_k}{L_k} \tag{3-23}$$

is the hydraulic gradient and represents the head loss per unit length of pipe. Under normal conditions (peak hour demand) with each primary link operating near capacity, J_k should be roughly the same for all links. A rule of thumb for estimating the flow capacity of a link [60] is

$$QMAX_k = 10\ D_k^2 \tag{3-24}$$

where $QMAX_k$ is in gallons per minute and D_k in inches. Letting all links operate most efficiently at their intended capacities we have

$$J_k = K_k\ 10^n\ D_k^{2n-m} = K_k\ 10^n\ D_k^{-.8} \tag{3-25}$$

for typical values of n and m. With D_k ranging from 6 to 20 inches $D_k^{-.8}$ ranges from .23 to .10. A link with an extremely high J_k (high flow rate versus diameter) is dissipating energy at an excessive rate and should be replaced with a larger, more efficient link. Likewise, an extremely low hydraulic gradient implies too low a flow in relation to link diameter and a smaller diameter link or no link at all is in order.

A common engineering design restriction is that the velocity of water in a link V_k remains within fairly narrow limits. Let A_k be the cross-sectional area of link k.

Then $Q_k = A_k\ V_k = \frac{\pi\ D_k^2}{4}\ V_k$ [1] and $J_k = (\frac{\pi}{4})^n\ \frac{K_k}{D_k^{m-2n}}\ V_k^n$. Thus, the assumption that J_k is uniform on all links is consistent with this design restriction on flow velocity. Furthermore, samples of the hydraulic gradient from several optimization runs of different tree

shaped systems lend further empirical support to this assumption.

Letting $J_k = \bar{J}$, equation (3-12) becomes

$$H_i = EL_s - EL_i + \sum_{k \in PATH_{si}} XS_k + \sum_{k \in PATH_{si}} XP_k - \bar{J} \sum_{k \in PATH_{si}} L_k \tag{3-26}$$

For each demand node we would like to minimize the internal frictional energy losses in lieu of costs. This results in the overall problem of minimizing

$$\sum_{i \in DNODE} \sum_{k \in PATH_{si}} L_k$$

where the decision variable is the path from the source node to each demand node $PATH_{si}$. This is the problem of finding the shortest path tree rooted at the source node.

A mathematical model of the problem formulated as a path selection problem is presented below.

PROBLEM P3

$$\text{Minimize} \quad \sum_{i \,\varepsilon\, DNODE} \; \sum_{j=1}^{NP_i} LP_{ij}\, y_{ij} \tag{3-27}$$

$$\sum_{j=1}^{NP_i} y_{ij} = 1 \quad i \,\varepsilon\, DNODE \tag{3-28}$$

$$y_{ij} = 0, 1$$

$$i \,\varepsilon\, DNODE$$

$$j = 1, \ldots, NP_i$$

where

NP_i--the number of different paths from the source node to node i

LP_{ij}--the length of j^{th} path from the source to node i.

$$y_{ij} = \begin{cases} 1 & \text{if path } j = 1, \ldots, NP_i \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

3.3.4.2 Solution Technique

Finding the shortest path tree in a network is simply the classical shortest path problem applied to finding the set of

shortest paths from a fixed root node (source) to all other nodes (demand) in the network. In the literature the shortest path tree is formulated as a minimum cost flow problem where each demand node has a requirement for a single unit of flow and the source node has NNODE - 1 units to supply. Problem P2 has been formulated as a more cumbersome 0-1 integer programming problem purely to illustrate the conceptual problem of selecting the set of NNODE - 1 shortest paths from the source node to the demand nodes. There are a variety of efficient techniques for finding the shortest path tree for a network with nonnegative link costs including dynamic programming, network flow programming, and Dijkstra's algorithm [59].

3.3.4.3 Multiple Source Application

The previous discussion and Bhave's work [49] were restricted to single source networks. To apply the shortest path approach to multiple source networks requires that each demand node be assigned to one of the sources. This assignment should be based on source capacities, nodal demands, and the distances between each source and demand node. The use of the uncapacitated linear minimum cost flow model appears appropriate to make this assignment. A statement of the model is presented below.

PROBLEM P4

$$\text{Minimize} \quad \sum_{k=1}^{NLINK} L_k \, Q_k$$

$$\text{subject to} \quad \sum_{k \varepsilon O_i} Q_k - \sum_{k \varepsilon \bar{T}_i} Q_k = b_i$$

$$i = 1, \ldots, NNODE - 1$$

$$Q_k \geqq 0 \qquad k = 1, \ldots, NLINK$$

Efficient network flow programming codes are available to solve this problem. It should be noted that no capacity constraints have been placed on the link flows. Water pipes are designed to withstand a certain amount of pressure depending on the pressure class of the pipe. It has been assumed that sufficiently large diameters are available to handle maximum flow rates in the distribution system. The maximum pipe diameter may be estimated using the flow capacity equation (3-24).

Solution of the linear minimum cost flow problem (Problem P4) should determine the demand nodes assigned to each source node. However, some demand nodes may be supplied by more than one source. In this case, the node can be arbitrarily assigned to either source.

Once the shortest path trees have been found for each source the trees are connected to form the single spanning core tree. The choice of connecting links is somewhat arbitrary. Good choices include the shortest link connecting the trees or the link that completes the shortest path between the two source nodes. Chapter 6 will illustrate the application of the above techniques to a two-source distribution system.

3.3.4.4 Empirical Support

To test the goodness of the shortest path tree model an extensive search of the literature was conducted for papers optimizing specific looped distribution systems. For each network the shortest path tree was found. By examining the results of the optimization algorithm, the primary links in the core tree were identified by eliminating the links from the network with minimum flow and diameters (redundant links). In every case the shortest path tree and the tree obtained by the optimization algorithm were identical. Summary information on the network problems surveyed is given in Table 3-1.

For the distribution system shown in Figure 3-1 consisting of 7 nodes, 8 potential links, and an elevated storage reservoir at node 1 all 15 spanning trees were enumerated (Figure 3-6) and the

TABLE 3-1

RESULTS OF CORE TREE LITERATURE SURVEY

Reference	No. Nodes	No. Links	No. Loops	No. Spanning Trees
Ceredese [47] and Mele	20	28	9	135,320
Watanadata [40]	4	25	2	8
Kally [35]	9	11	3	52
Jacoby [30]	6	7	2	15
Alperovits & [46] Shamir	7	8	2	18

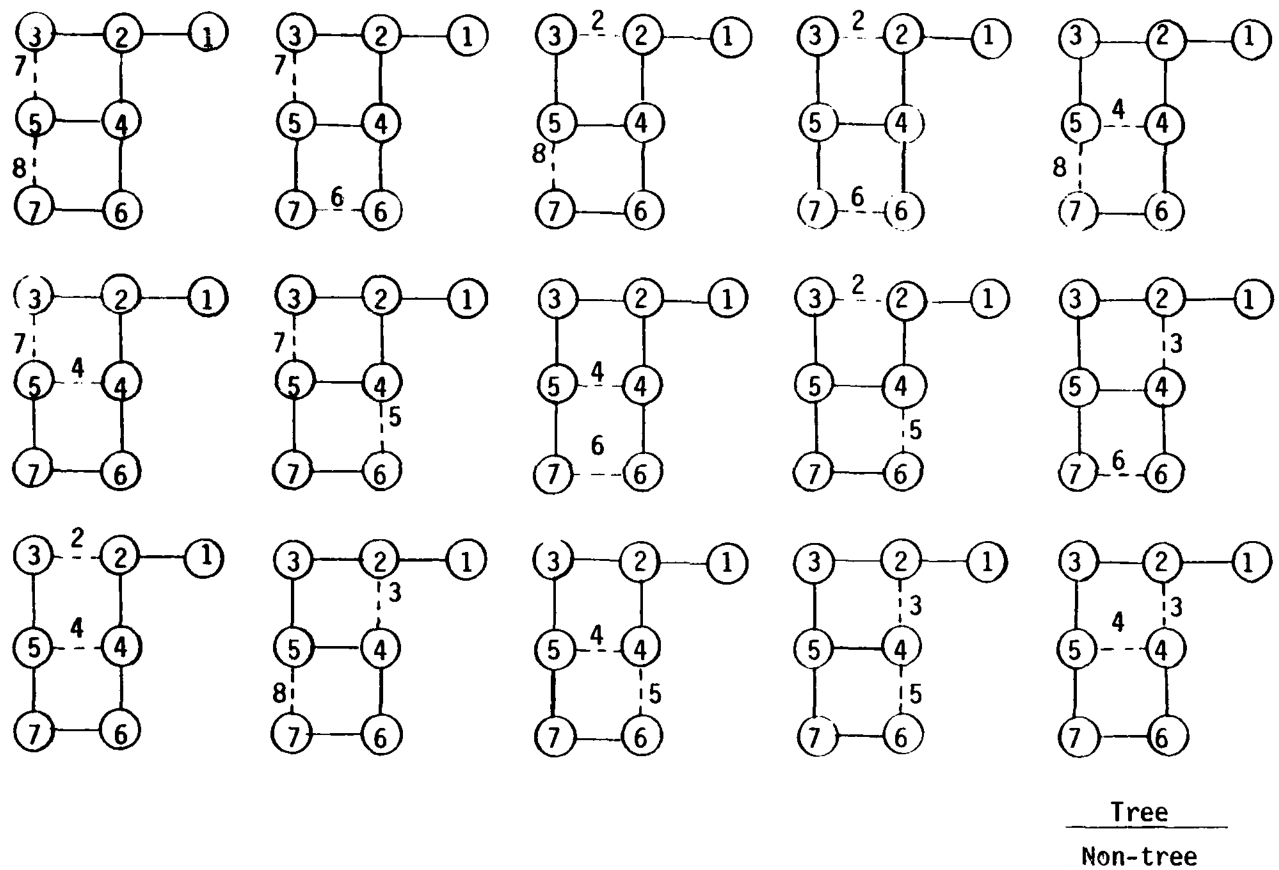

Figure 3-6

SPANNING TREES FOR NETWORK OF FIGURE 3-1

minimum cost design was found for each tree layout. Table 3-2 presents the minimum cost (column 2) and the tree path length (column 3), the total length of the tree paths from the source node to each demand node, for each tree layout. A linear least squares fit of the data yielded a coefficient of determination of .941 confirming the strong correlation between the actual minimum cost and the tree path length criteria. Columns 4 and 5 of Table 3-2 are results for linear and nonlinear flow models which will be discussed in section 3.3.5.

3.3.5 Nonlinear Minimum Cost Flow Model

3.3.5.1 Analytic Derivation

The shortest path tree model focused on the problem of minimizing internal energy losses thereby reducing the need for adding expensive external energy in the form of pumps and/or elevated storage. Without any regard for external energy costs the minimum cost tree layout would clearly be a minimal spanning tree with all links at minimal commercially available diameter. From a total cost viewpoint such a system would represent an extremely inefficient use of pipes since links with larger flows would have a very high hydraulic gradient J_k and would be dissipating excessive amounts of energy per unit length of pipe.

Table 3-2

EVALUATION OF SPANNING TREES

Links Missing	Minimum Cost ($)	Tree Path Length (ft)	Nonlinear flow cost (ft-gpm)	Linear Flow Cost (000 ft-gpm)
7,8	31,428	35,500	627,074	31,900
6,7	33,684	35,500	657,122	31,900
2,8	35,915	40,000	681,892	34,415
2,6	36,991	40,000	710,055	33,925
4,8	37,955	40,000	769,335	37,300
4,7	43,700	45,500	785,320	43,900
5,7	44,588	47,000	791,005	42,050
4,6	44,834	47,000	846,166	47,480
2,5	47,277	47,000	842,386	44,075
3,6	54,939	59,500	996,833	56,600
2,4	55,267	60,000	941,693	50,425
3,8	55,354	62,500	995,601	59,050
4,5	59,706	56,000	1,008,546	59,660
3,5	61,709	63,500	1,130,716	66,650
3,4	66,991	73,500	1,170,119	68,300

As in the derivation of the shortest path tree model assume that all candidate links in the system are operating at the same optimal hydraulic gradient $\bar{J}$. Thus, assuming all candidate links may have nonzero flow Problem P2 can be rewritten as follows:

PROBLEM P5

Minimize $$\sum_{k=1}^{NLINK} \bar{K}_k L_k Q_k^{\ell_3} \qquad (3\text{-}29)$$

subject to

$$\sum_{k \varepsilon O_i} Q_k - \sum_{k \varepsilon T_i} Q_k = b_i \qquad (3\text{-}30)$$

$$i \varepsilon \text{ DNODE U SNODE}$$

The feasible region for Problem P5 is convex since all the constraints are continuous linear functions. The feasible region is closed since it contains all its boundary points [17] and bounded since

$$0 \leq Q_k \leq \sum_{s \varepsilon \text{ SNODE}} b_s \quad .$$

Since the objective function is continuous, by Weierstrass' Theorem it attains a minimum over the constraint set [17].

The objective function is concave since it is the sum of nonnegatively weighted concave functions. By Theorem 3 [17, p. 119] any convex (concave) function f defined on a closed, bounded set Ω which has a maximum (minimum) over Ω achieve this maximum (minimum) at an extreme point of Ω .

The linear constraint set is that of the general uncapacitated minimum cost flow problem. An extreme point of this constraint set corresponds to a spanning tree for the network [55]. In this case the optimal solution will be the core tree.

3.3.5.2 Solution Technique

Since the objective function of Problem P5 has the form

$$\text{Minimize} \quad \sum_{k=1}^{NLINK} f_k(Q_k) \tag{3-31}$$

$$\text{subject to} \quad \sum_{k=1}^{NLINK} g_{ik}(Q_k) = b_i \tag{3-32}$$

$$i \in SNODE \cup DNODE$$

where

$$f_k(Q_k) = \bar{K}_k L_k Q_k^{\ell_3}$$

$$g_{ik}(Q_k) = \pm Q_k$$

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the problem is separable in Q_k.

Instead of solving the problem directly, an approximation is made in order that linear programming can be utilized. Two types of approximations, called the δ-method and the λ-method, are generally used [55]. The objective function is linearized using a piecewise-linear approximation. Since the problem involves minimizing a concave function, restricted basis entry rules must be incorporated in the simplex method to insure that the proper sections of the piecewise-linear approximation are used. Appendix B fully describes the λ-method of approximation used in the research.

3.3.5.3 Empirical Support

Applying separable programming to solve Problem P5 for the distribution system of Figure 3-1 resulted in identifying the minimal cost tree consisting of links 1-6. Letting $\overline{K}_k = 1$, i.e., all links have the same roughness coefficient, the nonlinear objective function value (3-29) was evaluated for the remaining spanning trees and the results are presented in column 4 of Table 3-2. A least squares fit of the data with the computed total minimum cost (column 2) had a coefficient of determination of .972. Column 5 of Table 3-2 shows the results of letting the exponent ℓ_3 of Q_k in the objective function (3-29) equal 1. Problem P5 then becomes a

linear minimum cost flow problem (Problem P3). A linear least squares fit of the data with the total minimum cost (column 2) gave a coefficient of determination of .959.

3.4 Comparison of Alternative Core Tree Models

Examination of exhaustive enumeration, steady state network analysis, and direct optimization methods has revealed serious deficiencies in these three techniques for selecting the core tree. This section will present a comparison of the two most promising techniques for selecting the core tree--the shortest path tree and nonlinear minimum cost flow models.

Both the shortest path tree and the nonlinear minimum cost flow models were analytically derived from the minimum cost distribution system model using the simplifying assumption that the hydraulic gradient J_k is uniform in all links. However, the shortest path tree model focuses on the less direct objective of minimizing total internal frictional energy loss on the path from the source node to each demand node whereas the nonlinear flow model is directly concerned with minimizing total link costs. The shortest path tree model implicitly assumes a uniform flow distribution for all nodes which may affect the results for widely varying nodal demands whereas the nonlinear flow model takes the actual flow distribution

into account. Furthermore, the nonlinear flow model can handle multiple source systems directly without the need to partition the system into disconnected trees. Based on the results of Table 3-2, the nonlinear flow model and its objective function is more discriminating than the shortest path tree model and its objective function. However, the set up and computer solution time for finding the core tree in a network is somewhat less for the shortest path tree model.

As discussed earlier the distribution system cost includes the cost of external energy added by pumps and elevated storage to insure heads at demand nodes exceed minimum levels, i.e.,

$$H_i = EL_s - EL_i + \sum_{k \in PATH_{si}} XS_k + \sum_{k \in PATH_{si}} XP_k - \sum_{k \in PATH_{si}} J_k L_k \geq HMIN_i \tag{3-33}$$

where EL_s is the elevation of the reference source node for demand node i.

The quantity $EL_s - EL_i - HMIN_i$ represents the maximum amount of internal frictional energy (head) loss before external energy is needed for demand node i. This quantity is independent of the

tree path to node j. The quantity

$$HMIN_i + \sum_{k \in PATH_{si}} J_k L_k + EL_i - EL_s$$

represents the amount of external energy required at demand node i if positive or the excess head available at node i if negative. Letting $J_k = \bar{J}$, if we compute the quantity

$$\Delta ENERGY = \underset{i \in DNODE}{\text{Maximum}} \left(\sum_{k \in PATH_{si}} \bar{J} L_k + HMIN_i + EL_i - EL_s \right) \qquad (3\text{-}34)$$

where $PATH_{si}$ is the tree path between source node s and demand node i, we have an estimate of the external energy that must be added to the system.

Both models developed implicitly take into account the requirement to minimize the quantity of external energy added to the system. However, in the process of generating different spanning trees for Table 3-2 certain discrepancies occurred between the order of costs predicted by the models and the order of actual minimum costs:

1. Shortest path tree length for the tree formed by dropping links 4 and 5.

2. Nonlinear flow cost for the tree formed by dropping links 2 and 5.

3. Nonlinear flow cost for the tree formed by dropping links 2 and 4.

For the first two cases the longest tree path is to node 6 which has the highest elevation of any demand node. For the third case the longest tree path is to node 3, the demand node with the second highest elevation. The combination of maximum

$$\sum_{k \varepsilon PATH_{si}} \bar{J} L_k$$

and maximum $HMIN_i + EL_i - EL_s$ ($HMIN_i$ = 90 for all demand nodes) resulted in ΔENERGY for each of the three trees to be considerably higher than trees with similar tree path lengths and nonlinear flow costs. Thus, because of the unusually high requirement for expensive external energy, the models underestimated the relative minimum cost of the tree.

Although these cases may appear somewhat pathological, they represent a limitation on the accuracy of both models over the entire range of possible tree layouts. Thus, it appears worthwhile to estimate ΔENERGY using (3-35) and the resulting minimum nodal head to check for any irregularities that may occur. If the

minimum nodal head is significantly lower than tree layouts with similar estimates, the estimate could be adjusted with the ΔENERGY term to compensate for the additional external energy required.

3.5 Generation of Alternative Low Cost Tree Layouts

The solution of Problems P3 or P5 provides the water distribution system design engineer with a single low cost tree to use as the basis for the network layout. The capability to efficiently identify and rapidly evaluate alternative low cost tree layouts appears especially useful. Perhaps, equally important is the need to avoid inherently expensive network layouts.

The results of Table 3-2 indicate a high linear correlation between the value of the objective function (shortest path tree and nonlinear flow) for each tree and the actual minimum cost of the layout. Given any spanning tree layout, the sum of the lengths of the NNODE-1 paths from the source node to each demand node (the tree path length) can be computed with simple arithmetic. Likewise, given the tree layout and the external flows, the link flows can be computed by solving the nodal conservation of flow equation (1-8) with $Q_k = 0$ for non-tree links. Because of its triangularity, this linear system of equations may be easily solved using backward substitution without the need to compute any basis inverse. With

the link flows Q_k the nonlinear objective function

$$\sum_{k=1}^{NLINK} \bar{K}_k L_k Q_k^{\ell_3}$$

is easily evaluated. Thus, once a candidate tree layout is generated, cost evaluation is almost immediate.

The problem becomes one of generating appropriate candidate tree layouts. Three possible methods for generating alternative spanning trees include:

1. Exhaustive enumeration
2. Expansion about the core tree
3. Expansion about randomly generated spanning trees.

3.5.1 Exhaustive Enumeration

Application of the Matrix Tree Theorem to the network of potential links results in the number of trees to be enumerated. If the number of spanning trees is not excessive, the spanning trees may be generated using existing algorithms [62] and evaluated as described above. Ranking the resulting objective function evaluations in increasing order will give the network designer a complete picture of the relative costs of potential network layouts. This aids the designer in selecting a set of layouts for further

evaluation that have desirable but not easily quantifiable design characteristics and are inherently economical.

3.5.2 Expansion About Core Tree

Cembrowicz and Harrington [36] noted in their studies of numerical examples a strong correlation between costs and similar tree structures. A close examination of the tree layouts (Figure 3-6) and the associated costs in Table 3-2 confirms this observation. Thus, it appears reasonable to consider using the core tree as a seed to generate other low cost tree layouts.

Consider the minimum cost tree for the distribution system of Figure 3-1 shown in Figure 3-7 and the corresponding optimal shortest path tree or linear minimum cost flow solution. There are two non-tree links, 7 and 8, not in the network and each link can have flow in two directions. Thus, ignoring the possibility of existing tree links reversing flow direction, there are 4 nonbasic variables (nontree) (Q_{7A}, Q_{7B}, Q_{8A}, and Q_{8B}) that can enter the basis (network). Since there can be only NNODE - 1 basic variables (tree links) and there are no upper bound flow capacity constraints, entrance of Q_{7A} , Q_{7B}, Q_{8A}, or Q_{8B} must force another basic variable (tree link), Q_2, Q_4, or Q_6 to zero and out of the basis (tree). Let nonbasic (non-tree) variable Q_j enter the basis (tree) forming

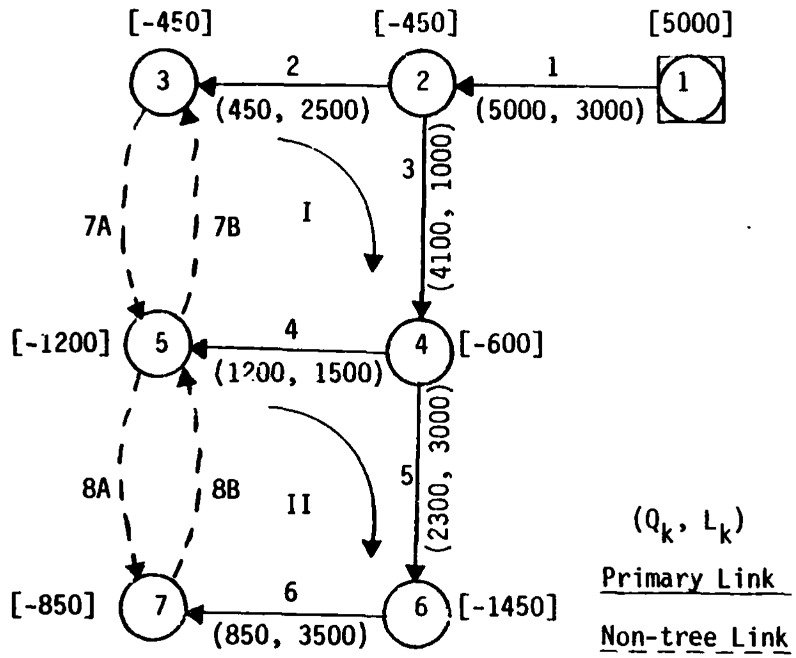


Figure 3-7

CORE TREE LAYOUT WITH NON-TREE LINKS

loop i. The increase in the objective function value, Δz, can be computed exactly as

$$\Delta z = \overline{C}_j \, \Delta \, Q_i \qquad (3\text{-}35)$$

where $\overline{C}_j$ is the reduced cost of nonbasic (non-tree) variable j and ΔQ_i is the change in loop i's flow resulting from link j entering the tree. ΔQ_i is equal to both the flow in the primary link that is leaving the tree (basis) and the external demand at the node being serviced by the entering link. For the shortest path tree problem external demands are all equal to one unit of flow. The value of $\overline{C}_j$ can be computed directly from the lengths of the links in the unique loop formed by link j entering the network and the direction of flow on the link. Assuming there are NLINK total links, the estimated cost of 2 (NLINK - NNODE + 1) tree layouts, i.e., two per unique loop, differing from the core tree by a single link can be exactly evaluated with little computational effort. For the nonlinear cost objective function the cost estimates can be performed using the reduced costs in the approximation linear program but clearly the results are not exact.

In a similar manner, the more promising of the 2 (NLINK - NNODE + 1) can be used to generate more alternative layouts.

However, care should be taken to avoid regenerating trees previously examined and creating a cycle.

3.5.3 Expansion About Random Tree

Instead of expanding only about the core tree (an inherently low cost tree) other spanning trees can be considered. Either systematically or randomly a set of spanning trees can be generated and the expansion process described above can be performed with each tree in the initial set acting as a seed for generating other potential trees. This tree generation and evaluation process can terminate when the designer feels he has considered the major types of tree structures in the potential layout.

CHAPTER 4

SELECTION OF REDUNDANT LINKS

4.1 Introduction

Given the layout of the core tree from the top level model, the next level in our hierarchical system of models is concerned with selecting the loop-forming redundant links to complete the network layout. This chapter examines the role of the redundant links in the operation of a water distribution system, discusses the major factors in redundant link selection and presents two alternative models developed to assist the water distribution system designer in selecting the redundant links. To simplify the presentation the first part of the chapter assumes a single source distribution system. Section 4.4.4 discusses extension of the models developed to multiple source systems.

4.2 Role of Redundant Links

Considering only the capital and operating costs of a water distribution system, the results of Theorem I appear to imply that redundant links serve little use except to add cost to the system.

However, such is not the case. The loops formed by the addition of redundant links serve the following functions:

1. Reduce water stagnation by providing for improved circulation of water in the network.
2. Retard accumulation of sediment in the pipes.
3. Facilitate cleaning of pipe sediment thereby increasing the smoothness of the pipe and reducing frictional energy losses.
4. Provide an alternate path from the source node to the demand nodes in case of primary link failure.

While not attempting to minimize the maintenance-related benefits of loops, the principal function of redundant links is to maintain continuity of service to demand nodes cut off from the source by failure of a primary link. Failure of water mains are usually attributed to one or more factors, which occur either by themselves or, more often, in combination. Some of these factors are improper installation, external corrosion, internal corrosion, soil movement, temperature changes, manufacturing defects, water hammer, and miscellaneous impacts [63]. Water hammer is extremely high pressure caused by the sudden closing of a valve or the shutdown of a pump. Impacts are usually the result of excavation.

In a fully looped water distribution system (usually found in municipalities) upon detection of a broken link, the shutoff

valves adjacent to the break are closed. This isolates the broken section and prevents any further loss of water and property damage. Depending on the particular system and the type of area (residential, mercantile, or industrial), isolation valves may be spaced several hundred to a few thousand feet apart. Because of the redundant links, water service is cut off to no more than a limited number of users. For example, the failure of link 3 in the looped distribution system of Figure 4-1 results in the two isolation valves on link 3 being shut and the rerouting of 3650 GPM along links 2, 7, and 4.

In a tree shaped water distribution system (usually found in rural areas) the failure of a water main can have a considerably greater impact on water service. For example, consider the tree-shaped distribution system of Figure 4-2 derived from Figure 4-1 by deleting links 7 and 8. The same failure on link 3 would cut off demand to nodes 4, 5, 6, and 7 or more than 80% of system demand.

4.3 Redundant Link Selection Factors

Prior to formulating a detailed mathematical model to select the redundant links to complete the network layout, we will examine the following major factors that influence the selection decision:

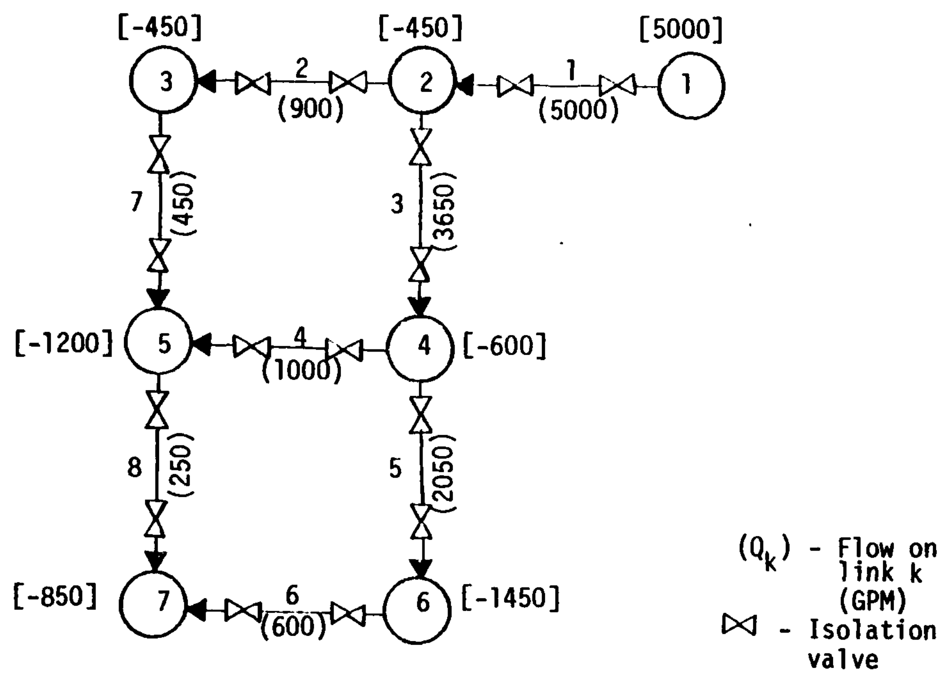


Figure 4-1

FULLY LOOPED DISTRIBUTION SYSTEM

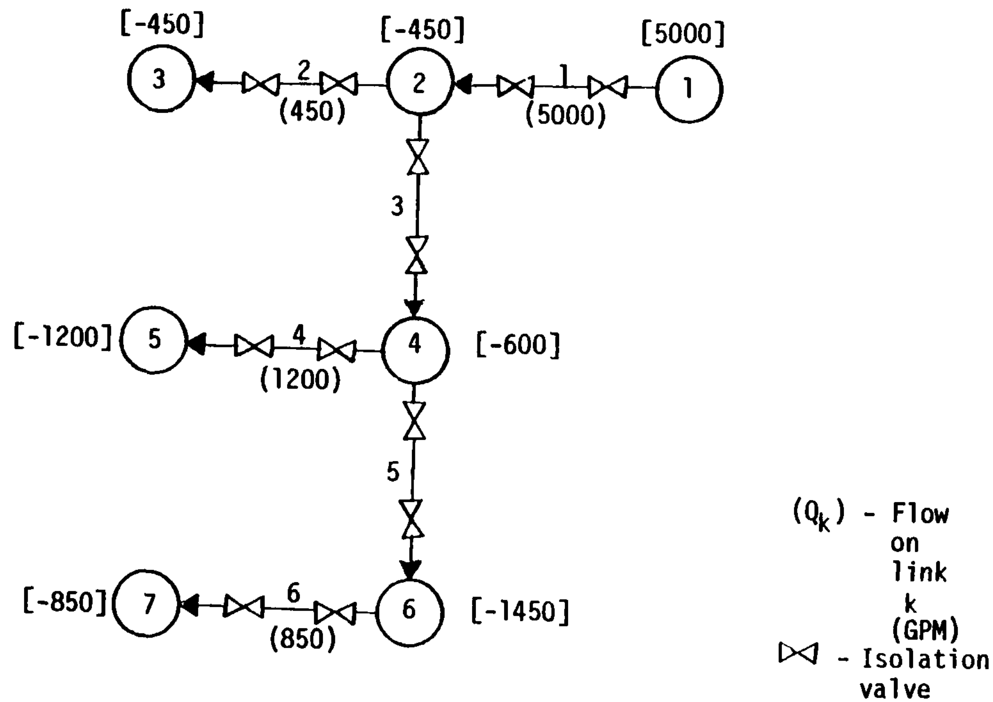


Figure 4-2

TREE DISTRIBUTION SYSTEM

1. Impact of primary link failure.
2. Likelihood of primary link failure.
3. Capability of redundant links to maintain service in case of primary link failure.
4. Cost of redundant link.

4.3.1 Impact of Primary Link Failure

The total impact of failure of the larger diameter primary link can be divided into three areas:

1. Cost of water lost prior to discovery of the break.
2. Value of water damage to surrounding public and private property.
3. Unsatisfied water demand while the failed link is being repaired which can lead to loss of goodwill.

The amount of water lost due to failure of a water main depends on several factors including the nature of the failure, the flow rate in the pipe, and the time it takes to detect the break. Leakage from water mains is readily discovered because water bubbles to the surface or can be detected by leak detection surveys [63]. In any case, the amount of water lost in a break is not especially relevant to selection of a redundant link but more closely related to operation and control of the water distribution system. Likewise,

property damage caused by escaping water depends on the location of the primary links and the particular operational and control scheme selected.

After the broken link has been detected and the appropriate valves closed to prevent further water loss and property damage, the network layout and the time to repair the broken section determine the extent of unsatisfied water damand. Given a single source tree-shaped distribution system, computation of the expected amount of unsatisfied demand resulting from failure of primary link i is straightforward.

Let us define the following terms:

$\overline{Q}_i$--the average daily flow rate in gallons per minute on primary link i

t_i--the expected repair time for restoring service on primary link i in minutes.

Then for the core tree:

$u_i = t_i \overline{Q}_i$--the expected amount of unsatisfied demand resulting from each failure of primary link i

Water distribution systems are usually designed to handle peak hourly demands which respresent 2 to 4 times the average daily flow rate [26]. The average daily flow rate is used to compute the volume of unsatisfied demand since the expected repair time is

24-48 hours depending on the location of the failure and the availability of replacement parts [64].

When service is frequently interrupted by broken link failures, undesirable customer reactions and public relations result. Although loss of customer goodwill is an intangible consideration, Stacha [63] performed an empirical cost analysis of service interruptions due to link failure and assigned an inconvenience value in dollars based on the number of service interruptions per year. However, Stacha makes no attempt to support his figures.

Thus, it appears that the most appropriate measure of the impact of failure of a primary link is the expected amount of unsatisfied demand. Ideally, one would desire to assign utility values to varying levels of unsatisfied demand to use in making appropriate cost/reliability tradeoffs. However, because of the lack of any widely accepted measure of the value of interruptions in water service [51], such an approach is highly speculative and lacks firm empirical support.

4.3.2 Likelihood of Primary Link Failure

As discussed above, there are several factors which alone or in combination can account for link failure. Prior to installation it is extremely difficult to accurately predict the individual

failure rates of each primary link. Other than theoretical analyses of pipe failure under well defined flow and pressure conditions, no work has been done to correlate the multiple factors involved in pipe failure with the failure rate. The only available information is aggregate historical data for real systems and is usually given in the number of link failures per year per length of pipe in the distribution system [25, 63]. Thus, it appears reasonable to assume that the number of link failures per year for the core tree obeys a Poisson probability law with parameter

$$\lambda' \sum_{i \,\varepsilon\, PL} L_i$$

where λ' is the number of failures per year per length of pipe and

$$\sum_{i \,\varepsilon\, PL} L_i$$

is the total length of the core tree. Therefore, assuming the failure rate of each primary link i is also proportional to its length, the number of failures per year for each primary link also obeys a Poisson probability law with parameter $\lambda' L_i$ (the expected number of failures per year on link i). Then, $\bar{u}_i = \lambda' L_i u_i = \lambda' L_i t_i \bar{Q}_i$

is the expected amount of yearly unsatisfied demand (gallons) resulting from failure of primary link i .

4.3.3 Redundant Link Capability

The capability of a potential redundant link to maintain service to nodes cut off by failure of a primary link depends on the location and capacity of the redundant link. In the core tree layout the failure of each primary link disconnects a unique set of nodes from the source. For example, in Figure 4-3 the failure of primary link 3 disconnects nodes 4 and 6 from the source at node 1 and a total of 350 GPM of flow. Each candidate redundant link can be classified according to its ability to reconnect the set of nodes disconnected by failure of each primary link. For example, non-tree links 8 and 10 can reconnect demand node 6 cutoff by failure of primany link 5, while non-tree links 6, 7, and 9 cannot. Non-tree links 6, 7, and 8 can solve the failure of primary link 2 while links 9 and 10 cannot.

For a single source distribution system the combined flow capacity of the redundant links serving the set of demand nodes cutoff from the normal primary link supply path determines the level of service during the broken link emergency loading condition. As a rule of thumb [60] the flow capacity of link k in gallons per

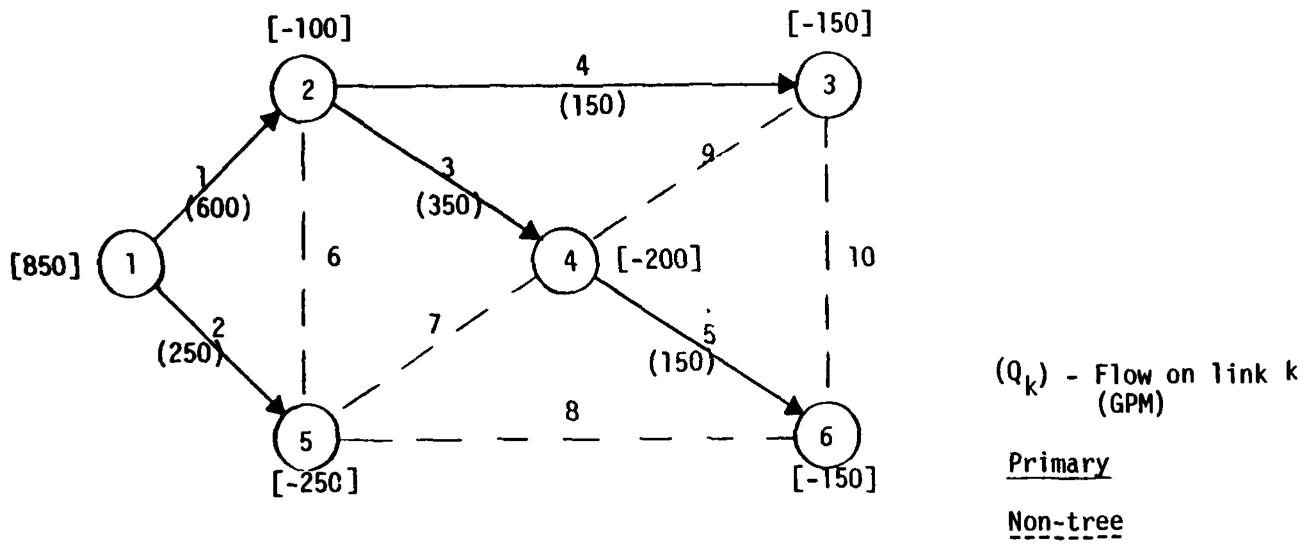


Figure 4-3

SINGLE-SOURCE TREE SYSTEM PLUS NON-TREE LINKS

minute with diameter D_k in inches as

$$QMAX_k = 10\ D_k^2 \tag{4-1}$$

As in the derivation of the expression for expected unsatisfied demand, it will be assumed that all nodal demands are average daily demands.

Let us consider, for example, the distribution system of Figure 4-3. Assume the core tree consisting of primary links 1-5 has been installed and average daily demand rates are shown. Table 4-1 presents a failure analysis for the primary links in the core tree. Column 2 shows the demand node disconnected as a result of failure of each primary link, column 3, the total unsatisfied demand rate, and column 4, the candidate redundant links capable of reconnecting the failure of each primary link. To provide continuing service for all failure modes a minimum of two links (8 and 9, 6 and 10, 7 and 10, or 8 and 10) must be in the network. Minimum pipe diameters installed in municipal water distribution systems in the United States are usually 6 or 8 inches in diameter. Thus, one feasible solution for covering expected unsatisfied demand would be to install an 8" pipe (640 GPM capacity) on link 8 and a 6" pipe (360 GPM capacity) on link 9.

Table 4-1

PRIMARY LINK FAILURE ANALYSIS

Failure of Primary Link No.	Nodes Disconnected	Total Unsatisfied Flow Rate (GPM)	Redundant Links Reconnecting
1	2, 3, 4, 6	600	6, 7, 8
2	5	250	6, 7, 8
3	4, 6	350	7, 8, 9, 10
4	3	150	9, 10
5	6	150	8, 10

4.3.4 Redundant Link Cost

As discussed in section 3.3.5.1 the capital cost of link k is

$$c_k = \ell_1 D_k^{\ell_2} L_k \tag{4-2}$$

Since flow capacity is a function of diameter, i.e.,

$$QMAX_k = 10 D_k^2 \tag{4-3}$$

the cost of a link can be expressed as a function of its capacity

$$c_k = \ell_1 \left(\frac{QMAX_k}{10}\right)^{\frac{\ell_2}{2}} L_k \tag{4-4}$$

This result is similar to the separable terms of the nonlinear minimum cost flow objective function (Problem P5) where a uniform hydraulic gradient J_k was assumed. Thus, a redundant link's cost increases nonlinearly with its capacity and linearly with its length.

Since in properly designed systems redundant links function at capacity only under emergency loading conditions (high fire demand or broken link), the diameter of these links are usually set to some minimal diameter. Usually there are state regulations [65] or municipal design standards [66] setting mimimum pipe diameters.

For fire insurance ratings the state board of insurance will not count links below a certain diameter (6" or 8") as part of a city's fire protection system thus increasing the cost of fire insurance.

4.4 Optimization Models

If we consider the failure of each primary link as a separate emergency loading condition, the problem of selecting redundant links becomes how to best maintain continuity of service to the various sets of disconnected nodes. One approach would be to assume a certain amount of funds were specifically allocated for redundant links and to formulate a 0-1 knapsack problem for selecting the set of redundant links with maximum capability. However, this approach places an unrealistic burden on the system designer to properly allocate his total budget between redundant links and all other system components. Another potential knapsack-type formulation would be to select the best k redundant links where the objective function could be the number of broken link loading conditions covered. Although this approach is somewhat more realistic than the previous one, it still assumes that the user already knows the best level of looping for the system. If k is set too high, the total system costs will be inflated by the costs of installing the excess redundant links at minimum diameter.

The optimization approach taken in the two models that were developed was to minimize the costs of the redundant links subject to satisfying all the broken link emergency loading conditions, i.e., providing continuity of water service in case of failure of each primary link. This approach was selected for the following reasons:

1. The continuity of water service requirements and redundant link costs are well defined.
2. The minimum cost approach is consistent with the selection of the minimum cost spanning tree in the first level model.
3. The resulting network layout for the final detailed design model is economical for operating under both normal and broken link emergency loading conditions.

4.4.1 Set Covering Model

4.4.1.1 Model Formulation

Let us consider the following integer programming model for selecting the set of redundant links:

PROBLEM P6

$$\text{Minimize} \quad \sum_{k \varepsilon \overline{PL}} c_k \, y_k \tag{4-5}$$

subject to

$$\sum_{k \varepsilon \overline{PL}} e_{ik} \, y_k \geq r_i \quad i \varepsilon PL \tag{4-6}$$

$$y_k = 0, 1 \qquad k \varepsilon \overline{PL}$$

where

$$y_k = \begin{cases} 1 & \text{if candidate redundant link } k \text{ is in the network} \\ 0 & \text{otherwise} \end{cases}$$

c_k--the total estimated cost of including redundant link k in the system at minimum diameter

$$e_{ik} = \begin{cases} 1 & \text{if candidate redundant link } k \text{ is incident to a node in the set of demand nodes disconnected by failure of primary link } i \\ 0 & \text{otherwise} \end{cases}$$

r_i--the minimum number of redundant links required to reconnect the set of demand nodes disconnected due to failure of primary link i.

PL--the set of primary links in the core tree

$\overline{PL}$--the set of candidate redundant links

The objective function (4-5) minimizes the total cost of installing redundant links at some specified diameter. Because of the 0-1 decision variable any fixed right of way costs can be directly incorporated in the cost coefficients. It is assumed that all redundant links have a common diameter. The set covering constraints (4-6) require that there are at least r_i redundant links in the network to cover the failure of primary link i. Problem P6 is formulated below for the network of Figure 4-3 with $r_i = 1$ for failure of primary link i and redundant link cost proportional to link length L_k. The value of r_i is set to 1 based on an 8" link diameter for all redundant links. Assuming no abnormal excavation or right of way costs, the cost of links of the same diameter is directly proportional to its length.

$$\text{Minimize} \quad L_6\, y_6 + L_7\, y_7 + L_8\, y_8 + L_9\, y_9 + L_{10}\, y_{10}$$

$$\begin{aligned}
\text{subject to} \quad & y_6 + y_7 + y_8 && \geq 1 \\
& y_6 + y_7 + y_8 && \geq 1 \qquad (4\text{-}7) \\
& y_7 + y_8 + y_9 + y_{10} && \geq 1 \\
& y_9 + y_{10} && \geq 1 \\
& y_8 + y_{10} && \geq 1
\end{aligned}$$

$$y_6, y_7, y_8, y_9, y_{10} = 0, 1$$

4.4.1.2 Solution Technique

Setting $r_i = 1$ for all primary links requires that there be at least two different paths to each demand node, i.e., fully looped network. For $r_i = 1$ for all primary links, Problem P6 is the classical weighted set covering problem which has been used for a variety of applications including airline crew scheduling (Drabeyre et al. [69]), political redistricting (Garfinkel and Nemhauser [70]), optimal attack and defense of a military communications network (Jarvis [71]), and information retrieval (Day [72]). Efficient search enumeration techniques are available for handling the size of problem under consideration (50 rows, 100 decision variables) [73]. For at least one r_i greater than 1 and all redundant link costs equal, Problem P6 is a multiple set covering problem for which Rao [74] developed an efficient specialized solution technique. For at least one r_i greater than 1 and redundant link costs not all equal Problem P6 becomes a weighted multiple set covering problem. Its form is that of a general 0-1 integer program but with a 0-1 coefficient matrix and all greater than or equal to constraints. The resulting problem can be viewed as a simple generalization of either the weighted set covering problem

(all $r_i = 1$) or of the multiple set covering problem (all c_j equal). Based on Forrest, Hirsch and Tomlin's computational experience [75] using the Dakin branch and bound technique with penalty calculations in which problems with up to 4000 rows and 130 0-1 variables were solved in times on the order of multiples of two or three of the first linear program solution time, it appears that existing general 0-1 integer programming algorithms are adequate to solve the size of problem under consideration. Because of the adequacy of existing general purpose 0-1 algorithms, development of a specialized algorithm for the general weighted multiple set covering problems appears to be unneeded. However, the algorithm of Lemke, Salkin, and Spielberg [73] for the weighted set covering problem and Rao's algorithm [74] for the multiple set covering problem might be modified to provide a more efficient algorithm for solving Problem P6.

Problem P6 requires the user to select r_i, the minimum number of redundant links needed to cover the failure of primary link i. The selection of r_i is based on the impact of failure of primary link i. For each primary link, the expected amount of unsatisfied demand per year, $\bar{u}_i$, can be calculated and used as a guide for selecting r_i. A relatively large $\bar{u}_i$ implies the need for a higher number of redundant links covering the failure of

primary link i. However, because of the limited availability of funds $\bar{u}_i$ can be an especially useful tool in ordering priorities for covering primary link failures. Based on a very low value of $\bar{u}_i$ compared to other primary links and a high cost of installing redundant links to solve the failure of primary link i, the decision could be made to set $r_i = 0$ and not require that failure of link i be covered. This situation might arise for a small development located far from the other concentrations of demand. Looping of that section of the network would be delayed until surrounding areas were developed.

4.4.2 Flow Covering Model

4.4.2.1 Model Formulation

Let us consider Problem P6 in terms of the flow capacity to the disconnected set of demand nodes that the satisfaction of the set covering constraints (4-6) implies. Assuming that all candidate redundant links have diameter D, then all have capacity $10D^2$. Multiplying both sides of (4-6) by the link capacities gives us

$$\sum_{k \varepsilon \overline{PL}} 10D^2 e_{ik} y_k \geq 10D^2 r_i \qquad (4\text{-}8)$$

$$i \varepsilon PL$$

Thus, satisfying the set covering contraints (4-6) in Problem P6 implies that the flow capacity of the redundant links serving the set of demand nodes disconnected due to failure of primary link I is 10 $D^2 r_i$ GPM.

Next, let us assume that instead of a single diameter each candidate redundant link k has a set S_k of candidate diameters to draw from. Further, based on the peak hourly demand for each node, we can compute the average total demand rate d_i for the set of demand nodes disconnected by failure of primary link i in the core tree. Expanding on Problem P6, we have the following 0-1 integer programming problem:

PROBLEM P7

Minimize $$\sum_{k \varepsilon \overline{PL}} \sum_{j \varepsilon S_k} c_{kj} \, y_{kj} \qquad (4\text{-}9)$$

$$\sum_{k \varepsilon \overline{PL}} \sum_{j \varepsilon S_k} e_{ikj} \; y_{kj} \geq d_i \qquad i \varepsilon PL \qquad (4\text{-}10)$$

$$\sum_{j \varepsilon S_k} y_{kj} \leq 1 \qquad k \varepsilon \overline{PL} \qquad (4\text{-}11)$$

$$y_{kj} = 0, 1 \qquad k \varepsilon \overline{PL}$$
$$\qquad\qquad\qquad j \varepsilon S_k$$

where

c_{kj}--the total estimated cost of including candidate diameter redundant link $j \varepsilon S_k$ in the network

$$y_{kj} = \begin{cases} 1 & \text{if candidate redundant link } k \text{ with diameter } D_{kj}, j \varepsilon S_k \text{ is in the network} \\ 0 & \text{otherwise} \end{cases}$$

$$e_{ikj} = \begin{cases} 10\, D_{kj}^2 & \text{if candidate redundant link } k \text{ is incident to a node in the set of demand nodes disconnected by failure of primary link } i \text{ where } j \varepsilon S_k \\ 0 & \text{otherwise} \end{cases}$$

d_i--the minimum total flow capacity of redundant links serving the set of demand nodes disconnected due to failure of primary link i

S_k--the set of candidate diameters for candidate redundant link k

D_{kj}--candidate diameter $j \varepsilon S_k$

The flow covering constraint (4-10) serves the same function as the set covering constraint (4-6) of Problem P6. The inequality

constraint (4-11) insures that at most one pipe diameter is chosen for each candidate redundant link.

Problem P7 is formulated below for the network of Figure 4-3 with $S_k = \{6, 8\}$ $k = 6, 7, 8, 9, 10$, i.e., $D_{k1} = 6$ and $D_{k2} = 8$,

and $c_{kj} = \ell_1 (D_{kj})^{\ell_2} L_k$.

$$\text{Minimize} \quad c_{66}\, y_{66} + c_{68}\, y_{68} + c_{76}\, y_{76} + c_{78}\, y_{78} + c_{86}\, y_{86} + c_{88}\, y_{88} + c_{96}\, y_{96} + c_{98}\, y_{98} + c_{10,6}\, y_{10,6} + c_{10,8}\, y_{10,8}$$

subject to

$$360\, y_{66} + 640\, y_{68} + 360\, y_{76} + 640\, y_{78} + 360\, y_{86} + 640\, y_{88} \geq 600$$

$$360\, y_{66} + 640\, y_{68} + 360\, y_{76} + 640\, y_{78} + 360\, y_{86} + 640\, y_{88} \geq 250$$

$$360\, y_{76} + 640\, y_{78} + 360\, y_{86} + 640\, y_{88} + 360\, y_{96} + 646\, y_{98} + 360\, y_{10,6} + 640\, y_{10,8} \geq 350$$

$$360\, y_{96} + 640\, y_{98} + 360\, y_{10,6} + 640\, y_{10,8} \geq 150$$

$$360\, y_{86} + 640\, y_{88} + 360\, y_{10,6} + 640\, y_{10,8} \geq 150$$

$$y_{66} + y_{68} \leq 1$$

$$y_{76} + y_{78} \leq 1$$

$$y_{86} + y_{88} \leq 1$$

$$y_{96} + y_{98} \leq 1$$

$$y_{10,6} + y_{10,8} \leq 1$$

$$y_{66},\ y_{68},\ y_{76},\ y_{78},\ y_{86},\ y_{88},\ y_{96},\ y_{98},\ y_{10,6},\ y_{10,8} = 0,\ 1$$

As before the average daily flow rate was chosen as the value for d_i because this is the expected flow over the length of the emergency loading condition. However, the system designer has the flexibility to adjust the d_i values based on any special conditions that may coincide with failure of a specific primary link.

It should be noted that although sufficient flow capacity may be designed into the redundant links, there is no guarantee that a primary link failure will not result in some reduction in water pressure to the disconnected set of demand nodes. The lower head results from both the higher frictional losses incurred by increasing flow rates on other primary links and the fact that

some of the water is no longer traveling to each demand node on the shortest path. If there is a special concern about the precise performance of the system due to the failure of a specific primary link (see de Neufville et al. [50]), this failure may be formulated as an emergency loading condition to be handled in the detailed design phase (see Chapter 5). If deterioration of nodal heads is sufficiently severe, it may become necessary to have additional standby pumping.

4.4.2.2 Solution Technique

Unlike the set covering model (Problem P6), the flow covering model (Problem P7) does not have any special form and must be classified as a general 0-1 integer program. A variety of general 0-1 integer programming algorithms are available to solve this problem including cutting plane, branch and bond, search enumeration, and group theoretic algorithms [68].

4.4.3 User Design Constraints

Because both redundant link selection models are integer programs, there is considerable flexibility for incorporating various user supplied design constraints into the model. For the set

covering model (Problem P6) with $r_i = 1$ (the simple weighted set covering problem) Roth [76] has demonstrated a simple technique to incorporate conditional constraints of the form

$$-y_k + \sum_{j \in \overline{PL}_k} y_j \geq 0 \qquad (4\text{-}12)$$

where $\overline{PL}_k$ is a nonempty subset of $\overline{PL}$ and $k \notin \overline{PL}_k$.

Constraint (4-12) requires that if link k is in the network then at least one link from the set $\overline{PL}_k$ must also be in the network. The technique replaces the full set of constraints (4-6 and 4-12) with an equivalent set of constraints having the same form as normal set covering constraints (4-6). Thus, in this special case efficient set covering algorithms may still be used.

Constraint (4-12) is a special case of the general set constraint which can be useful in refining the system design. Let $\overline{PL}'$ be any subset of candidate redundant links that have some common property, e.g., the set of candidate redundant links incident to a specific node or a set of nodes. Constraints of the form

$$\sum_{j \in \overline{PL}'} y_j \quad \left\{ \begin{matrix} \geq \\ = \\ \leq \end{matrix} \right\} \quad k \qquad (4\text{-}13)$$

where k is a positive integer, may be incorporated in either the set or flow covering models. Although slightly increasing the computational burden of solving the problem (since only rows are added), such constraints allow the system designer to explicitly incorporate various realistic design restrictions into the problem. It also aids in accurately assessing the impact on total cost arising from such design restrictions which were formerly only handled implicitly.

4.4.4 Multiple Source Application

Our prior analysis had assumed a single source distribution system. Properly located additional sources can reduce the requirement for redundant links and provide protection in case of source outages. To illustrate this situation let us consider the 7-node, 6-link, two-source system in Figure 4-4. Node 1 is the principal supplier for demand nodes 2, 3, and 4, and node 5, the principal supplier for demand nodes 6 and 7. Failure of a primary link on the source-to-source path, links 1, 4, and 5, still leaves a path of primary links from the alternate source to the set of demand nodes cutoff from their principal source. Thus, the redundant link requirements of the set and flow covering models must be appropriately reduced. The purpose of this section is to present a

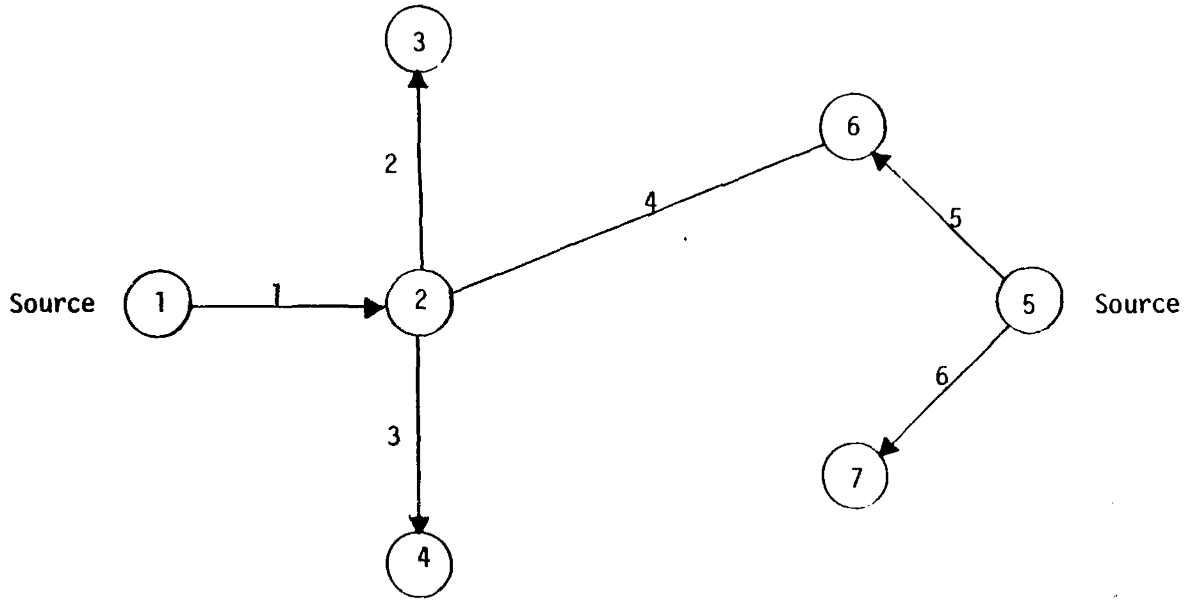


Figure 4-4

TWO-SOURCE TREE LAYOUT

procedure for assessing the impact of the alternate source on the redundant link requirements and incorporating this impact into the redundant link selection models.

Given the core tree for a multiple source network, consider the path of primary links connecting any two adjacent sources, $SOURCE_j$ and $SOURCE_k$. From Chapter 3 we know that each demand node on the source-to-source path or on a branch from it has as its principal source $SOURCE_j$ or $SOURCE_k$, while the other source serves as its alternate source. Failure of primary link i on the source-to-source path disconnects a unique set of demand nodes from their principal source.

Let us examine the problem of supplying some of the unsatisfied demand due to failure of primary link i from the alternate source via the existing source-to-source path of primary links. It will be assumed in this analysis that the capacity of the alternate source is not a limiting factor.

To assist in this analysis we will define the following terms:

SSP_i--the set of primary links on the source-to-source path from the alternate source to primary link i

Q_{k_i}--the average flow rate on link $k \varepsilon SSP_i$ subsequent to the failure of primary link i

$QMAX_k$--the total flow capacity of link $k \varepsilon SSP_i$ when empty

Then, the average excess primary link flow capacity available in case of failure of primary link i from the alternate source via the links of SSP_i is

$$EQCAP_i = \min_{k \varepsilon SSP_i} \left[QMAX_k - Q_{k_i} \right] \qquad (4\text{-}14)$$

i.e., the minimum of the primary link excess flow capacities. The quantity $QMAX_k - Q_{k_i}$ is the excess flow capacity on primary link $k \varepsilon SSP_i$. The value of Q_{k_i} is computed by finding the core tree flow distribution for average daily demands at each node and then simulating failure of link i. To determine $QMAX_k$ an estimate of link k's optimal diameter is required. An accurate estimate can be obtained by solving Problem P1 with no redundant links, i.e., solving the minimum cost optimization problem for the core tree under the normal (peak hour) loading condition.

The resulting $EQCAP_i$ is then subtracted from d_i (4-10) computed using the standard method of failure analysis. The result is that the minimum total flow capacity that must be provided by the redundant links, d_i, in the flow covering model (Problem P7) is reduced. Similarly, r_i, the minimum number of redundant links

required to cover failure of primary link i in the constraints (4-6) of the set covering model (Problem P6), may be appropriately reduced. If either d_i or r_i becomes nonpositive, the contraint is trivially satisfied and can be dropped from the constraint set. The above procedure is repeated for each primary link on all source-to-source paths in the core tree.

The primary link where $EQCAP_i$ is attained is the limiting component or bottleneck for alternate source supply. It may be less expensive to build additional flow capacity into an existing source-to-source primary link than to install a new or larger capacity redundant link. Next, we will discuss how the alternative of setting minimum capacities (diameters) for primary links on the source-to-source path can be incorporated into the flow covering model (Problem P7).

Let link k be the bottleneck link for primary link i and link j be the link having the second least excess capacity in case of primary link i failure, i.e., the secondary bottleneck. Assuming we fix the capacity of link j, the secondary bottleneck, the quantity

$$QMAX_j - Q_{j_i} - EQCAP_i = QMAX_j - Q_{j_i} - QMAX_k + Q_{k_i}$$

is the maximum additional flow capacity that can be added to link k

for link k to remain the bottleneck for link i failure. To determine the exact associated increase in diameter of link k, ΔD_k, we can solve the quadratic equation

$$10\,(D_k + \Delta D_k)^2 = QMAX_j - Q_{j_i} + Q_{k_i} \qquad (4\text{-}15)$$

where D_k is the estimated diameter of link k obtained from the minimum cost core tree optimization. However, since the pipe diameters are discrete and pipe cost is a nonlinear function of diameter (capacity), consider increasing the diameter of link k to each commercially available diameter between the current diameter D_k and the next commercially available diameter above $D_k + \Delta D_k$. For each of these diameters, D_{kj}, $j \varepsilon S_k$, the gain in flow capacity is equal to

$$10\,D_{kj}^2 - QMAX_k \quad \text{if} \quad D_{kj} < D_k + \Delta D_k \quad \text{and}$$

$$QMAX_j - Q_{j_i} - QMAX_k + Q_{k_i} \quad \text{if} \quad D_{kj} \geq D_k + \Delta D_k$$

The additional cost of replacing a link of diameter D_k with a link of diameter D_{kj} is

$$\ell_1 \left(D_{kj}^{\ell_2} - D_k^{\ell_2}\right) L_k \,.$$

To allow us to compute the correct value of the additional flow capacity on the source-to-source path to link i we had to assume that the capacity of the secondary bottleneck, link j, the reference link, remains constant. If link j is not a bottleneck link for the failure of some other primary link or the increases in D_k are limited such that link k remains the bottleneck link for primary link i, then, using the added flow capacities and costs defined above, primary link k may be treated just like any other redundant link and included directly in the flow covering constraint for primary link i.

The case in which link j, the secondary bottleneck link, is a bottleneck for another primary link greatly complicates the problem; the reference capacity for the bottleneck link becomes a decision variable. Attempts to incorporate this case into the flow covering model result in constraints that are the product of two 0-1 variables. Separability of the resulting $y_k\, y_j$ terms can be induced by the substitution $y_k\, y_j = \bar{y}_k{}^2 - \bar{y}_k{}^2$, and adding the constraints $\bar{y}_k = \frac{1}{2}\,(y_k + y_j)$ and $\bar{y}_j = \frac{1}{2}\,(y_k - y_j)$. The new decision variables $\bar{y}_k$ can only assume discrete values of $0, \frac{1}{2}$, and 1 and $\bar{y}_j$ of $0, \frac{1}{2}, -\frac{1}{2}$, and 1. Thus, the flow covering model (Problem P7) would become a nonlinear integer program.

However, because of the relatively small number of decision variables affected by this case and the considerable additional difficulty and effort to develop an algorithm to solve this problem, it appears that selective enumeration is the most appropriate solution technique. This procedure involves systematically fixing the diameters of links that were both primary and secondary bottlenecks at current or higher diameters, solving the resulting flow covering model (Problem P7) and finally comparing the optimal objective values taking into account the added cost and capacity of links set above current diameters.

Let us consider applying the above procedure to the 11-node, 21-link network of Figure 4-5 supplied from nodes S1 and S2. The core tree consists of links 1-10 and the candidate redundant links 11-21. The average daily flow distribution depicted in the figure shows that S1 is the principal source for nodes 1, 2, 3 and 5 and S2 for the remaining 5 demand nodes. Assume that minimum cost optimization of the core tree results in optimal diameters of 14, 10, 6, and 12 inches for links 1, 4, 6, and 10, respectively. Table 4-2 shows the calculation of $EQCAP_i$. Based on the results of Table 4-2, the alternative to increase the minimum diameters of the bottleneck links, 4 and 6, should be incorporated into the flow covering model (Problem P7).

Table 4-2

PRIMARY LINK BOTTLENECK ANALYSIS

LINK k	D_k (IN)	$QMAX_k$ (GPM)	$QMAX_k - Q_{k_1}$	$QMAX_k - Q_{k_4}$	$QMAX_k - Q_{k_6}$	$QMAX_k - Q_{k_{10}}$
1	14	1960	--	--	1160	1160
4	10	1000	1000	--	700	700
6	6	360	260	260	--	360
10	12	1440	640	640	--	--
Bottle-neck Link			6	6	4	6
$EQCAP_i$			260	260	700	360

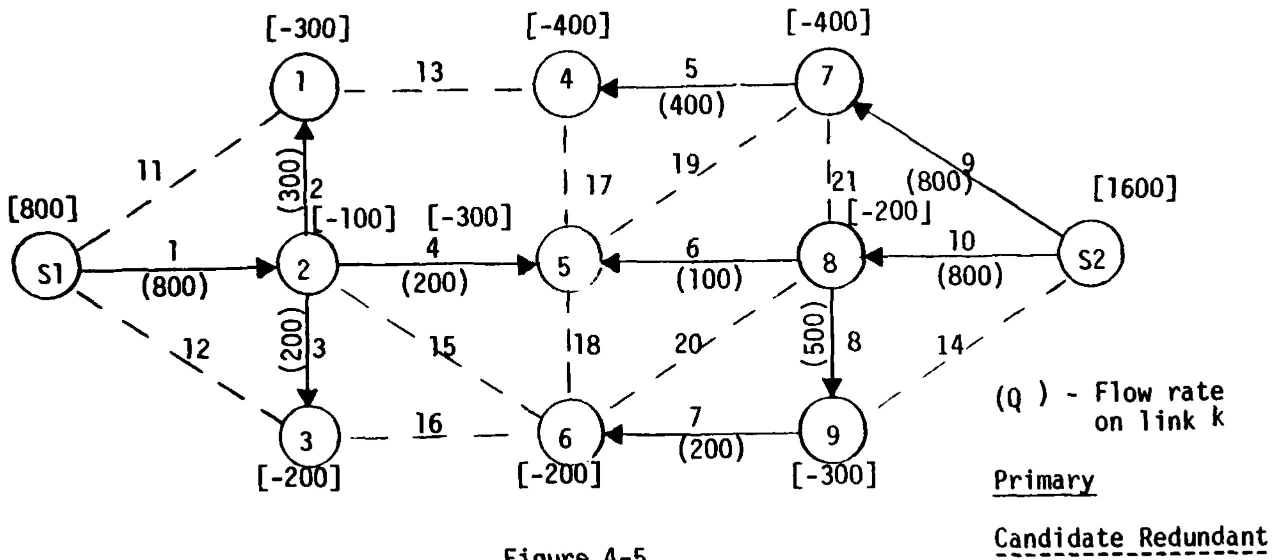


Figure 4-5

TWO-SOURCE TREE SYSTEM PLUS NON-TREE LINKS

4.4.5 Comparison of Models

The set and flow covering models will be compared on the basis of utility, ease of formulation, and ease of solution.

Problem P6, the set covering model, handles the problem of covering the failure of primary links by explicitly focusing on the quantity of redundant links required and only implicitly considering the flow capacity provided by the redundant links. On the other hand, Problem P7, the flow covering model, explicitly takes into account the minimum flow capacities which the redundant links must provide in case of each primary link failure. Consequently, the flow covering model, provides a solution which specifically addresses the concerns expressed by previous researchers (Wantanadata [40] and Alperovits and Shamir [46]) over what diameter to select for the redundant links in order to provide a well-defined level of reliability. Thus, the solution of Problem P7 which provides both the optimal redundant links and their minimum diameters is significantly more useful to the system designer.

The formulation of the coefficients of the covering constraints for both models, (4-6 and (4-10), is similar since the basic failure analysis is the same. The flow covering model elaborates upon the 0-1 covering matrix of the set covering model by incorporating capacities of redundant links and allowing a choice

of redundant link diameters (capacities). Because of the more precise nature of the flow covering approach to solving the broken link emergency condition, the selection of the minimum flow capacity requirement for each primary link failure, d_i, in the flow covering model can be defined in a much less arbitrary manner than the minimum number of redundant links required, r_i, in the set covering model.

Analysis of the structure and size of the constraint sets reveals that, in general, the set covering model, is somewhat easier to solve than the flow covering model. The constraint set of Problem P6 is identical to the standard set covering problem and as previously discussed may be solved efficiently using special techniques. Except in the special case where it is equivalent to a set covering problem, i.e., each candidate redundant link has only a single candidate diameter, the flow covering model is a general 0-1 integer programming problem requiring more complex solution techniques. Furthermore, the flow covering model requires an additional $|\overline{PL}|$ equality constraints (4-11). More important from a computational viewpoint a total of

$$\sum_{k \in \overline{PL}} |S_k|$$

decision variables are needed for the flow covering model whereas

only $|\overline{PL}|$ are required for the set covering model. The computational results of applying a general purpose 0-1 integer programming code using both models on a realistic size problem will be presented in Chapter 6. Thus, the question of which is the superior model hinges on the value of the additional information obtained from the flow covering model (Problem P7) versus the increased computational cost of solving this more complex problem.

CHAPTER 5

DETAILED SYSTEM DESIGN

5.1 Introduction

Given the total network layout (including the minimum diameters for all redundant and certain primary links), the purpose of the third level model of the hierarchical system is to assist the water distribution system designer in the detailed system design. The detailed system design involves selecting

1. Link diameters
2. Pump capacity and arrangement
3. Height of elevated storage reservoirs.

After discussing emergency loading conditions, we will present the mathematical model developed to solve the detailed design problem including the solution technique and its application to a small example problem.

5.2 Emergency Loading Conditions

To insure reliable water distribution the system must be designed to accommodate the range of expected emergency loading

conditions. The major types of emergency loading conditions to be considered are:

1. Broken primary links
2. Fire demands
3. Pump/power outages

Each of the above conditions will be examined with an emphasis on describing its impact on the system, developing relevant measures of system performance, and designing into the system the capability to handle the emergency loading condition.

5.2.1 Broken Primary Link

As discussed in Chapter 4 the major impact of a broken primany link is the interruption or reduction in flow to the set of demand nodes serviced by the primary link. The set covering model (Problem P6) and the flow covering model (Problem P7), developed in Chapter 4, insure that sufficient flow capacity is built into the critical links of the system, redundant and primary, to provide acceptable performance at minimum cost in case of primary link failure.

A secondary measure of performance, first used by de Neufville et al. [50], is the pressure at the demand nodes. Theoretically, the detailed design model could also consider the failure

of each primary link as a separate loading condition and use some function of nodal pressures as the measure of performance. However, the computational burden of solving such a large problem would be prohibitive and the potential for distortion of the link design under such a multitude of diverse, unusual flow conditions is considerable. Nevertheless, to illustrate its proper treatment we will analyze and solve a detailed design problem with a single primary link failure in Section 5.5.4.

5.2.2 Fire Demand

The performance of a water distribution system during a fire is critical because of its impact on loss of life and property. The potential for property loss is best reflected in the cost of fire insurance. In most U.S. cities fire insurance rates are a function of the level of fire protection as defined by the Insurance Services Office (ISO). Most municipalities are graded by the ISO and classified according to the quality of their fire protection. The ISO's grading schedule [77] rates the following five areas:

1. Water distribution system
2. Fire department
3. Fire service communications

4. Fire safety control

5. Miscellaneous additional areas

The water distribution system accounts for 30 percent of the rating. Municipalities which the ISO assesses as having better fire protection benefit from lower insurance rates. Total fire protection cost is the sum of both the tax dollars spent for fire protection services (public expenditures) and fire insurance premiums paid by residences and businesses (private expenditures). Seward, Plane and Hendrick [78] developed a 0-1 integer programming model for allocating public funds among various fire service projects to achieve a specified ISO rating at minimum cost. Thus, the performance of the water distribution system under the expected fire demand loading is a major concern of the system designer.

A fire requires a high flow rate of water concentrated at a single demand node for several hours. The major concern and principal measure of performance in the fire demand loading condition is delivering the required flow rate at sufficient pressure to be used by the fire fighting equipment. The ISO [77] provides guidelines for estimating fire-flow requirements and duration at various locations throughout a municipality. Their formulas for computing fire-flow requirements, originally based strictly on population, have in recent years been modified to take into account the varying fire-

flow requirements of the commercial, industrial, warehousing, institutional, apartment, and dwelling districts in a city. The pressure requirements at the fire demand node may vary considerably based on the type of fire pumping equipment used and the height of the buildings in the particular district.

To deliver the required fire demand flow rate over the expected period of time requires sufficient water in storage over and above normal peak hour demands and for pumping systems may require additional standby pumps. Three possible methods exist for the distribution system to provide the necessary pressure [24]:

1. The maintenance of sufficient pressure in the mains at all times for direct hydrant service for hose streams.
2. The use of emergency fire pumps to boost the pressure in the distribution system during fires.
3. The use of a separate high-pressure distribution system for fire protection only.

Typically, municipalities [66] and state regulations [65] set minimum pressure levels (e.g., 46 feet), that the distribution system must maintain under all expected emergency loading conditions.

5.2.3 Pump/Power Outage

The horizontal centrifugal pump is the most commonly used pump for waterworks duty because of its low cost and the great variety of designs available to meet a wide range of pumping conditions [25]. Unscheduled shutdowns are usually due to problems with the pump's seals, packing, bearings, or balancing [79]. Unlike other industrial equipment there is little published data on the mathematical availability of pumping equipment [79]. Messina [79] suggests using an availability of 99.3 percent for centrifugal pumps for the purpose of evaluating alternative pumping arrangements.

The impact of unscheduled pump shutdowns on a water distribution system depends on the system demand, the number of pumps and their arrangement, and the time to repair the failed pump. The potential impacts of pump failure include shortfalls in water supply and/or reduction in nodal pressures. Damelin, Shamir, and Arad [51] have concluded that for municipal water distribution systems, the economic value of shortfalls in supply cannot be determined as a function of their magnitude and time of occurrence. Therefore, based on the lack of adequate pump failure data, the difficulty in evaluating the economic impact of pump failures, the great variety of possible series and parallel pumping arrangements, and the

inherent uncertainty in the design of a new distribution system, standard guidelines [26] were consulted to determine the initial number of primary pumps for normal (peak hour) demand. The number and capacity of standby pumps will be determined by applying the basic fire demand loading with selected pump(s) out of service in accordance with standard fire insurance rating requirements [80]. Both the number of primary and standby pumps and their capacities can be varied parametrically to properly assess the appropriate tradeoff between cost and reliability.

The possibility of an electrical power outage for the distribution system heavily dependent on pumping demonstrates the need for standby pumping that uses an alternate power source such as gasoline or diesel fuel. The motors for these standby pumps are less efficient than the electrical motors normally used, thus reducing the overall efficiency of the pump-motor combination and increasing their costs.

5.3 Description of Mathematical Model

In order to fully describe the detailed design model we will formulate the mathematical model for a small example distribution design problem. The distribution system and the associated normal

and emergency loading conditions were selected to illustrate the full capability of the model. This section will conclude with a formal statement of the mathematical model.

5.3.1 Example Distribution System

The layout of the example distribution system is pictured in Figure 5-1.

5.3.1.1 Nodes

The system consists of 8 nodes, 6 demand nodes and 2 source nodes. The source at node 1 is an elevated storage reservoir and there is a pumping station at the source at node 8.

5.3.1.2 Links

The lengths of the 9 links are also given in Figure 5-1. Applying the shortest path tree model (Problem P3) with source capabilities and normal nodal demands as shown in Figure 5-2, the core tree for source node 1 and demand nodes 2, 3, 4, and 5 consists of links 1, 2, 3, and 4. For source node 8 and demand nodes 6 and 7 the core tree consists of links 6 and 9. Connecting the separate trees using link 5, the shortest link between the two trees, we have the core tree for the total system consisting of primary links 1, 2,

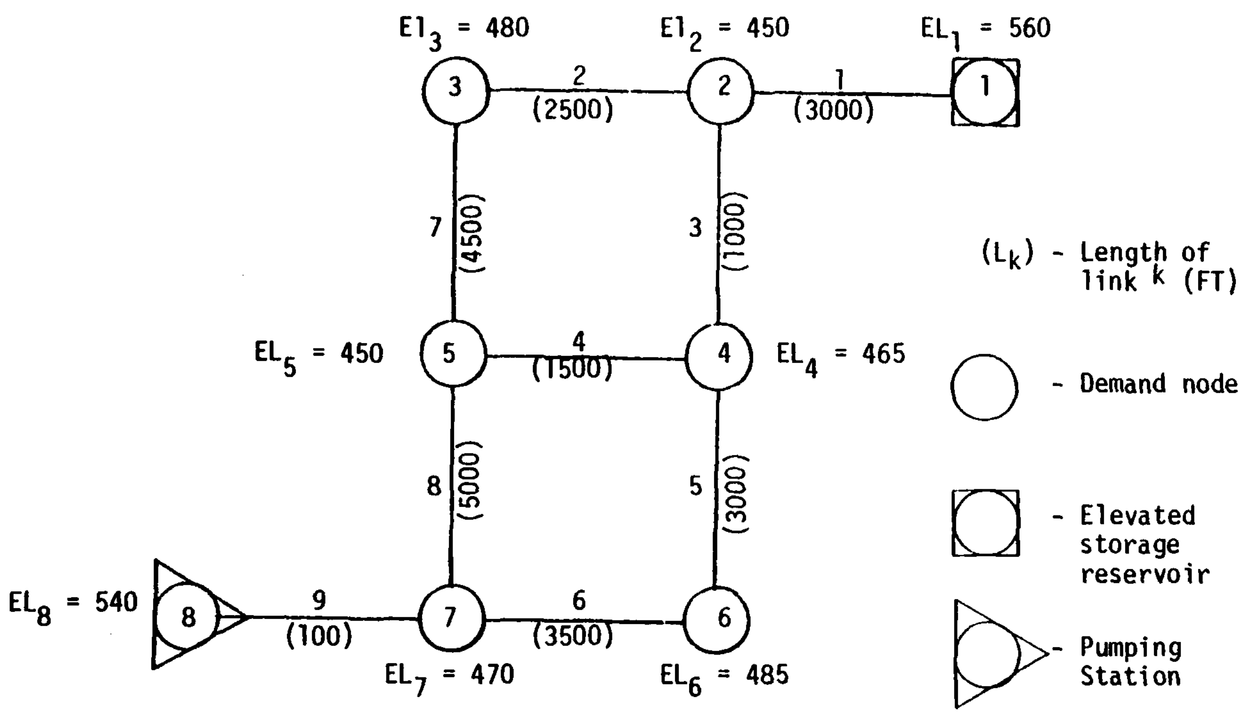


Figure 5-1

EXAMPLE DISTRIBUTION SYSTEM TOPOLOGY

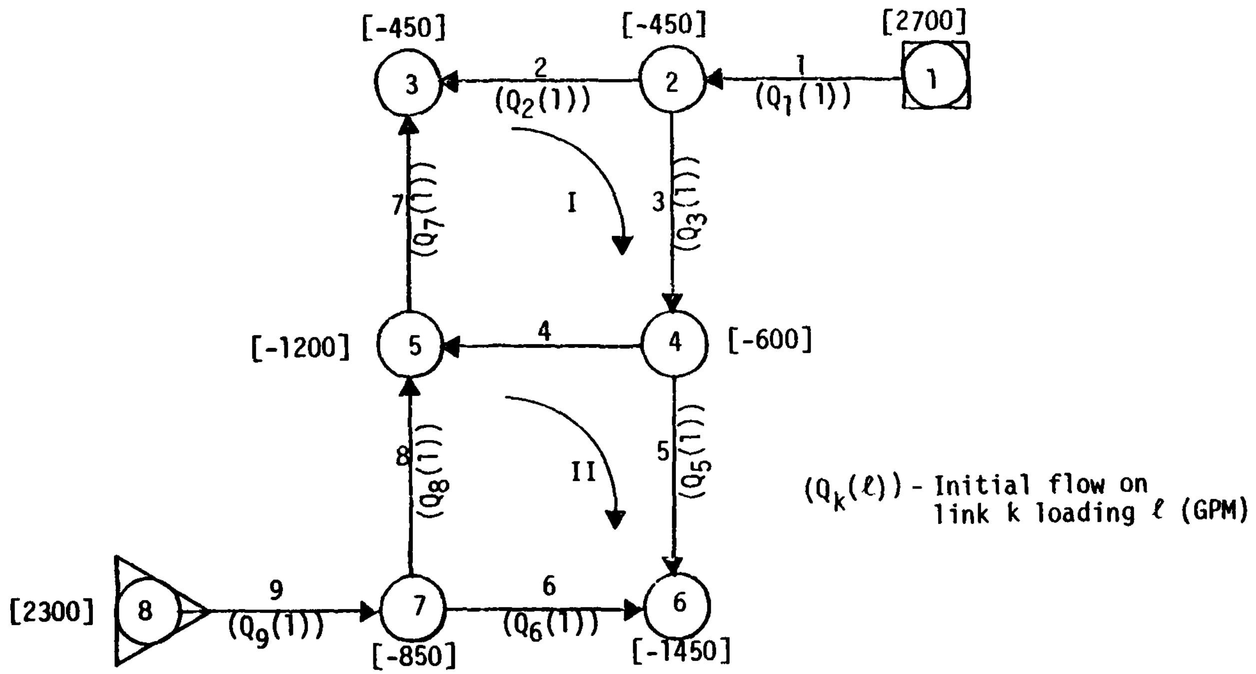


Figure 5-2

NORMAL LOADING CONDITION

3, 4, 5, 6, 9, and the redundant links 7 and 8. The same results are obtained using the nonlinear minimum cost flow model (Problem P5). Identification of the core tree, even in the case where the network layout is given, is very useful in selecting a good initial flow distribution for the normal loading condition for the solution algorithm, i.e., by concentrating the majority of flow in the primary links.

5.3.1.3 Pumps

Based on guidelines from Al-Layla et al. [26], a total of 4 pumps, 3 fixed speed pumps with identical flow and head lift capacities and a variable speed (flow) standby pump are used at node 8. All pumps are designed to operate in parallel with each other; thus, the total flow output of the pump station is the sum of the flows of each of the pumps and the pumps operate at a common head lift. Pumps operating in series add their head lifts and each pump has the same flow rate. The standby pump must be designed to be capable of replacing the normal pumps under normal loading condition and provide the additional flow requirements of the fire demand loading conditions. These two flow/head lift operating points can be used to develop the standby pump's operating

characteristic curve. A typical pump characteristic curve is shown in Figure 5-3.

5.3.1.4 Elevated Storage

The capacity of the elevated storage reservoir at node 1 has been designed to satisfy demand at its associated demand nodes and provide a certain amount of fire demand flow to assist in fighting fires at all demand nodes. The elevation at node 1 is the height of the water level in the reservoir which varies over the course of the day. The assumed elevation of node 1 for each of the normal and emergency loading conditions is based on the nature of the loading condition. For example, for the broken link loading condition a time weighted average value can be used. The maximum height that the storage can be elevated is 50 feet.

5.3.1.5 Loading Conditions

5.3.1.5.1 Normal

The peak hour demand loading, shown in Figure 5-2, is the single normal loading condition. There are several good references to assist the designer in estimating normal demand requirements [1,

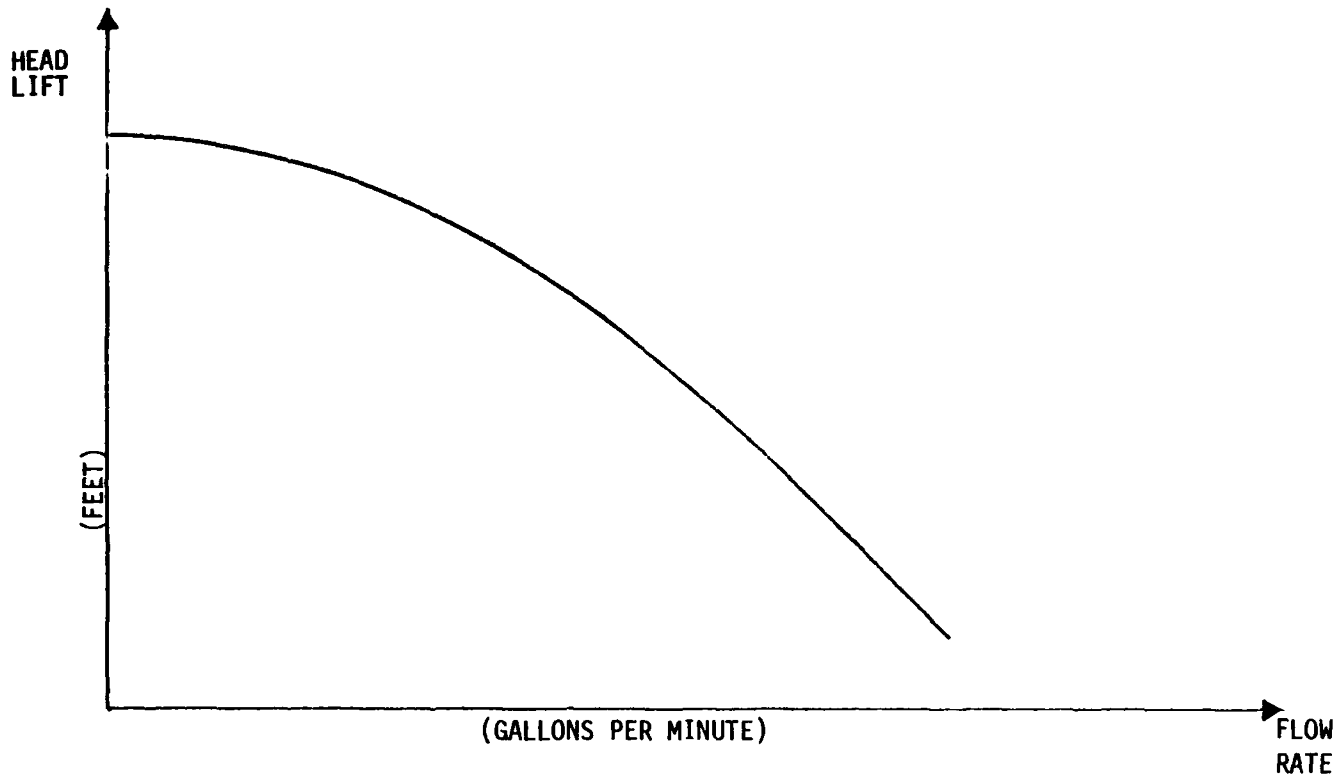


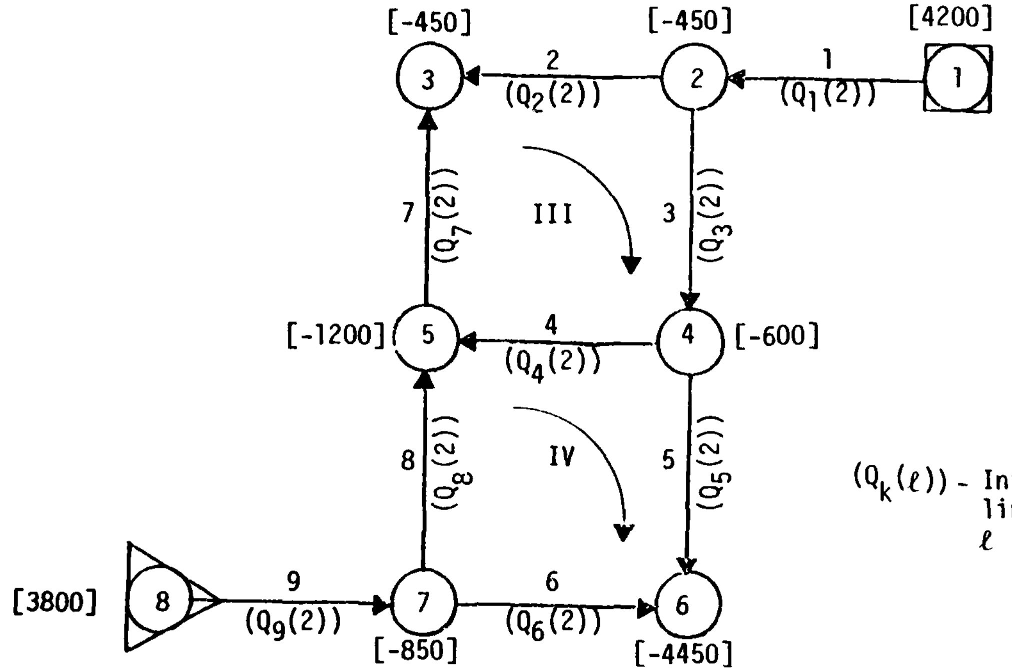
Figure 5-3

PUMP CHARACTERISTIC CURVE

24, 25, 26, 27, 28, 65]. The three parallel pumps are assumed to be operating at maximum flow/head lift capacity.

5.3.1.5.2 <u>Emergency</u>

The model formulation presented in the following sections is based on the single fire demand emergency loading condition shown in Figure 5-4. An additional emergency loading condition, failure of primary link 3, shown in Figure 5-5, will be added to the model during solution of the example problem in section 5.5.4. Using zoning maps and ISO guidelines [77], the required fire flow at each demand node can be estimated. A comparison of the severity of fire demand at each node taking into account the fire flow demands, the proximity of the node to a source, and the relative nodal elevation allows the system designer to select the appropriate fire demand loading condition(s) for the detailed design model. For a municipality the controlling fire demand requirement is usually located in the downtown district. Consistent with fire insurance guidelines [80], the fire flow requirements are added to the peak hourly demand loading and one of the normal pumps is assumed to be out of service. Thus, the variable speed standby pump must be capable of replacing the flow normally provided by the out-of-service pump and node 8's share of the 3000 GPM fire demand at node 6.



$(Q_k(\ell))$ - Initial flow on link k loading ℓ (GPM)

Figure 5-4

FIRE DEMAND LOADING CONDITION

3000 GPM FIRE DEMAND AT NODE 6

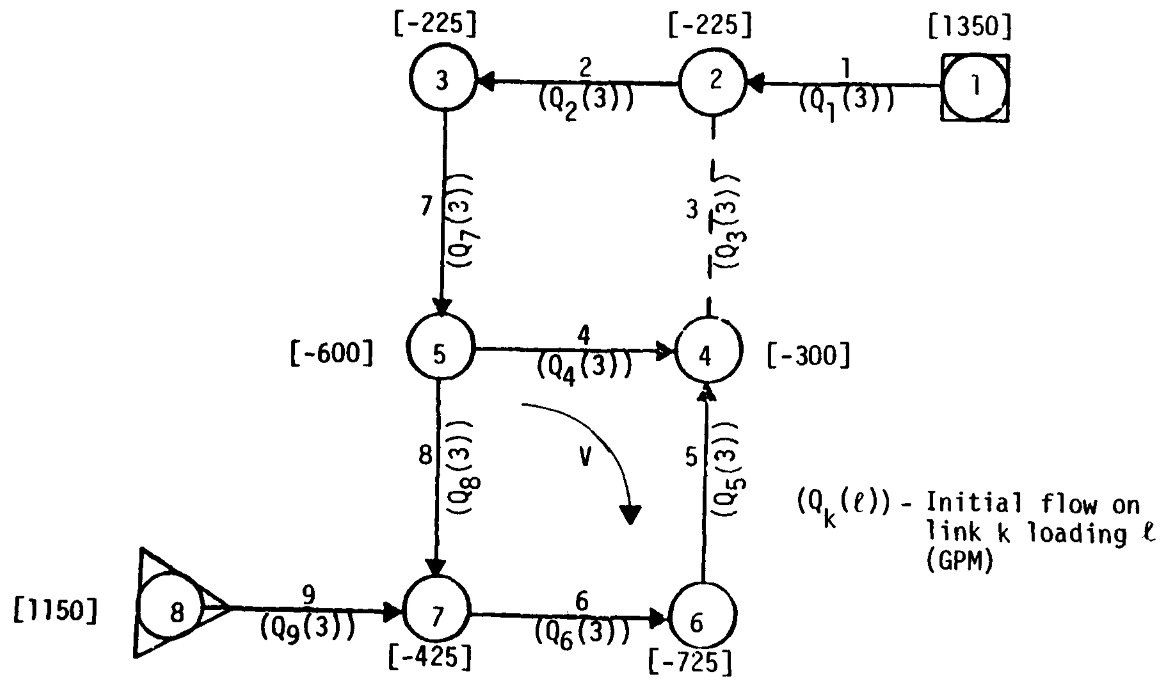


Figure 5-5

BROKEN PRIMARY LINK LOADING CONDITION

LINK 3 BROKEN

5.3.2 Constraints

In this section each type of constraint will be illustrated by deriving the corresponding constraint for the mathematical model of the example distribution system design problem shown in Table 5-1.

5.3.2.1 Normal Loading Pressure Constraints

Under normal loading conditions, the pressure or head at each demand node i must exceed a minimum level $HMIN_i$. Municipal [66] and state [65] regulations mandate this requirement. The minimum pressure level (usually 85-105 feet) is assumed to provide adequate water pressure to the individual consumer. Because of individual consumer needs, minimum pressure requirements may vary within the same system.

To define the head at each demand node, i.e., nodes 2-7, a head path constraint must be written starting at a node with a known head, i.e., nodes 1 or 8, describing the head losses and gains along the path of nodes and links to each demand node.

We know that the head loss on link k on loading ℓ is

$$\Delta HF_k(\ell) = \frac{K_k \, [Q_k(\ell)]^n \, L_k}{D_k^m} \qquad (5\text{-}1)$$

Table 5-1

MATHEMATICAL MODEL FOR EXAMPLE PROBLEM

XS_1	$XP_1(1)$	$XP_2(2)$	$XL_{1,16}$	$XL_{1,18}$	$XL_{2,6}$
-1			$K_{1,16}[Q_1(2)]^n$	$K_{1,18}[Q_1(2)]^n$	
-1			$K_{1,16}[Q_1(2)]^n$	$K_{1,18}[Q_1(2)]^n$	
-1			$K_{1,16}[Q_1(1)]^n$	$K_{1,18}[Q_1(1)]^n$	
1			$-K_{1,16}[Q_1(1)]^n$	$-K_{1,18}[Q_1(1)]^n$	
					$-K_{2,6}[Q_2(1)]^n$
					$-K_{2,6}[Q_2(2)]^n$
-1	1		$K_{1,16}[Q_1(1)]^n$	$K_{1,18}[Q_1(1)]^n$	
-1		1	$K_{1,16}[Q_1(2)]^n$	$K_{1,18}[Q_1(2)]^n$	
			1	1	
					1
1					
	-1	1			
STC_1	$PU[QP_1(1), XP_1(1)]$	$PU[QP_2(2), XP_2(2)]$	$CL_{1,16}$	$CL_{1,18}$	$CL_{2,6}$

Table 5-1 continued

$XL_{2,3}$	$XL_{3,14}$	$XL_{3,16}$	$XL_{4,10}$	$XL_{4,12}$	$XL_{5,6}$
	$K_{3,14}[Q_3(2)]^n$	$K_{3,16}[Q_3(2)]^n$			
	$K_{3,14}[Q_3(2)]^n$	$K_{3,16}[Q_3(2)]^n$			$K_{5,6}[Q_5(2)]^n$
	$K_{3,14}[Q_3(1)]^n$	$K_{3,16}[Q_3(1)]^n$			
	$-K_{3,14}[Q_3(1)]^n$	$-K_{3,16}[Q_3(1)]^n$			$-K_{5,5}[Q_5(1)]^n$
$-K_{2,8}[Q_2(1)]^n$	$K_{3,14}[Q_3(1)]^n$	$K_{3,16}[Q_3(1)]^n$	$K_{4,10}[Q_4(1)]^n$	$K_{4,12}[Q_4(1)]^n$	
			$-K_{4,10}[Q_4(1)]^n$	$-K_{4,12}[Q_4(1)]^n$	$K_{5,6}[Q_5(1)]^n$
$-K_{2,8}[Q_2(2)]^n$	$K_{3,14}[Q_3(2)]^n$	$K_{3,16}[Q_3(2)]^n$	$K_{4,10}[Q_4(2)]^n$	$K_{4,12}[Q_4(2)]^n$	
			$-K_{4,10}[Q_4(2)]^n$	$-K_{4,12}[Q_4(2)]^n$	$K_{5,6}[Q_5(2)]^n$
	$K_{3,14}[Q_3(1)]^n$	$K_{3,16}[Q_3(1)]^n$	$K_{4,10}[Q_4(1)]^n$	$K_{4,12}[Q_4(1)]^n$	
	$K_{3,14}[Q_3(2)]^n$	$K_{3,16}[Q_3(2)]^n$	$K_{4,10}[Q_4(2)]^n$	$K_{4,12}[Q_4(2)]^n$	
1					
	1	1			
			1	1	
					1
$CL_{2,8}$	$CL_{3,14}$	$CL_{3,16}$	$CL_{4,10}$	$CL_{4,12}$	$CL_{5,6}$

Table 5-1 continued

$XL_{5,8}$	$XL_{6,12}$	$XL_{6,14}$	$XL_{7,6}$	$XL_{7,8}$	$XL_{8,6}$
$K_{5,8}[Q_5(2)]^n$					
$-K_{5,8}[Q_5(1)]^n$					
			$K_{7,6}[Q_7(1)]^n$	$K_{7,8}[Q_7(1)]^n$	
$K_{5,8}[Q_5(1)]^n$	$-K_{6,12}[Q_5(1)]^n$	$-K_{6,14}[Q_6(1)]^n$			$K_{8,6}[Q_8(1)]^n$
			$K_{7,6}[Q_7(2)]^n$	$K_{7,8}[Q_7(2)]^n$	
$K_{5,8}[Q_5(2)]^n$	$-K_{6,12}[Q_6(2)]^n$	$-K_{6,14}[Q_6(2)]^n$			$K_{8,6}[Q_8(2)]^n$
					$-K_{8,6}[Q_8(1)]^n$
					$-K_{8,6}[Q_8(2)]^n$
1					
	1	1			
			1	1	
					1
$CL_{5,8}$	$CL_{6,12}$	$CL_{6,14}$	$CL_{7,6}$	$CL_{7,8}$	$CL_{8,6}$

Table 5-1 continued

$XL_{8,8}$	$XL_{9,16}$	$XL_{9,18}$	Z			CONSTRAINT
			1	$\leq$	95	(1)
			1	$\leq$	75	(2)
				$\leq$	5	(3)
				$\leq$	15	(4)
				=	0	(5)
$K_{8,8}[Q_8(1)]^n$				=	0	(6)
				=	0	(7)
$K_{8,8}[Q_8(2)]^n$				=	0	(8)
$-K_{8,8}[Q_8(1)]^n$	$-K_{9,16}[Q_9(1)]^n$	$-K_{9,18}[Q_9(1)]^n$		=	20	(9)
$-K_{8,8}[Q_8(2)]^n$	$-K_{9,16}[Q_9(2)]^n$	$-K_{9,18}[Q_9(2)]^n$		=	20	(10)
				=	1000	(11)
				=	2500	(12)
				=	1000	(13)
				=	1500	(14)
				=	3000	(15)
				=	3500	(16)
				=	4500	(17)
1				=	5000	(18)
	1	1		=	100	(19)
				$\leq$	50	(20)
				$\leq$	0	(21)
$CL_{8,8}$	$CL_{9,16}$	$CL_{9,18}$		$\leq$	BMAX	(22)

where link k has a single diameter D_k .

Head gains are provided by elevated reservoirs and pumps. The additional head XS_k provided by elevated reservoir k at a source node represents the height added by the structure supporting the reservoir. Likewise, $XP_k(\ell)$ is the head lift added by pump k on loading ℓ . The resulting combination of flows and head lifts of a pump over all loading conditions can be used to define the pump's desired characteristic curve.

Thus, from Figure 5-2 the head at node 4 under the normal loading (loading 1) is

$$\begin{aligned} H_4(1) &= H_1(1) - EL_4 - \Delta HF_1(1) - \Delta HF_3(1) \\ &= EL_1 + XS_1 - EL_4 - \Delta HF_1(1) - \Delta HF_3(1) \qquad (5\text{-}2) \\ &= (EL_1 - EL_4) + XS_1 - \frac{K_1\,[Q_1(1)]^n L_1}{D_1^m} - \frac{K_3\,[Q_3(1)]^n L_3}{D_3^m} \end{aligned}$$

The quantity $EL_1 - EL_4$ is the potential energy of water at node 4 referenced to node 1.

The head at node 4 could instead be referenced to node 8 as follows:

$$H_4(1) = (EL_8 - EL_4) + XP_1(1) - \frac{K_9\,[Q_9(1)]^n L_9}{D_9^m}$$

$$- \frac{K_6\,[Q_6(1)]^n L_6}{D_6^m} + \frac{K_5[Q_5(1)]^n L_5}{D_5^m} \tag{5-3}$$

where $XP_1(1)$ is the common head provided by the three parallel pumps at the pump station. Since the three identical pumps are operating in parallel, each pump provides one-third of the total flow capacity at the same head lift.

Instead of each link having a pipe of only a single diameter, define S_k as the set of candidate diameters that segments of link k may assume. Standard adaptors can be used to connect pipes of different diameters. For example, for link 3, segments of pipe with 14 or 16 inch diameter may be combined to make up its 1000 foot length.

Let XL_{kj} be the length of pipe of diameter $j \varepsilon S_k$ to place on link k. S_k is a subset of the commercially available pipe diameters. S_k may be restricted to satisfy the minimum diameter requirements for broken link emergency loading conditions, statutory regulations [65], and minimum and maximum normal hydraulic gradient (velocity) limits on normal loading link flow. Furthermore, due to

computational considerations the specific link diameters from S_k used in the model at any instant may be limited and changed as necessary to find an improved solution.

The head loss on a link with segments of different diameters is the sum of the head losses on each of the separate segments of the link. Thus, the head loss on link 3 for loading 1 is

$$\Delta HF_3(1) = \frac{K_3[Q_3(1)]^n L_3}{D_3^m} = \frac{K_3[Q_3(1)]^n XL_{3,14}}{(14)^m} + \frac{K_3[Q_3(1)]^n XL_{3,16}}{(16)^m} \tag{5-4}$$

where

$$XL_{3,14} + XL_{3,16} = L_3 = 1000$$

D_3 can be considered to be the diameter of a single equivalent pipe 1000 feet long that would provide the same frictional loss as the segments of the set of candidate diameters.

To simplify notation let

$$K_{kj} = \frac{10.471}{(HW_k)^n (D_{kj})^m} = \frac{K_k}{(D_{kj})^m} \tag{5-5}$$

computational considerations the specific link diameters from S_k used in the model at any instant may be limited and changed as necessary to find an improved solution.

The head loss on a link with segments of different diameters is the sum of the head losses on each of the separate segments of the link. Thus, the head loss on link 3 for loading 1 is

$$\Delta HF_3(1) = \frac{K_3[Q_3(1)]^n L_3}{D_3^m} = \frac{K_3[Q_3(1)]^n XL_{3,14}}{(14)^m} + \frac{K_3[Q_3(1)]^n XL_{3,16}}{(16)^m} \tag{5-4}$$

where

$$XL_{3,14} + XL_{3,16} = L_3 = 1000$$

D_3 can be considered to be the diameter of a single equivalent pipe 1000 feet long that would provide the same frictional loss as the segments of the set of candidate diameters.

To simplify notation let

$$K_{kj} = \frac{10.471}{(HW_k)^n (D_{kj})^m} = \frac{K_k}{(D_{kj})^m} \tag{5-5}$$

where D_{kj} is a diameter from the candidate set S_k. For notational purposes we will let $j = D_{kj}$. The head loss on link 3 on loading 1 is now written

$$\Delta HF_3(1) = K_{3,14}[Q_3(1)]^n XL_{3,14} + K_{3,16}[Q_3(1)]^n XL_{3,16} \quad (5\text{-}6)$$

where the quantity $K_{3j}[Q_3(1)]^n$ is the hydraulic gradient. Letting $HMIN_i(1) = 90$ feet for all demand nodes, we have for node 4

$$\begin{aligned} H_4(1) = (EL_1 - EL_4) + XS_1 - K_{1,16}[Q_1(1)]^n XL_{1,16} \\ - K_{1,18}[Q_1(1)]^n XL_{1,18} - K_{3,14}[Q_3(1)]^n XL_{3,14} \quad (5\text{-}7) \\ - K_{3,16}[Q_3(1)]^n XL_{3,16} \geq HMIN_4(1) \end{aligned}$$

Substituting for constants, multiplying both sides by -1, and moving the constants to the right hand side we have

$$\begin{aligned} -XS_1 + K_{1,16}[Q_1(1)]^n XL_{1,16} + K_{1,18}[Q_1(1)]^n XL_{1,18} \\ + K_{3,14}[Q_3(1)]^n XL_{3,14} + K_{3,16}[Q_3(1)]^n XL_{3,16} \leq 5 \quad (5\text{-}8) \end{aligned}$$

Inequality (5-8) corresponds to constraint (3) of Table 5-1. To

illustrate the structure of the model in an economical manner only two candidate diameters are shown for each link and only 2 of the 6 possible minimum head constraints (nodes 4 and 6) for the normal loading condition (inequalities (3)-(4)) are shown in Table 5-1.

Head constraints for the emergency loading are constructed in a similar manner to those for the normal loading. However, instead of serving as a constraint for defining the feasible region, these constraints are used to define the objective function. Constraints (1) - (2) of Table 5-1 are the head constraints for loading 2 and will be discussed at length in section 5.3.3.

5.3.2.2 Loop/Source Constraints

For the steady state conditions three requirements must be satisfied:

1. The sum of flows entering a node must equal the sum of flows leaving a node.
2. The sum of frictional head losses around any closed loop must equal zero.
3. The sum of the head losses between any two fixed head nodes, e.g., reservoirs or other sources, must equal the difference between the fixed heads at these nodes.

Condition 1, nodal conservation of flow, is satisfied in the model by the user selecting an initial link flow distribution that satisfies this requirement. Subsequent flow changes are made so as to maintain the initial conservation of flow.

Condition 2, conservation of energy around a loop, is satisfied by writing loop equations for each independent loop in the network. Loop equations are written in the same manner as head path constraints except that the starting and ending nodes are the same. Head changes due to booster pumps or elevated reservoirs located along the loop path are ignored.

For the example distribution system there are four loop equations--two for each loading condition. The loops and their initial flows are shown in Figures 5-2 and 5-4. The clockwise arrows indicate the positive flow direction. The loop equation for the normal loading loop I is

$$\begin{aligned} &-K_{2,6}[Q_2(1)]^n XL_{2,6} - K_{2,8}[Q_2(1)]^n XL_{2,8} + K_{3,14}[Q_3(1)]^n XL_{3,14} \\ &+ K_{3,16}[Q_3(1)]^n XL_{3,16} + K_{4,10}[Q_4(1)]^n XL_{4,10} \qquad (5\text{-}9) \\ &+ K_{4,12}[Q_4(1)]^n XL_{4,12} + K_{7,6}[Q_7(1)]^n XL_{7,6} \\ &+ K_{7,8}[Q_7(1)]^n XL_{7,8} = 0 \end{aligned}$$

The loop equations for the example problem are constraints (5) - (8) in Table 5-1.

Condition 3 represents the physical requirement that external energy added to the system (potential energy due to elevation and pressure energy from pumps) is conserved. The source equations establish a common reference point among all fixed head nodes allowing nodal head constraints to be written starting at any fixed head node in the network. Since there are two source nodes, source equations have been written--one for each loading. The source equation for the normal loading condition is

$$
\begin{aligned}
& - XS_1 + XP_1(1) + K_{1,16}[Q_1(1)]^n XL_{1,12} + K_{1,18}[Q_1(1)]^n XL_{1,18} \\
& + K_{3,14}[Q_3(1)]^n XL_{3,14} + K_{3,16}[Q_3(1)]^n XL_{3,16} \\
& + K_{4,10}[Q_4(1)]^n XL_{4,10} + K_{4,12}[Q_4(1)]^n XL_{4,12} \qquad (5\text{-}10) \\
& - K_{8,6}[Q_8(1)]^n XL_{8,6} - K_{8,8}[Q_8(1)]^n XL_{8,8} \\
& - K_{9,16}[Q_9(1)]^n XL_{9,16} - K_{9,18}[Q_9(1)]^n XL_{9,18} = EL_1 - EL_8 = 20
\end{aligned}
$$

Constraints (9) -(10) of Table 5-1 are the source path equations for both loadings.

5.3.2.3 Length Constraints

For each link a length constraint of the form

$$\sum_{j \varepsilon S_k} XL_{kj} = L_k \qquad (5\text{-}11)$$

$$k = 1, \ldots, NLINK$$

must be written to insure that each link is fully defined. Constraints (11) - (19) of Table 5-1 are the length constraints.

5.3.2.4 Storage Height Constraints

By increasing the height of elevated storage, the head at each node in the system on all loadings is increased by the elevation of the structure XS_k. Depending upon the size of the storage reservoir, the topography of the area, and safety considerations, it may not be possible or desirable to build a supporting structure for the reservoir above a certain height. Also, elevating a balancing storage reservoir too high may hinder its filling during periods of low demand. Thus, a constraint of the form

$$XS_k \leq SHMAX_k \qquad (5\text{-}12)$$

must be included in the model where XS_k is the number of feet to add to elevated storage k and $SHMAX_k$ is the storage height

limitation. Constraint (20) of Table 5-1 is the 50-foot height limit on the elevated storage at node 1.

5.3.2.5 Pump Capacity Constraint

Likewise, there may be limitations on the capacity of a pump due to

1. Capacity of an existing pump
2. Limitation on the capacity of available pumps
3. Pump operating level constraints arising from
   a. Operation of the same pump on different loadings
   b. Operation of pumps in parallel

The first two types of constraints involve comparison of the pump capacity against a known upper or lower bound. These constraints may be written in terms of either a head or a horsepower limit as follows:

$$PHMIN_k \leq XP_k(\ell) \leq PHMAX_k \tag{5-13}$$

$$HPMIN_k \leq \frac{\gamma\ QP_k(\ell)\ XP_k(\ell)}{550\ \eta_k} \leq HPMAX_k \tag{5-14}$$

where

$PHMIN_k$--the minimum head for pump k

$PHMAX_k$--the maximum head for pump k

$HPMIN_k$--the minimum horsepower for pump k

$HPMAX_k$--the maximum horsepower for pump k

$QP_k(\ell)$--the flow rate through pump k under loading ℓ

η_k--the combined pump-motor efficiency of pump k

γ--the specific weight of water at the known temperature.

Constraint type 3.a arises from the need to establish pump capacity limits which may be used to properly assess the cost of a pump which operates on more than one loading condition. The cost of a pump is a function of its maximum flow rate and head lift [45]. Although a pump may operate on multiple loading conditions, each pump can be associated with a particular loading condition, its critical loading condition, for which the pump is being primarily designed to operate. For example, the set of three parallel pumps in the example problem are principally designed for efficient, economical operation during the normal loading condition. On the other hand, the critical loading condition for the variable speed standby pump is the fire demand loading condition. The flow rate and head on the critical loading condition determine both its cost and the

capacity limits for its operation on other non-critical loading conditions. The general form of constraint type 3.a is

$$XP_k(\ell) \leqq XP_k\left({}^{\ell}c_k\right) \tag{5-15}$$

$$\frac{\gamma QP_k(\ell)XP_k(\ell)}{550\ \eta_k} \leqq \frac{\gamma QP_k\left({}^{\ell}c_k\right)XP_k\left({}^{\ell}c_k\right)}{550\ \eta_k} \tag{5-16}$$

where ${}^{\ell}c_k$ is pump k's critical loading condition and loading ℓ is any other loading for which the pump operates. In the example problem the set of normal parallel pumps operates on both loading conditions with loading 1 as the critical loading. Since parallel pumps operate at the same head and the pumps are operating at the same maximum flow capacity on both loadings, the constraint

$$XP_1(2) \leqq XP_1(1) \tag{5-17}$$

applies.

Constraint 3.b arises from the requirement that pumps operating in parallel must work at a common head lift. Thus, for the standby pump, pump 2, operating in parallel with the two remaining normal pumps we have

$$XP_2(2) = XP_1(2) \tag{5-18}$$

However, since pump 1 is already costed out in loading 1, and both pump 1 and pump 2 (which is costed out on loading 2) deliver the same nonadditive head on loading 2, constraints (5-17) and (5-18) may be replaced by

$$XP_2(2) \leqq XP_1(1) \qquad (5\text{-}19)$$

which corresponds to constraint (21) of Table 5-1.

5.3.2.6 Budget Constraint

This section examines the individual cost components of the budget constraint (constraint (22) of Table 5-1) some of which have been introduced in Chapters 3 and 4 and briefly addresses some considerations in selecting the maximum budget level. However, discussion of an analytical method for selecting BMAX, the maximum budget limit, had been deferred until section 5.4.2 after development of the necessary analytical tools.

There are two major classes of costs associated with water distribution systems--capital and operating costs. The distinction between capital and operating costs is important because of the different method of calculating and financing each cost.

5.3.2.6.1 Capital Costs

Capital costs are the investment costs of the water distribution system. Capital costs represent the complete cost of acquiring and installing links, pumps, and elevated structures for storage reservoirs. Because of the high initial capital costs of either installing a new water distribution system or making a major expansion to an existing water distribution system, municipalities generally finance the capital costs by issuing bonds. Although the face value of the bonds may represent the total capital costs of the distribution system, because of the time value of money, the capital costs must be converted using present value analysis to a stream of equivalent uniform annual costs to allow capital costs to be be combined with annual operating costs.

5.3.2.6.1.1 Pipe Capital Cost

The expression for the capital cost per foot of pipe

$$\ell_1 \, (D_{kj})^{\ell_2} \qquad (5\text{-}20)$$

was covered in section 3.2.2.1. The graph of this convex function for $\ell_1 = 1.01$ and $\ell_2 = 1.29$ is shown in Figure 5-6. This expression assumes a cast-iron pipe of a specific tensile strength

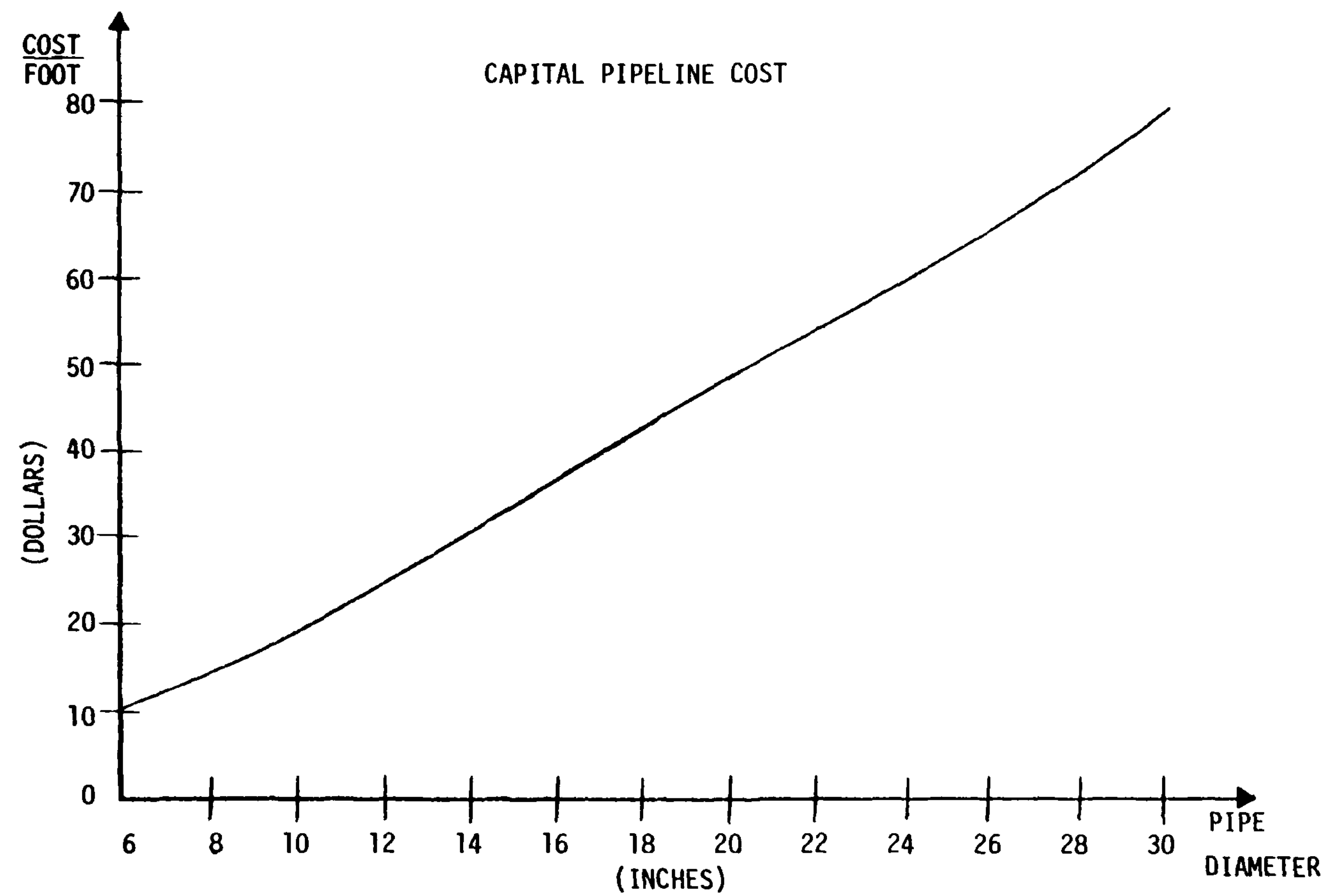

COST
FOOT
(DOLLARS)
CAPITAL PIPELINE COST
80
70
60
50
40
30
20
10
0
6
8
10
12
14
16
18
20
22
24
26
28
30
PIPE
DIAMETER
(INCHES)

Figure 5-6

(pressure class). Certain links in the system may require pipes in a higher pressure class due to unusual pressure conditions.

5.3.2.6.1.2 Pump Capital Cost

The capital cost of installed pump k in dollars is [48]

$$\ell_4 \, [QP_k(\ell_{c_k})]^{\ell_5} \; [XP_k(\ell_{c_k})]^{\ell_6} \tag{5-21}$$

where ℓ_4, ℓ_5, and ℓ_6 are constants. Per section 5.3.2.5 ℓ_{c_k} is the loading condition for which pump k is principally designed to operate. The graph of this concave function for a fixed flow of 1500 GPM and $\ell_4 = 16.14$, $\ell_5 = .453$ and $\ell_6 = .642$ (1976 prices) is shown in Figure 5-7. For identical pumps operating in parallel each pump shares an equal part of the total flow rate on the link and has the same operating head. Thus, for pump k composed of $NPPUMP_k$ parallel pumps the total capital cost is

$$NPPUMP_k \cdot \ell_4 \left[\frac{QP_k(\ell_{c_k})}{NPPUMP_k}\right]^{\ell_5} \left[XP_k\,(\ell_{c_k})\right]^{\ell_6} \tag{5-22}$$

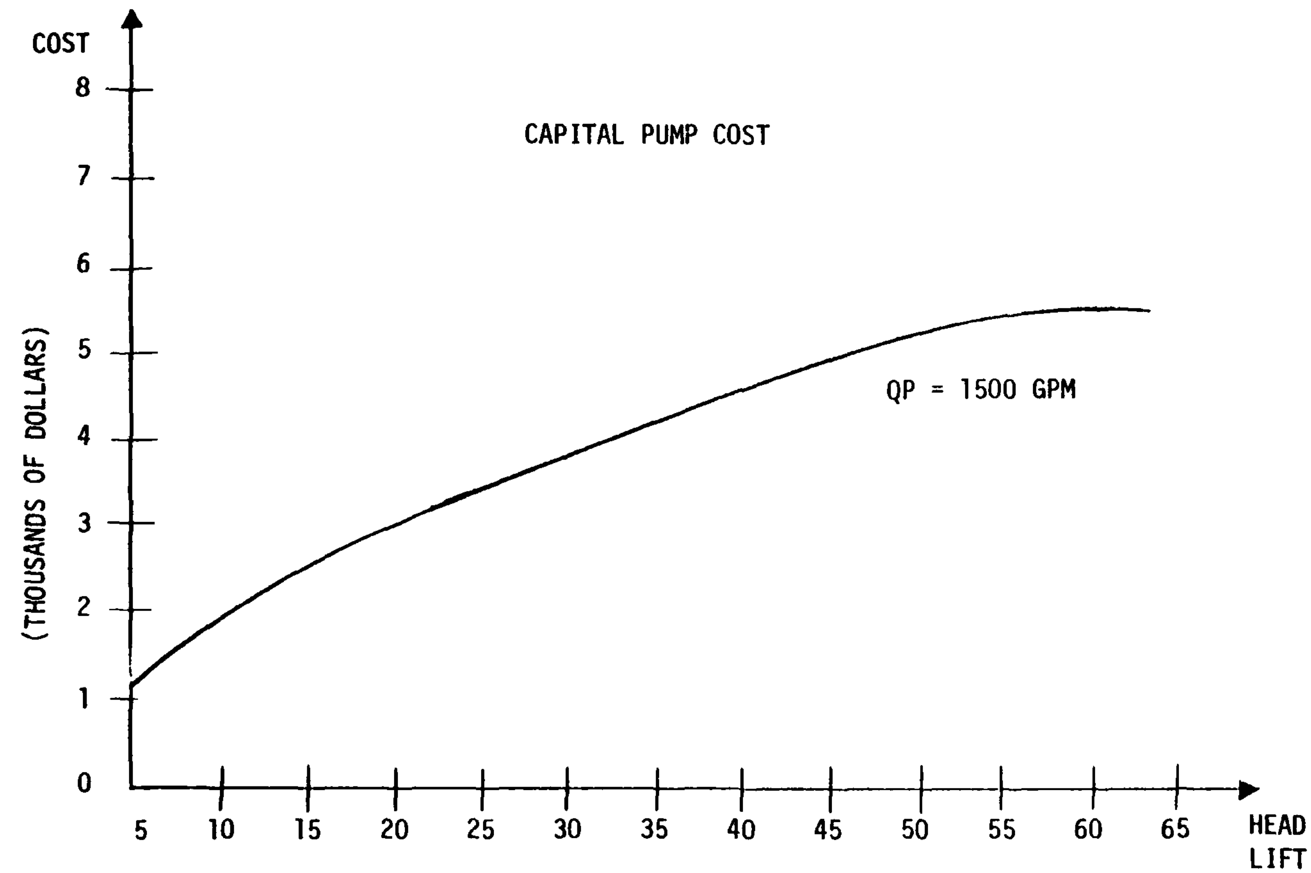

CAPITAL PUMP COST
COST
(THOUSANDS OF DOLLARS)
0
1
2
3
4
5
6
7
8
QP = 1500 GPM
5
10
15
20
25
30
35
40
45
50
55
60
65
HEAD
LIFT

Figure 5-7

5.3.2.6.1.3 Storage Height Capital Cost

Although the total capital cost of an elevated storage reservoir depends on its capacity, type of design, and elevation, since the reservoir design is fixed in our model, we are concerned only with the cost of building a structure to elevate the reservoir. From section 3.2.2.1 we have that the cost of elevating the reservoir is directly proportional to its height [46], i.e.,

$$STC_k \quad XS_k \tag{5-23}$$

5.3.2.6.2 Annualizing Capital Costs

Before discussing the operating cost, we will discuss a method for converting capital costs to equivalent uniform annual costs (EUAC) which can then be combined directly with annual operating costs [56]. Assuming that capital costs are to be repaid in equal annual installments over the useful life of the capital equipment (NYEAR) at an interest rate of I with SV as the ratio of the initial value of the investment to its salvage value, the annual capital recovery factor is

$$CRF = \left(\frac{I\ (1+I)^{NYEAR}}{(1+I)^{NYEAR}-1} \right) (1-SV) + I(SV) \tag{5-24}$$

The values of NYEAR used in the model are 30 years for pipe and reservoir capital costs and 15 years for pump capital costs [48]. An interest rate of .06 and salvage ratio of .1 [48] were used for all capital equipment. The appropriate CRF value multiplies the pipe, reservoir, and pump capital costs derived in the previous sections to form the capital cost component of the budget constraint.

5.3.2.6.3 Operating Costs

Operating costs are associated with running and maintaining the water distribution system. Unlike capital costs, operating costs are incurred continuously during the lifetime of the system. Thus, operating costs can be computed on an annual basis and directly combined with the annualized capital cost to arrive at the total equivalent uniform annual cost.

5.3.2.6.3.1 Pipeline Operating Cost

The efficient operation of water distribution system pipelines requires periodic maintenance and inspection. The annual cost of this operation is proportional to the diameter and the length of the pipe. At 1976 price levels, the proportionality factor is $4/in of diameter/mile/year [61].

5.3.2.6.3.2 Pump Operating Cost

5.3.2.6.3.2.1 Energy Cost

The energy required to operate a pump is directly proportional to its maximum horsepower and is given by [24]

$$E = \frac{\gamma QP_k(\ell_{c_k})\ XP_k(\ell_{c_k})}{737.6\ \eta_k} \tag{5-25}$$

$$= .746\ HP_k \tag{5-26}$$

where E is in kilowatt-hours and HP_k is the maximum horsepower of pump k. As noted above, only energy associated with normal operation is included. The annual pumping cost in dollars is

$$\begin{aligned} &24 \cdot 365 \cdot U \cdot C_E \cdot .746 \cdot HP_k \\ &= 6535 \cdot U \cdot C_E \cdot HP_k \end{aligned} \tag{5-27}$$

where

C_E--the electricity cost per kilowatt-hour in dollars

U--the utilization or load factor for the pump

In the model $C_E = \$.04$. The utilization factor takes into account the fact that the peak pumping rate is not pumped 24 hours a day. For residential demand U ranges from .097 to .26 [81].

5.3.2.6.3.2.2 Maintenance Cost

The general maintenance cost for a pump station is directly proportional to its maximum horsepower. A cost of $4/horsepower in 1976 prices was used [61].

5.3.2.6.4 Budget Level Selection

A major consideration in selecting the maximum budget level is the ability of the municipality to finance the system. Municipalities usually issue bonds to cover the capital costs of the system. The budget level may depend on the financial rating of the municipality, its borrowing capacity, and most importantly on the willingness of voters and/or officeholders to approve costly bond issues. Because of budget limitations certain performance/reliability features such as loops may have to be delayed until additional funds are available. A method for selecting the range of budget levels, which takes into account the expected emergency loading conditions, will be discussed in section 5.4.2.

5.3.3 Objective Function

5.3.3.1 Selection

The purpose of the model's objective function is to measure the performance of the distribution system under emergency loading conditions. As previously discussed, the principal physical impacts of the emergency loading conditions are deficiencies in the required flow rates and nodal pressures. Providing adequate flow rates for the expected duration of the emergency loading condition has been taken into account by setting minimum diameters for redundant and selected primary links, acquiring sufficient standby pumping flow capacity, and properly sizing the storage capacity of reservoirs. Thus, consistent with de Neufville et al.'s pioneering work [50], a function of the heads at the demand nodes will be used to measure system performance under emergency loading conditions.

Three functions of nodal heads were considered for the objective function:

1. Maximize a weighted sum of the nodal heads throughout all emergency loading conditions (MAXWNODE).
2. Maximize the minimum nodal head over all emergency loading conditions (MAXMIN).

3. Maximize a weighted sum of each emergency loading condition's minimum nodal head (MAXWMIN).

de Neufville et al. [50] used the MAXWNODE as a measure of performance to manually evaluate alternative network configurations under expected emergency loading conditions. The weight for each nodal head was based on the ratio of each node's demand to total system demand. However, this author's own results using the MAXWNODE objective function in the optimization algorithm for small problems proved unsatisfactory; some nodes had extremely high heads while others had extremely low heads.

Noting this inherent inadequacy in their measure of performance, de Neufville et al. [50] also suggested the need for a distributional measure of performance. They used the nodal head at the extreme end of the supply network which in their case would inevitably be the lowest.

This led us to the MAXMIN objective function which focuses on maximizing the minimum head over all emergency loading conditions. A similar criterion is often applied in decision theory [83] and game theory [84], i.e., the minimax criterion--minimize the maximum loss--and represents a very conservative strategy.

The MAXWMIN objective function incorporates the good points while avoiding the weaknesses of the MAXWNODE and MAXMIN objective functions. MAXWMIN avoids the difficult task of weighting individual nodes and the uneven results of MAXWNODE, but still allows the user the flexibility to weigh each emergency loading condition based on the importance or likelihood of its occurrence. The MAXWMIN is less conservative than the MAXMIN objective function where performance on a single emergency loading condition can control the optimization. Furthermore, MAXWMIN is more realistic than the MAXMIN since MAXWMIN focuses on each emergency loading condition individually rather than the minimal head over all nodes over all emergency loadings. Except perhaps in a disaster situation, rarely are distribution systems simultaneously exposed to several emergency loading conditions.

5.3.3.2 Implementation

Although the concept of the MAXWMIN objective function may appear complex, its formulation as a mathematical program is fairly simple. In compact form the mathematical program may be written

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UNCLASSIFIED AFIT-CI-79-234D NL

PROBLEM P8

$$\underset{\hat{X} \varepsilon F}{\text{Maximize}} \left[\sum_{\ell \varepsilon LE} w_\ell \underset{i \varepsilon DNODE}{\text{Minimum}} \left\{ H_i(\ell) \right\} \right]$$

where

F--the feasible region defined by the constraints of section 5.3.2

$\hat{X}$--the vector of all decision variables

LE--the set of emergency loading conditions

DNODE--the set of demand nodes

$H_i(\ell)$--the head at node i under emergency loading condition ℓ (a function of $\hat{X}$)

w_ℓ--the weight assigned to emergency loading ℓ

Let us consider the case where there is a single emergency loading condition. Problem P8 above simplifies to

PROBLEM P9

$$\underset{\hat{X} \varepsilon F}{\text{Maximize}} \left[\underset{i \varepsilon DNODE}{\text{Minimum}} \left\{ H_i \right\} \right]$$

where H_i is the head at demand node i . Problem P9 involves maximizing the minimum of a finite number of functions over a common

domain and is called the Chebyshev problem. The Chebyshev problem is a common one arising in mathematical contexts, game theory, and statistical analysis and has been examined by several researchers including Minieka [85], Sobel [86], Wagner [87], Zangwill [88], and Blau [89]. Thus, Problem P8 could be classified as a weighted Chebyshev problem.

Let z be the value of the objective function. Problem P9 can be written in the following equivalent form:

PROBLEM P10

$$\underset{\hat{X} \in F}{\text{Maximize}} \quad z$$

subject

$$z \leq H_i(1) \quad i \in \text{DNODE}$$

Let z_ℓ be the minimum head on emergency loading condition ℓ. Then, Problem P8 can be written

PROBLEM P11

$$\underset{\hat{X} \varepsilon F}{\text{Maximize}} \quad \sum_{\ell \varepsilon LE} w_\ell \; z_\ell$$

$$z_\ell \leqq H_i(\ell) \quad \begin{array}{l} i \varepsilon \text{DNODE} \\ \ell \varepsilon \text{LE} \end{array}$$

The minimum nodal head on each emergency loading condition serves as a ceiling for the objective function component z_ℓ.

Thus, for the example problem the objective function constraint for node 4 on the fire demand emergency loading condition (number 2) can be written as

$$\begin{aligned} z \leqq H_4(2) \;\; = \;\; & EL_1-EL_4+XS_1-K_{1,16}[Q_1(2)]^n XL_{1,16} \\ & -K_{1,18}[Q_1(2)]^n XL_{1,18}-K_{3,14}[Q_3(2)]^n XL_{3,14} \\ & -K_{3,16}[Q_3(2)]^n XL_{3,16} \end{aligned} \tag{5-28}$$

Substituting constants and moving all decision variables to the left hand side we have

$$-XS_1+K_{1,16}[Q_1(2)]^n XL_{1,16}+K_{1,18}[Q_1(2)]^n XL_{1,18}$$

$$+K_{3,14}[Q_1(2)]^n XL_{3,14} + K_{3,16}[Q_3(2)]^n XL_{3,16}$$

$$+z \leqq 95 \tag{5-29}$$

Inequality (5-29) corresponds to constraint (1) of Table 5-1. Treating z as a nonnegative decision variable is consistent with the physical requirement that for water to reach a demand node it must have nonnegative pressure. Constraints (1)-(2) of Table 5-1 correspond to objective function constraints for nodes 4 and 6. The constraints for the other four demand nodes have been omitted to allow the model to be presented in an economical manner.

5.3.4 Formal Statement of Mathematical Model

This section presents a formal statement of the mathematical model, a summary of the constraints, and definitions of new parameters.

PROBLEM P12

$$\text{Maximize} \quad \sum_{\ell \in LE} w_\ell \; z_\ell \tag{5-30}$$

subject to

$$EL_s - EL_i + \sum_{k \in PATH_{si}} XS_k + \sum_{k \in PATH_{si}} XP_k(\ell)$$

$$\pm \sum_{k \in PATH_{si}} \sum_{j \in S_k} K_{kj} \, [Q_k(\ell)]^n XL_{kj} \geqq z_\ell \tag{5-31}$$

$\ell \in LE$

$i \in DNODE$

any $s \in SNODE$

$$EL_s - EL_i + \sum_{k \in PATH_{si}} XS_k + \sum_{k \in PATH_{si}} XP_k(\ell)$$

$$\pm \sum_{k \in PATH_{si}} \sum_{j \in S_k} K_{kj} \, [Q_k(\ell)]^n XL_{kj} \geqq HMIN_i(\ell)$$

$\ell \in LN$ (5-32)

$i \in DNODE$

any $s \in SNODE$

$$\pm \sum_{k \in LOOP_i(\ell)} \sum_{j \in S_k} K_{kj} \, [Q_k(\ell)]^n XL_{kj} = 0 \tag{5-33}$$

$i = 1, \ldots, NLOOP(\ell)$

$\ell \in LN \cup LE$

$$\pm \sum_{k \,\varepsilon\, PATH_{st}} XS_k \;\pm \sum_{k \,\varepsilon\, PATH_{st}} XP_k(\ell)$$

$$\pm \sum_{k \,\varepsilon\, PATH_{st}} \sum_{j \,\varepsilon\, S_k} K_{kj}\, [Q_k(\ell)]^n XL_{kj} \;=\; EL_s - EL_t \qquad (5\text{-}34)$$

$s,\ t \,\varepsilon\, SNODE$

$s \neq t$

$\ell \,\varepsilon\, LN \cup LE$

$$\sum_{k=1}^{NST} STC_k\, XS_k \;+\; \sum_{k=1}^{NPUMP} PU \left[XP_k\left(\ell_{c_k}\right),\ QP_k\left(\ell_{c_k}\right)\right]$$

$$+ \sum_{k=1}^{NLINK} \sum_{j \,\varepsilon\, S_k} CL_{kj}\, XL_{kj} \;\leq\; BMAX \qquad (5\text{-}35)$$

$$\sum_{j \,\varepsilon\, S_k} XL_{kj} \;=\; L_k \qquad (5\text{-}36)$$

$k = 1, \ldots, NLINK$

$$0 \;\leq\; XS_k \;\leq\; SHMAX_k \qquad (5\text{-}37)$$

$k = 1, \ldots, NST$

$$PHMIN_k \;\leq\; XP_k(\ell) \;\leq\; PHMAX_k \qquad (5\text{-}38)$$

$k = 1, \ldots, NPUMP$

$$\ell \in LN \cup LE$$

$$XL_{kj} \geqq 0 \quad k = 1, \ldots, NLINK$$

$$j \in S_k$$

$$z_\ell \geqq 0 \quad \ell \in LE$$

$$Q_k(\ell) \geqq 0 \quad k = 1, \ldots, NLINK$$

$$\ell \in LN \cup LE$$

Objective function (5-30) and the objective function constraints (5-31) combine to implement the MAXWMIN objective function. Constraint (5-32) is the requirement that the pressure at each demand node exceed minimum acceptable levels under normal loading conditions. Equality constraint (5-33) requires conservation of frictional head loss on all loops on all loading conditions. Equality constraint (5-34) requires conservation of energy between all pairs of sources on all loading conditions. Inequality (5-35) is the budget constraint. Equality (5-36) is the link length constraint. Inequalities (5-37) and (5-38) represent bounds on storage height and pump size, respectively.

The following new parameters are included in the model:

LN--the set of normal loading conditions

$LOOP_i(\ell)$--the set of links in loop i on loading condition ℓ

$NLOOP(\ell)$--the number of loops in loading condition ℓ

5.4 Analysis of the Model

5.4.1 Constraint Set

This section will analyze various important characteristics of the constraint set essential to selecting the proper solution algorithm and evaluating the results of the chosen algorithm.

5.4.1.1 Nonlinearity

The frictional head loss relationship is, in general, nonlinear in both flow rate and link diameter. However, by allowing each link to assume only a discrete set of candidate diameters, S_k, the head loss terms in the model, $K_{kj}\ [Q_k(\ell)]^n XL_{kj}$ are only nonlinear in flow rate. Likewise, the capital pipe cost function,

$$\ell_1\ (D_{kj})^{\ell_2}\ ,$$

is nonlinear in diameter but becomes linear in XL_{kj} since each XL_{kj} is associated with a single diameter $j \varepsilon S_k$.

The capital pump cost function,

$$\ell_4 \left[QP_k\left({}^{\ell}c_k\right)\right]^{\ell_5} \left[XP_k\left({}^{\ell}c_k\right)\right]^{\ell_6}$$

where ${}^{\ell}c_k$ is pump k's critical loading condition, is nonlinear in both flow rate and head lift. However, in most cases the pump, unless it is an in-line booster pump, will not be located on a loop and its flow rate will be fixed. Assuming a fixed pump flow rate, since $\ell_6 < 1$, the capital pump cost term is a nonlinear concave function of its head lift.

5.4.1.2 Nonconvexity

Since $n > 1$, the head loss term $+ K_{kj} [Q_k(\ell)]^n XL_{kj}$ is convex while the term $- K_{kj} [Q_k(\ell)]^n XL_{kj}$ is concave. For loops the sum of the head loss terms must equal zero. Since not all of the head loss terms are zero (unless there is no flow in any links in the loop), the loop constraint is the sum of both convex and concave functions. Therefore, the intersection of the loop constraints forms a nonconvex set and the feasible region is nonconvex.

Consider the special case of optimizing a tree distribution system. Since for a tree system all link flows are fixed, the nonlinearities in the nodal constraints (5-31) and (5-32) and the

source constraints (5-34) are removed. The only remaining nonlinearity is the concave capital pump cost term in the budget constraint. Since the other terms in the budget constraint are linear, i.e., both concave and convex, and the sum of a finite number of concave functions is concave, the budget constraint is concave. Thus, the feasible region is still nonconvex. However, for a tree distribution system without pumps the constraint set is convex since all constraints are linear.

5.4.1.3 Structural Analysis

The purpose of a structural analysis of the constraints is to identify any special structure that could be exploited in the solution algorithm. Ideally, a large problem could be decomposed into independent subsystems whose subproblems could be independently solved. However, coupling constraints, such as a common resource, or coupling variables, i.e., common activities among the subsystems, are often present reflecting the interaction among subsystems. Figure 5-8 depicts some common structures.

A natural way to approach the MAXWMIN problem (Problem P12) is to treat each loading condition as a subsystem since each loading condition has its own unique flow distribution. However, each loading condition shares a large number of coupling variables with other

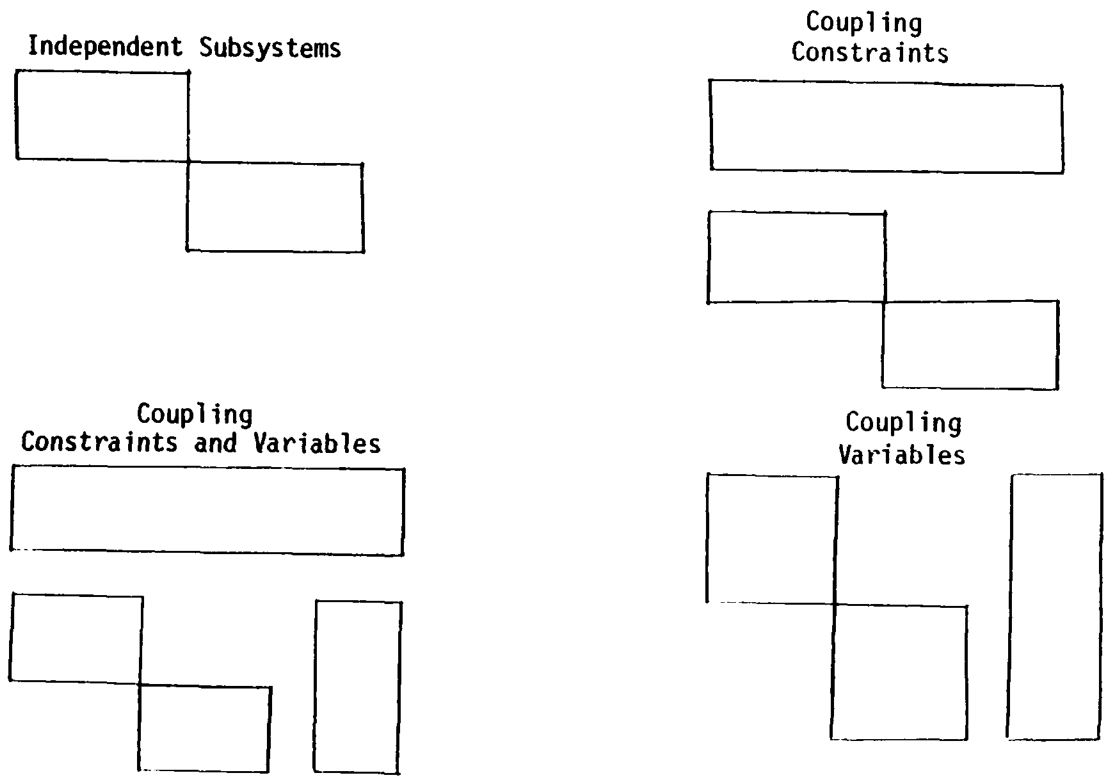


Figure 5-8

TYPICAL STRUCTURAL CONSTRAINT FORMS

loading conditions, i.e., link diameters and added storage height, in addition to important coupling constraints, i.e., budget, link lengths, bounds on storage height and pump capacity, and pump operating level constraints between various loading conditions. Thus, because of the tremendous amount of interaction among loading conditions, the constraint structure is not appropriate for decomposition based on loading conditions. Nevertheless, its structure does suggest the need for a central coordinator to allocate the available resources among the competing emergency loading conditions in an optimal manner.

5.4.2 Feasibility

Because of upper bounds on the budget level, the storage height, and the pump capacity, the MAXWMIN problem is not guaranteed to have a feasible solution. A way to check the feasibility of the MAXWMIN problem is to solve the following minimum cost optimization problem:

PROBLEM P13

$$\text{Minimize} \quad \sum_{k=1}^{NST} STC_k \, XS_k + \sum_{k=1}^{NPUMP} PU\left[XP_k\left(\ell_{c_k}\right), QP\left(\ell_{c_k}\right)\right]$$

$$+ \sum_{k=1}^{NLINK} \sum_{j \varepsilon S_k} CL_{kj}\ XL_{kj} \tag{5-39}$$

$$EL_s - EL_i + \sum_{k \varepsilon PATH_{si}} XS_k + \sum_{k \varepsilon PATH_{si}} XP_k(\ell)$$

$$\pm \sum_{k \varepsilon PATH_{si}} \sum_{j \varepsilon S_k} K_{kj}\ [Q_k(\ell)]^n XL_{kj} \geqq HMIN_i(\ell) \tag{5-40}$$

$i \varepsilon DNODE$

any $s \varepsilon SNODE$

$\ell \varepsilon LN \cup LE$

$$\pm \sum_{k \varepsilon LOOP_i(\ell)} \sum_{j \varepsilon S_k} [Q_k(\ell)]^n XL_{kj} = 0 \tag{5-41}$$

$i = 1, \ldots, NLOOP(\ell)$

$\ell \varepsilon LN \cup LE$

$$\pm \sum_{k \varepsilon PATH_{st}} XS_k \pm \sum_{k \varepsilon PATH_{st}} XP_k(\ell)$$

$$\pm \sum_{k \varepsilon PATH_{st}} \sum_{j \varepsilon S_k} K_{kj}\ [Q_k(\ell)]^n XL_{kj} = EL_s - EL_t \tag{5-42}$$

$s, t \varepsilon SNODE$

$s \neq t$

$$\ell \in LN \cup LE$$

$$\sum_{j \in S_k} XL_{kj} = L_k \qquad (5\text{-}43)$$

$$k = 1, \ldots, NLINK$$

$$XS_k \geqq 0 \quad k = 1, \ldots, NST$$

$$XP_k(\ell) \geqq 0 \quad k = 1, \ldots, NPUMP$$

$$\ell \in LN \cup LE$$

$$XL_{kj} \geqq 0 \quad k = 1, \ldots, NLINK$$

$$j \in S_k$$

$$Q_k(\ell) \geqq 0 \quad k = 1, \ldots, NLINK$$

$$\ell \in LN \cup LE$$

Problem P13 was derived from Problem P12 by replacing the MAXWMIN objective function with the left hand side of the budget constraint (5-35), replacing the z_ℓ variables with selected minimum pressure levels, and relaxing bounds on storage height and pumping head lift. By its construction with no bounds on external energy, Problem P13, the MINCOST problem, must have a feasible solution.

Proper selection of the minimum nodal head pressures, $HMIN_i(\ell)$, in the MINCOST problem allows us to obtain a range of feasible budget levels for the MAXWMIN problem. Setting $HMIN_i(\ell)$

for normal loadings equal to statutory minimum levels (usually 85-105 feet) and for emergency loading conditions equal to zero, we can obtain an absolute lower bound on BMAX. If instead $HMIN_i(\ell)$ for emergency loading conditions is set to minimum statutory requirements for emergency operation (usually 46 feet), the cost of satisfying government regulations can be evaluated. Setting $HMIN_i(\ell)$ for emergency loading conditions to the minimum normal pressures provides an upper bound for BMAX.

Analysis of the cost components in the optimal solution to the above MINCOST problems may indicate an excessive amount of funds have been implicitly allocated for redundant links. By careful analysis of the redundancy requirements of the set and flow covering models (Problem P6 and P7), appropriate adjustments in these requirements may be made freeing additional funds for handling detailed design emergency loading conditions.

5.4.3 Optimality

Due to the nonconvexity of the general constraint set of the MAXWMIN problem (Problem P12), any algorithm for solving Problem P12 can at most guarantee a local optimum. However, for the special case of a tree distribution system without pumping it can be shown that Problem P12 becomes a concave program, i.e., maximizing a

concave function over a convex set for which every local optimum is a global optimum. Since in the case of a tree all flows are fixed, the coefficients of the XL_{kj} terms in the normal loading minimum pressure constraints (5-32) and the source equations (5-34) are fixed, and the constraint set is linear in the remaining decision variables. For each emergency loading condition ℓ and demand node i let

$$f_{i\ell}(\hat{X}) = EL_s - EL_i + \sum_{k \varepsilon PATH_{si}} XS_k + \sum_{k \varepsilon PATH_{si}} XP_k(\ell)$$

$$\pm \sum_{k \varepsilon PATH_{si}} \sum_{j \varepsilon S_k} K_{kj} [Q_k(\ell)]^n XL_{kj} \qquad (5\text{-}44)$$

where $\hat{X}$ is the vector of all decision variables. Since $Q_k(\ell)$ is fixed, $f_{i\ell}(\hat{X})$ is linear (and thus concave). For every feasible $\hat{X}$ define the pointwise infimum of $\{f_{i\ell}(\hat{X})\}$ for each loading as

$$\overline{f}_\ell(\hat{X}) = \inf_{i \varepsilon DNODE} f_{i\ell}(\hat{X}) = \min_{i \varepsilon DNODE} f_{i\ell}(\hat{X}) \qquad (5\text{-}45)$$

By Theorem 4.13 p. 75 Avriel [90], then $\overline{f}_\ell(\hat{X})$ is a concave function. Figure 5-9 illustrates this situation for linear functions of a single variable. Multiplying $\overline{f}_\ell(\hat{X})$ by its appropriate positive weight ω_ℓ and summing over all emergency loading conditions

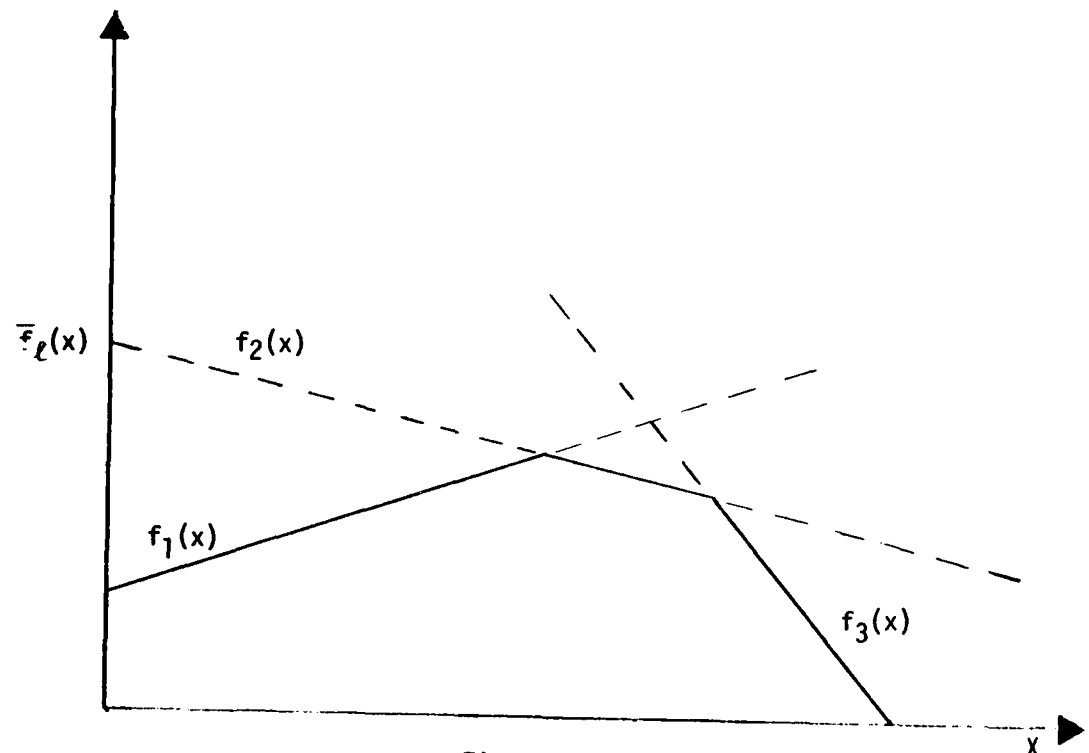


Figure 5-9

POINTWISE INFIMUM OF A SET OF LINEAR FUNCTIONS

we have the concave function

$$\sum_{\ell \varepsilon LE} \omega_\ell \, \overline{f}_\ell(\hat{X}) \qquad (5\text{-}46)$$

which is just the MAXWMIN objective function. Since we are maximizing a concave function over a convex set, any local optimum is a global optimum.

Let us next consider solving the general Problem P12 with loops and pumping when we fix all the link flows $Q_k(\ell)$. Now, the budget constraint becomes concave and Problem P12 is a noncovex program since the capital pump cost function is both nonlinear and concave. More specifically Problem P12 becomes a complementary convex or reverse convex program since the set of decision variables satisfying the budget constraint is the complement of an open convex set and the remaining constraints are convex [90]. For continuous functions of a single variable, $f_1(x)$ and $f_2(x)$, let $R = \{x : x \geqq 0,\ f_1(x) \leqq f_2(x)\}$ where $f_1(x)$ is concave and $f_2(x)$ is convex. For two example cases Figure 5-10 illustrates the resulting nonconvex sets. In Figure 5-10A $R = \{a_1 \leqq x \leqq a_2$ or $a_3 \leqq x \leqq a_4\}$ and is the complement of the open convex set $a_2 < x < a_3$. In Figure 5-10B $R = \{0 \leqq x \leqq a_1$ or $x \geqq a_2\}$ and is the complement of the open convex set $a_1 < x < a_2$. Unless

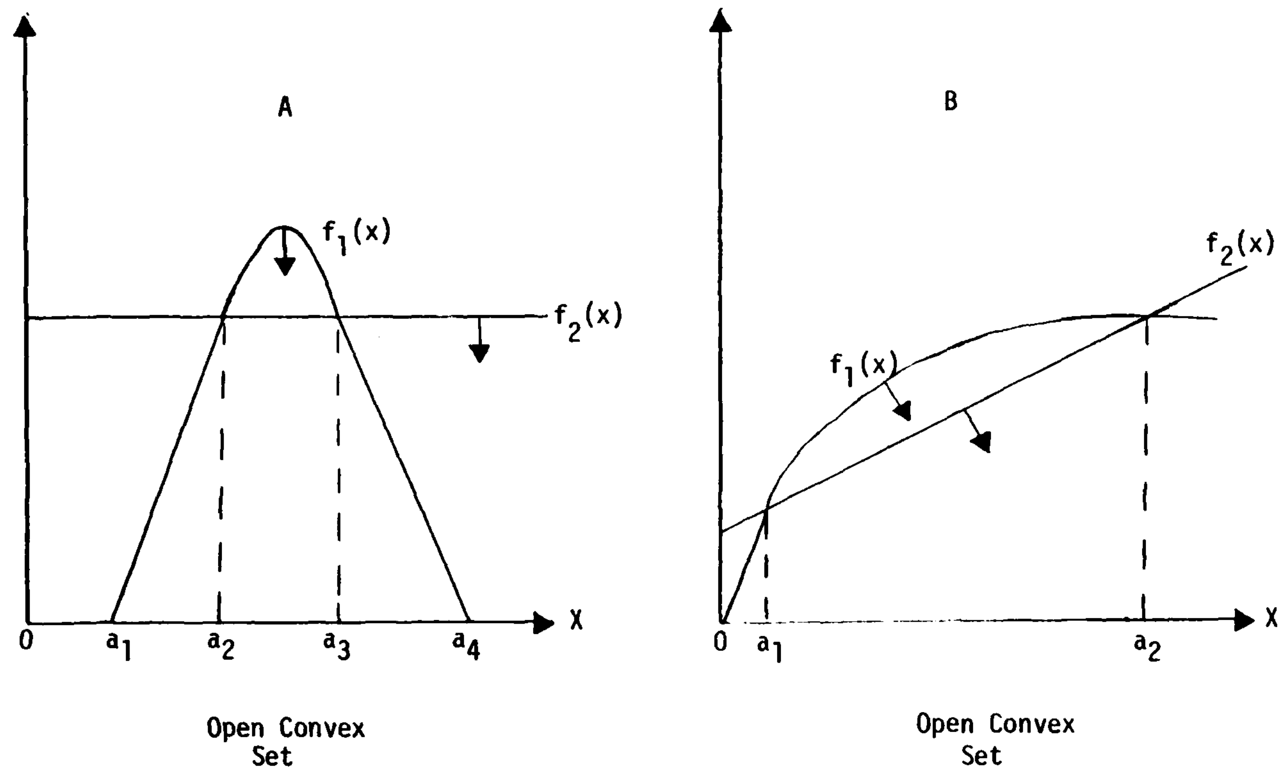


Figure 5-10

COMPLEMENTARY CONVEX SETS

specialized algorithms [91, 92] are used, convergence of the solution algorithm to the global optimum for the complementary convex program cannot be guaranteed.

Alperovits and Shamir [46] state without proof that the optimal solution for Problem P13 will have at most two segments with their diameters adjacent on the candidate diameter list for that link. Quindry, Brill, Liebman, and Robinson [94] offer an apparent counterexample. Appendix C presents a proof for Alperovits and Shamir's [46] statement including the exact conditions for which it is valid. Also, a linear programming model to find the minimum cost feasible solution for a given optimal continuous diameter solution is developed.

5.5 Solution Technique

5.5.1 Introduction

Alperovits and Shamir's [46] Linear Programming Gradient (LPG) approach was selected as the basis for the solution algorithm for the MAXWMIN problem. The LPG approach was developed to solve a simpler version of the MINCOST problem (Problem P13) for normal loading conditions only. Fixing the complicating variables $Q_k(\ell)$ in Problem P13, the constraint set is linear. Representing the

concave capital pump cost as a piecewise linear function, the LPG approach solves a series of linear programs linked by changes in the flow distribution resulting from loop flow changes. Loop flow changes are made so as to improve the objective value in the next program. The LPG approach has been specifically tailored to solve the MAXWMIN problem (Problem P12). We will first describe in detail the specific algorithm used with an emphasis on the major modifications to Alperovits and Shamir's LPG approach, present a formal statement of the algorithm and apply the solution algorithm to design of the example distribution system.

5.5.2 Description

5.5.2.1 Introduction

The solution algorithm involves partitioning the decision variables into two classes, the complicating variables and all others. When the values of the complicating $Q_k(\ell)$ variables are fixed, i.e., the vector $\hat{Q} = (Q_1(1), \ldots)$, the MAXWMIN problem becomes at worst a complementary convex program (CCP) which can be solved using a series of linear programs for an optimal objective value $CCP(\hat{Q})$ [90]. Using dual variables and derivatives of flow constraints, loop flow changes $\Delta\hat{Q} = (\Delta Q_1, \ldots)$ are computed in an

attempt to improve the current solution, i.e., $CCP(\hat{Q} + \Delta\hat{Q}) > CCP(\hat{Q})$. The general method is illustrated in Figure 5-11. The algorithm is terminated when a local optimum is reached. The remainder of this section will cover in detail important aspects of the algorithm.

5.5.2.2 Nodal Pressure Constraints

In theory, nodal pressure constraints, inequalities (5-31) and (5-32), must be written for each demand node and loading condition. However, the greater the number of constraints the more computational effort needed to solve the linear program and to update the coefficient matrix with changes in $Q_k(\ell)$ and S_k . Thus, by identifying demand nodes on each loading which are likely to experience lower pressures, e.g., nodes farthest from the source or fire demand nodes, we can perhaps reduce somewhat the number of nodal pressure constraints.

Shamir and Alperovits [46] suggest solving the problem for a small set of nodal constraints and then checking the relaxed nodal constraints at the optimal solution. If any of the relaxed nodal constraints are violated, the violated constraints are added and the total problem re-solved. To simultaneously minimize the number of nodal head constraints required and preclude the need to re-solve

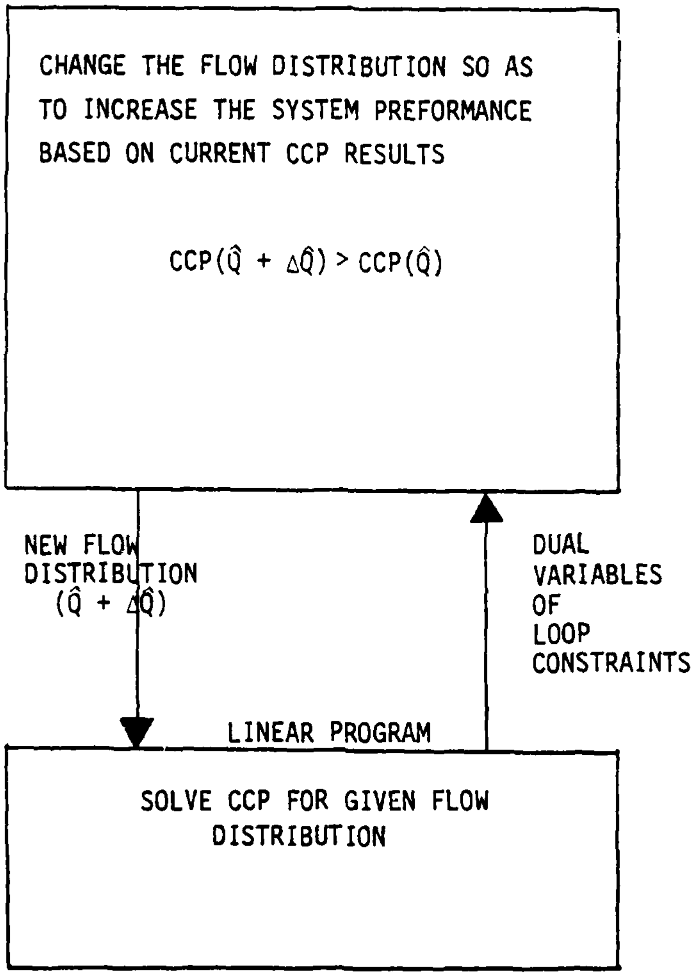


Figure 5-11

GENERAL SOLUTION ALGORITHM

the entire problem the following scheme was developed and incorporated in the solution algorithm:

1. Include the bare minimum number of nodal constraints for each loading in the model.
2. Solve the resulting complementary convex program and compute the heads at all demand nodes on each loading condition.
3. If none of the relaxed nodal constraints are violated, the set of enforced nodal constraints remains the same.
4. For each loading condition for which nodal constraints are violated compute the following lists:
   a. Relaxed nodal heads that have been violated in order of decreasing negative slack, i.e., the most violated constraints first.
   b. Enforced nodal heads in order of decreasing positive slack, i.e., the most satisfied constraints first.

   Slack for normal loading conditions is computed as $H_i(\ell) - HMIN_i(\ell)$ and for emergency loading condistions as $H_i(\ell) - z_\ell$ where z_ℓ is the minimum nodal head for loading ℓ.
5. Using the two lists, replace the enforced inactive nodal constraint with its corresponding violated constraint in the constraint set until all violated constraints are in

the set of enforced constraints.

The above scheme has proven especially useful when dealing with a new system or new loading conditions where critical nodes are not readily apparent.

5.5.2.3 Initial Flow Distribution

Alperovits and Shamir [46] state that the initial flow distribution for each loading condition is arbitrary. However, a poor choice of initial flow distribution for a large problem can waste considerable computation time reaching a feasible (balanced) let alone a local optimum solution (see section 6.5.3.3). Thus, it appeared worthwhile to develop efficient techniques for finding good distributions for either the MAXWMIN or MINCOST problem.

The author's extensive computational experience has indicated that the proper use of the following tools can significantly reduce both the total computational and programming effort necessary to solve the MAXWMIN problem in addition to providing valuable insight into the distribution system design:

1. Knowledge of the core tree
2. Network balancer
3. Preparatory MINCOST optimizations

As discussed in section 3.3.5.1, flow tends to concentrate in the primary links of the core tree. Thus, the initial flow distribution for the normal loading should place little flow, if any at all, in the redundant links. This frees the optimization algorithm to change the loop flow in the appropriate direction not burdened with overcoming an initial flow distribution with a large flow concentration placed incorrectly in a redundant link.

Even using the above procedure it can take several costly flow iterations for a large problem to reach a feasible (balanced) flow distribution using the crude balancing mechanism of the LPG method. Furthermore, in the meantime the solution algorithm is so concerned with removing the high penalty costs associated with the infeasibility that little real progress is made towards reaching optimality until feasibility is attained. Thus, a network balancer using the Hardy Cross loop method was incorporated as an integral part of the solution algorithm. After the initial complementary convex solution is obtained, using the resulting link design and the initial flow distribution, the network balancer balances the unbalanced loading conditions to within a specified imbalance level. For the next complementary convex problem the network balancing flow changes are used instead of the normally computed flow changes. The

subsequent complementary convex problem is almost always feasible and the solution algorithm proceeds as usual.

Let us consider the role of the normal loading condition in Problem P12, the MAXWMIN problem. Although the normal loading condition is not a part of the objective function, it seems reasonable to desire to minimize the costs of satisfying the normal loading condition constraints in order to maximize the portion of the budget available for system components explicitly designed for emergency operation such as booster fire pumps. Thus, solving the MINCOST problem subject to the normal loading condition only should provide an inherently economical flow distribution. The resulting optimal normal flow distribution, in turn, can be used as the initial normal loading flow distribution for the MINCOST problem with emergency loading conditions added and minimal nodal emergency pressures set at statutory minimum pressures (usually 46 feet) or at zero feet. Because emergency loading conditions vary so widely, it is difficult to formulate any definitive rules for selecting their initial flow distributions. The best rule of thumb is to concentrate the flow in the larger primary links where possible and to pattern the flow distribution after the MINCOST normal loading flow distribution. Finally, the initial flow distribution from the

optimal MINCOST solution for both normal and emergency loadings can be used as the initial flow distribution for the MAXWMIN problem.

Because of the importance of the initial flow distribution, the solution algorithm has been modified to automatically save the optimal flow distribution, candidate diameters, and pump cost coefficients that define the optimal solution. This enables the user to restart the same problem or a number of closely related problems, e.g., the alternative MINCOST or MAXWMIN formulation, with minimal effort.

5.5.2.4 Link Candidate Diameters

The selection of the set of initial candidate diameters for each link, S_k , depends on several factors:

1. Commercial availability
2. Minimum and maximum normal loading hydraulic gradients (velocity)
3. Minimum link diameters driven by broken link loading conditions
4. Status of link-existing or new
5. Problem size considerations
6. Initial flow distribution.

Depending on the type of pipe (cast iron, PVC, asbestos-concrete) and its pressure class, only certain pipe diameters are commercially available. In the United States, for example, cast iron pipes are generally available in 2" increments starting at 4" continuing to 20", and in 24" and 30" diameters.

As discussed in section 3.3.4.1, engineering design considerations restrict the range of permissible hydraulic gradients, J_k, on the normal loading. Excessively high J_k can result in burst pipes while excessively low gradients result in water stagnation. Such limits are usually included in statutory regulations in terms of maximum and minimum flow velocities. The results of the redundant link selection models of Chapter 4 will also provide minimum pipe diameters for all redundant and certain primary links. For analysis of capacity expansion of existing systems some of the links will already exist and S_k will be restricted to a single pipe diameter.

Theoretically, the set of diameters from which the solution algorithm could choose at any one time is the complete set of commercially available diameters within the minimum and maximum limits defined by the above constraints. However, computational considerations preclude this approach. Using a large number of candidate diameters for each link considerably increases the number of

decision variables in the linear program. More importantly, after each flow change, the flows in each of the diameter segments of each of the links in all of the flow equations must be updated including an updating of the basis inverse. Therefore, the initial set of candidate diameters in S_k has been restricted to from 3-5 diameters. The initial set is chosen based on the initial flow distribution in the links over all loading conditions.

Although the size of S_k during any linear programming optimization is fixed, the specific diameters in the set may change if the possibility of an improved solution is indicated. Assume that $S_k = \{6,8,10\}$ in the current complementary convex problem, minimum and maximum commercially available diameters are 6 and 20 inches with no other restrictions on pipe diameter and that $XL_{k,10} = L_k$ in the current LP solution, i.e., link k has a single segment of diameter 10 inches. Thus, link k is artificially constrained to a maximum diameter of 10 inches. By letting $S_k = \{8,10,12\}$ and re-solving the linear program, the optimal objective value could improve and, at worst, will remain the same. Alperovits and Shamir [46] also change S_k during the solution algorithm but instead of simply shifting the candidate set up or down the size of S_k is haphazardly reduced as the algorithm progresses, further limiting the choice of diameters.

Experience using the solution algorithm to solve the MAXWMIN problem led to a further restriction in allowing the set S_k to change. Because of the numerous, often conflicting flow distributions of the various loadings even after a feasible (balanced) solution was obtained, subsequent flow changes often led to a slightly infeasible (unbalanced) solution (see section 5.5.2.6). Allowing candidate link diameters to become larger to achieve balance significantly reduced the minimum nodal heads on the emergency loadings since funds were reallocated from the head producing pumps and storage reservoirs to the links. When feasibility was reached (usually by the next flow change) sets of candidate diameters that had become larger in an attempt to achieve feasibility had to be reduced. This erratic behavior greatly impeded progress towards a local optimum. Thus, once an initial feasible solution had been obtained, the set of candidate diameters could add larger diameters only if the current CCP solution is feasible. Implementation of this rule eliminated this counterproductive behavior and speeded up significantly convergence of the algorithm.

5.5.2.5 Nonlinear Pump Capital Cost

For systems with pumps, the budget is a nonlinear, concave function, the feasible region for a fixed flow distribution $(Q_k(\ell))$

is no longer convex, and a complementary convex program results. There are several potential techniques for solving this particular problem including the general techniques of separable programming and iterative linearization [93] which can guarantee only local optimal solutions and specialized algorithms developed by Soland [91] or Hillestad [92] which guarantee a global optimum. The specialized algorithms involve complicated infinitely [91] or finitely [92] convergent search procedures. Because the complementary convex program must be solved numerous times during the solution algorithm (at a minimum equal to the number of flow changes if S_k remains constant), the pump capital cost function is only mildly concave (see Figure 5-7), and the overall solution technique converges at best to a local optimum, the complex specialized algorithms were judged not worth the added computational effort. Iterative linearization was selected instead of separable programming because it requires no increase in the number of decision variables, the same level of solution accuracy can be obtained regardless of the value of the decision variable, and it is considerably simpler to implement than separable programming.

The iterative linearization algorithm for solving the complementary convex program is described next [90]. Let F be the feasible region, $\hat{X}$, the vector of all decision variables (link, pump,

and storage), and $g(\hat{X}) \leq$ BMAX, the concave budget constraint. At any point $\hat{X}^k \varepsilon F$ the nonlinear budget constraint is replaced by its first-order Taylor series approximation

$$\bar{g}(\hat{X}, \hat{X}^k) = g(\hat{X}^k) + (\hat{X} - \hat{X}^k) \nabla g(\hat{X}^k) \leq \text{BMAX} \qquad (5\text{-}47)$$

to obtain a convex (linear) program. The next point $\hat{X}^{k+1}$ is the optimal solution of the linear program at $\hat{X}^k$. Avriel [90] demonstrates that if the initial point $\hat{X}^0 \varepsilon F$ then each member of the sequence $\{\hat{X}^k\}$ converges to a Kuhn-Tucker point of the complementary convex program, i.e., a locally optimal solution.

The principal problem with using this approach is that the local optimum solution may be far from the global optimum. It is difficult to make any general statements about the convergence characteristics of the complementary convex program resulting from fixing $Q_k(\ell)$ in the MAXWMIN problem. For fixed flows and link candidate diameter sets the MAXWMIN problem was solved for the example problem for seven widely varying initial pump head values ranging from .1 to 6 times the optimal values. Each time the algorithm converged to within 1% of the true cost of each of the two pumps requiring at most 3 linear programming iterations. The maximum difference between the highest and lowest objective function values

was .02 feet. These results combined with the mild concavity of the capital pump cost function make the selected approach appear reasonable. However, if desired, one of the specialized global optimal algorithms [91, 92] may be applied to the MAXWMIN optimal solution.

5.5.2.6 Dummy Valves

Although the loop/source constraints are written as strict equalities in the MAXWMIN problem, additional slack and surplus variables are required for each of these constraints. Although the MAXWMIN problem may have a feasible solution, it is possible that for the current flow distribution $Q_k(\ell)$ and set of candidate diameters S_k that the complementary convex program is not feasible, i.e., not balanced. Thus, for each equality constraint in (5-33) and (5-34) two slack variables are added. For example, for each loop constraint we have

$$\pm \sum_{k \in LOOP_i(\ell)} \sum_{j \in S_k} K_{kj} [Q_k(\ell)]^n XL_{kj} + XV_i^+ - XV_i^- = 0 \tag{5-48}$$

where XV_i^+ and XV_i^- are the nonnegative slack and surplus variables respectively. These slack variables correspond to dummy valves that

provide resistance loss in the proper direction. These slack variables which are assigned high penalty costs operate somewhat like artificial variables by forming part of an initial basic solution and driving the linear program to find a feasible (balanced) solution. Also, as described in section 5.5.2.4 the current set of candidate diameters can be adjusted to attain feasibility. Further, the high penalty cost of a dummy valve in the basis impacts the dual variables ($\hat{\pi}$) since

$$\hat{\pi} = \hat{C}_B B^{-1} \tag{5-49}$$

where $\hat{C}_B$ is the vector of basic variable costs and B^{-1} the current basis inverse. The dual variables are used to compute the loop flow changes, thus driving the flow on unbalanced loops in the feasible direction. Thus, unlike artificial variables, the slack and surplus variables are allowed to reenter the basis when the current flow distribution cannot be balanced.

In some cases it may not be possible to eliminate the dummy valves and find a feasible (balanced) solution. This indicates that a real valve may be required to properly operate the system providing the same resistance as the dummy valve.

5.5.2.7 Loop Flow Change Vector

We will discuss how to compute the loop flow change vector $\Delta\hat{Q} = (\Delta Q_1, \ldots, \Delta Q_{NLOOP})$, where

$$NLOOP = \sum_{\ell \in LN \cup LE} NLOOP(\ell) \quad .$$

The loop flow change vector links together successive complementary convex programs. It should be remembered that the set of loop changes translates into flow changes on the individual links for each loading and preserves the initial nodal conservation of flow.

Given the optimal solution to the complementary convex program at iteration k and the associated link flow distribution, we want to find $\Delta\hat{Q}^k$ such that the optimal value of the new complementary convex program increases, i.e., $CCP(\hat{Q}^k + \Delta\hat{Q}^k) > CCP(\hat{Q}^k)$. The direction of change for loop i is found by calculating

$$G_i = \frac{\partial Z}{\partial(\Delta Q_i)} \quad , \tag{5-50}$$

the positive gradient for loop $i = 1, \ldots, NLOOP$ where Z is the objective function. Alperovits and Shamir use the expression

$$G_i = \frac{\partial Z}{\partial(\Delta Q_i)} = \left(\frac{\partial Z}{\partial h_i}\right)\left(\frac{\partial h_i}{\partial(\Delta Q_i)}\right) \tag{5-51}$$

where $\partial Z / \partial h_i = \pi_i$ is the dual variable of loop equation i in the current optimal CCP and $\partial Z / \partial(\Delta Q_i)$ is the partial derivative of loop equation i with respect to loop flow changes evaluated at the current flow distribution. Fixing the length decision variables, XL_{kj}, the right hand side of the loop equations (5-33) can be viewed as a function of the flow change on the loop ΔQ_i, i.e.,

$$h_i = \pm \sum_{k \varepsilon LOOP_i(\ell)} \sum_{j \varepsilon S_k} K_{kj}\, XL_{kj}\, [Q_k(\ell)]^n \tag{5-52}$$

Differentiating with respect to ΔQ_i we have

$$\frac{\partial h_i}{\partial(\Delta Q_i)} = \sum_{k \varepsilon LOOP_i(\ell)} \sum_{j \varepsilon S_k} \left| n\, K_{kj}\, XL_{kj}\, [Q_k(\ell)]^{n-1} \right| \tag{5-53}$$

$$= n \sum_{k \varepsilon LOOP_i(\ell)} \sum_{j \varepsilon S_k} \left| \frac{K_{kj}\, XL_{kj}\, [Q_k(\ell)]^n}{Q_k(\ell)} \right| \tag{5-54}$$

$$= n \sum_{k \in LOOP_i(\ell)} \left| \frac{\Delta H \; F_k(\ell)}{Q_k(\ell)} \right| \qquad (5\text{-}55)$$

Thus, $\partial h_i \,/\, \partial(\Delta Q_i)$ is nothing more than the same expression found in the denominator of the Hardy Cross equation for computing loop flow changes (1-19). The sign of π_i in the gradient expression, like the sign of the numerator of equation (1-19), the head imbalance term, determines the loop flow direction (clockwise or counterclockwise) needed to improve the objective value.

Quindry, Brill, Liebman, and Robinson [94] correctly note that Alperovits and Shamir [46] did not include the interaction of the loop constraints with the other loop, source, and nodal head constraints in their gradient expression (5-51). Interaction occurs when another flow constraint on the same loading condition has at least one link in common with the loop whose gradient is being computed. For example, in the example problem since both loops share link 4, there is interaction between both loops on each loading condition. Thus the gradient expression (5-51) becomes

$$G_i = \left(\frac{\partial Z}{\partial h_i}\right)\left(\frac{\partial h_i}{\partial(\Delta Q_i)}\right) + \sum_{j \in LC_i} \frac{\partial Z}{\partial h_j} \cdot \frac{\partial h_j}{\partial(\Delta Q_i)} \qquad (5\text{-}56)$$

where LC_i is the set of constraints that have links in common with the constraint for loop i. The added term is intended to take into account the impact on other constraints resulting from flow changes on loop i. Quindry et al. [94] apply the corrected gradient to a small minimum cost optimization problem solved by Alperovits and Shamir [46] and obtained an 8% reduction in total cost. The author duplicated Quindry et al.'s results [94]. However, applying Quindry et al.'s correction to another small problem in [46], minimum total costs increased by 7%. Since these results were only for small problems, computational tests on a realistic size problem were performed. The formal results, presented in section 6.5.3.3, indicate that Quindry et al.'s gradient expression offers no advantage and is somewhat less consistent than Alperovits and Shamir's gradient expression.

Once the gradient has been computed the magnitude of the flow change ΔQ_i must be determined. Because of the high computational expense of evaluating the function at different points, i.e., changing the constraint matrix and solving the new CCP, a step length method is used rather than attempting to compute the optimal step size. Let $GMAX^k$ be the absolute value of the maximum loop gradient and α^k the step length at iteration k. Then, the flow change for loop i at interation k is

$$\Delta Q_i^k = \frac{G_i}{GMAX^k} \alpha^k \tag{5-57}$$

The step length is fixed at an initial value and reduced by a constant factor $\beta < 1$ if the objective value worsens on consecutive complementary convex problems. To reduce the considerable computational effort associated with insignificant loop flow change quantities only loop flow changes above a certain magnitude $\Delta QMIN^k$ (proportional to α^k) are implemented in the constraint matrix.

5.5.2.8 Termination Criterion

In the case of the tree distribution system the solution algorithm terminates when the CCP is solved since no flow changes are involved. For the looped distribution system termination occurs when a local optimum solution is reached, i.e., when α^k falls below a specified value α_{min}(5 GPM), or when the maximum number of flow iterations is exceeded (MAXFLOIT).

5.5.3 Formal Statement of Solution Algorithm

The following is a formal statement of the solution algorithm:

STEP 1. Initialize

a. Flow iteration $k = 1$

b. Flow distribution $\hat{Q}^1$

c. Candidate diameter set

d. Nodal head constraint set

e. Capital pump cost coefficient

f. Step length α^0

g. Optimal objective value $z^* = -\infty$

h. Previous objective value $CCP(\hat{Q}^0) = -\infty$

STEP 2. For flow iteration k solve the linear program for $CCP(\hat{Q}^k)$.

STEP 3. Check for convergence of capital pump cost coefficient and change if necessary.

STEP 4. Check set of candidate diameters and change if necessary.

STEP 5. Check for violation of relaxed nodal head constraints and change if necessary.

STEP 6. Update constraint matrix if changes made in STEPS 3, 4, or 5 and GO TO STEP 2. Otherwise go to STEP 7.

STEP 7. If $CCP(\hat{Q}^k) > z^*$, $z^* = CCP(\hat{Q}^k)$.

STEP 8. If $CCP(\hat{Q}^k) < CCP(\hat{Q}^{k-1})$, $\alpha^k = \beta\,\alpha^{k-1}$, otherwise $\alpha^k = \alpha^{k-1}$.

STEP 9. If $\alpha^k < \alpha_{min}$ or $k >$ MAXFLOIT, GO TO STEP 12.

STEP 10. Compute loop flow change vector $\Delta\hat{Q}^k$.

STEP 11. Change flows in constraint matrix, i.e.,

$\hat{Q}^{k+1} = \hat{Q}^k + \Delta\hat{Q}^k$. Let $k = k + 1$. GO TO STEP 2.

STEP 12. STOP.

Appendix D presents the user's manual and source listing of the computer model developed to implement the solution algorithm.

5.5.4 Application to Example Problem

5.5.4.1 Introduction

In this section we apply the solution algorithm for the lowest level model of the hierarchical system to the detailed design of the small example distribution system of Figure 5-1. First, to illustrate the cost of redundant links and to assist in establishing a cost baseline, the minimum costs of alternative network layouts for the normal loading condition (Figure 5-2) are computed. Next, using the normal and fire demand emergency condition (Figure 5-3),

the core tree and the fully looped layout are designed over a range of alternative budget levels. Finally, a broken primary link emergency loading condition (Figure 5-5) is added and the detailed system design is reaccomplished.

5.5.4.2 Minimum Cost Optimization of Alternative Network Layouts

In section 5.3.1.2 we identified the core tree for the example distribution system (Figure 5-12) which consists of primary links 1, 2, 3, 4, 5, 6, and 9 with links 7 and 8 as the redundant links. Separately adding either redundant link to the core tree result in a single loop layout (Figures 5-13 and 5-14) while adding both redundant links gives the fully looped layout (Figure 5-15).

The MINCOST problem was solved for each of the four network layouts for the normal loading only. In addition to the data in Figures 5-1 and 5-2, other major parameters common to each optimization are summarized in Table 5-2. The initial and optimal flow distribution along with the optimal nodal heads for each of the four network layouts are illustrated in Figures 5-12 to 5-15. A summary of the results of each optimization is presented in Table 5-3. The detailed link design for the core tree and the fully looped layouts are presented in Tables 5-4 and 5-5, respectively.

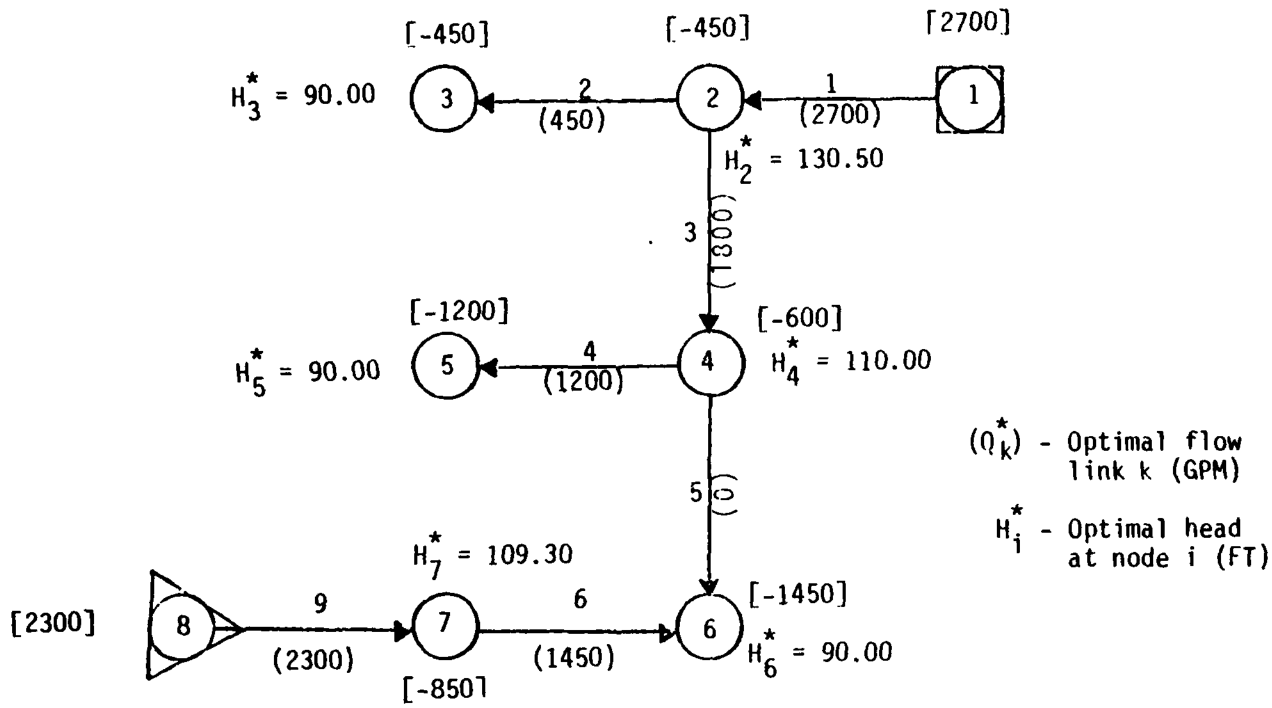


Figure 5-12

CORE TREE LAYOUT
OPTIMAL FLOW AND NODAL HEAD DISTRIBUTION

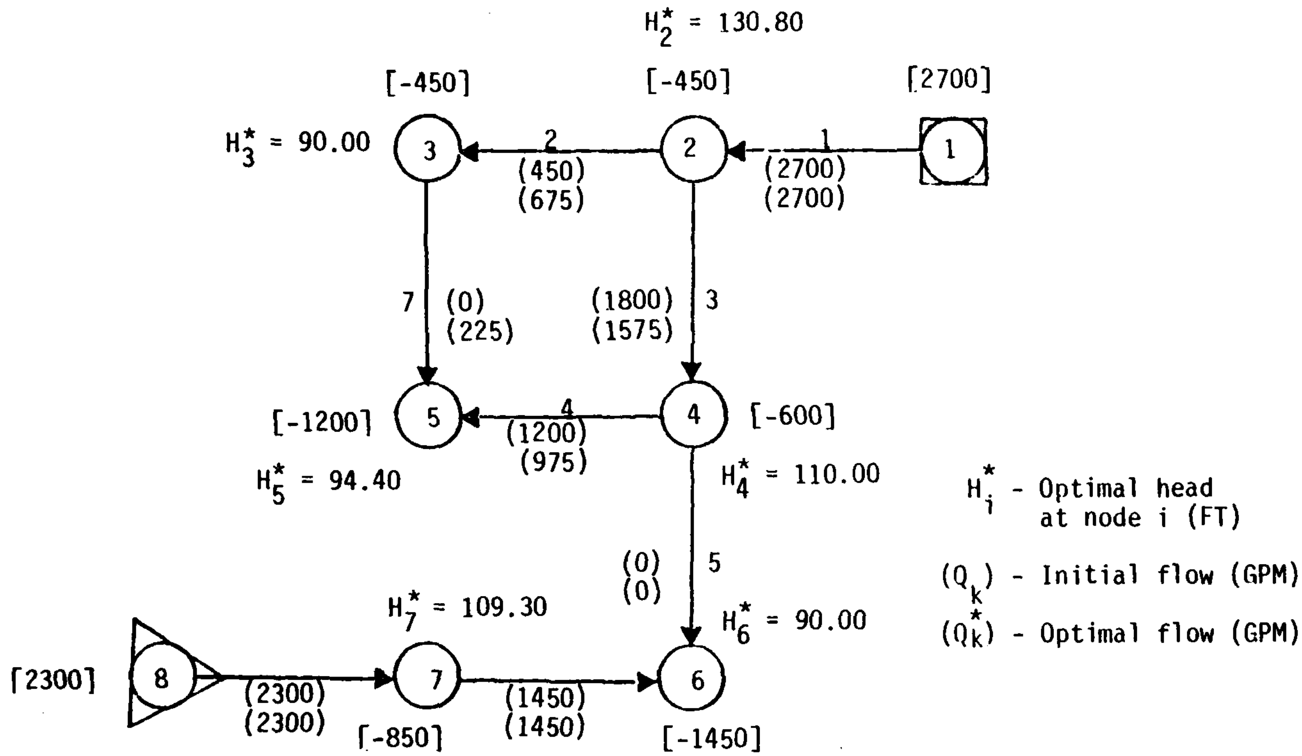


Figure 5-13

SINGLE-LOOP LAYOUT (REDUNDANT LINK 7 ADDED)
OPTIMAL FLOW AND NODAL HEAD DISTRIBUTION

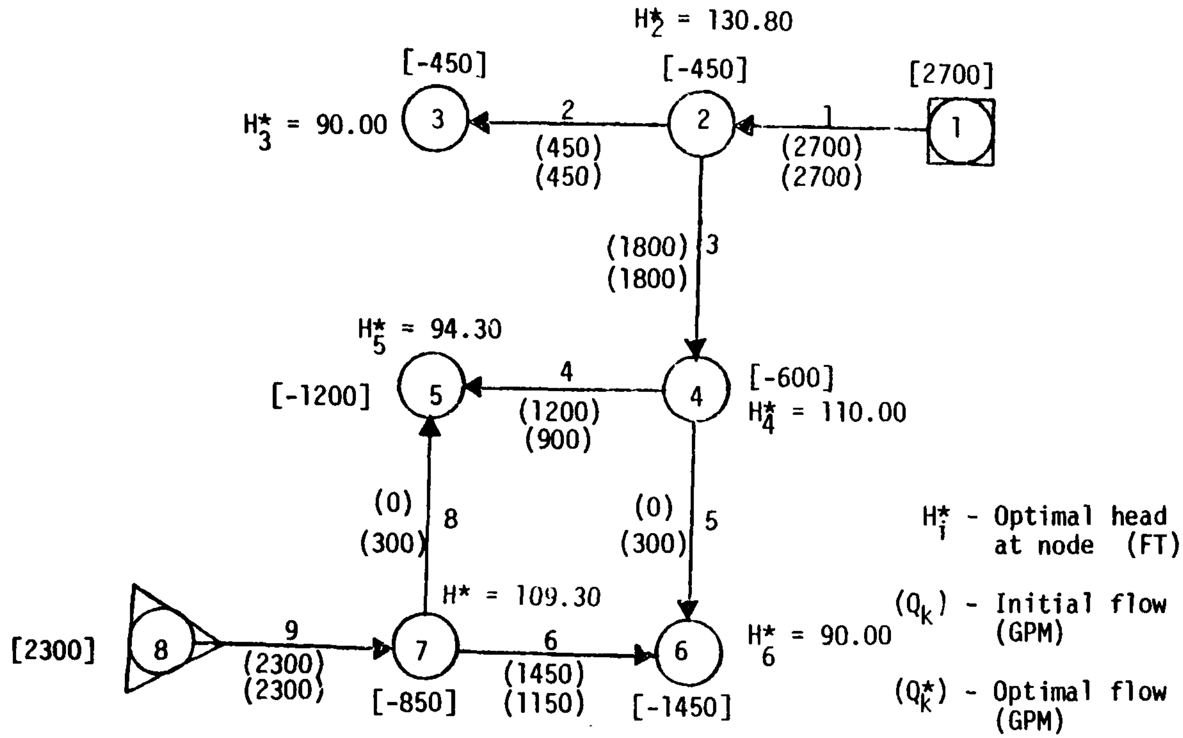


Figure 5-14

SINGLE LOOP LAYOUT (REDUNDANT LINK 8 ADDED)

OPTIMAL FLOW AND NODAL HEAD DISTRIBUTION

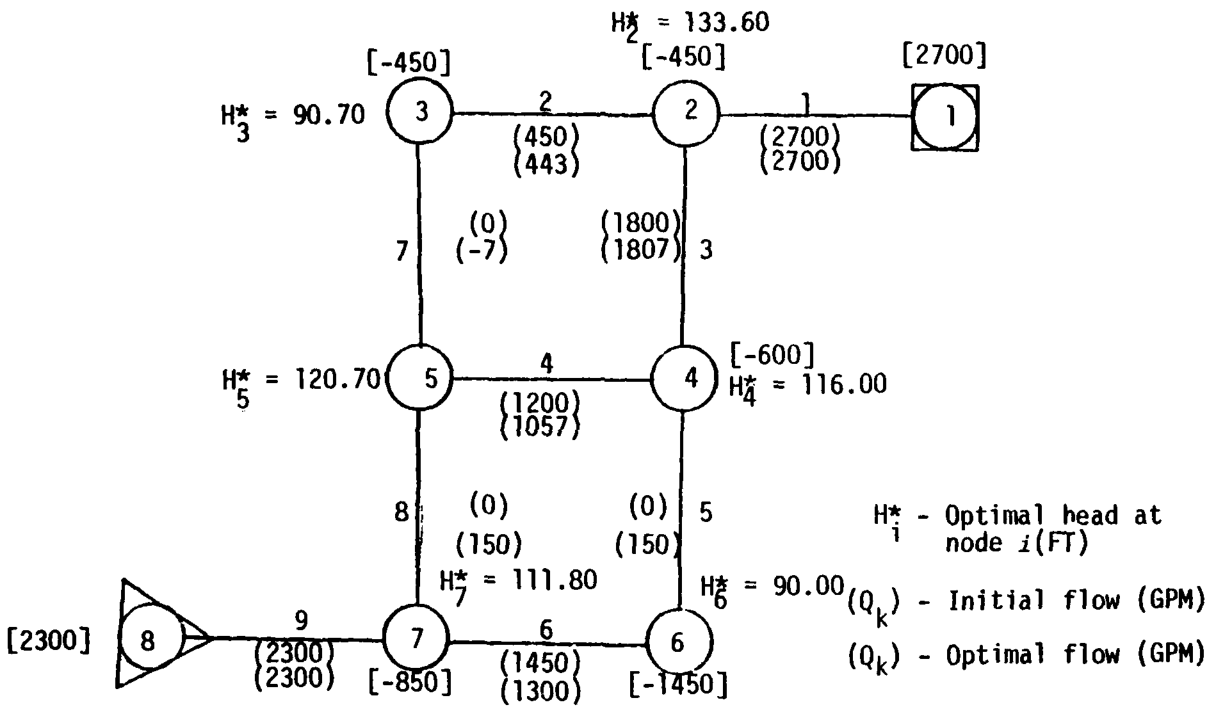


Figure 5-15

FULLY LOOPED LAYOUT

OPTIMAL FLOW AND NODAL HEAD DISTRIBUTION

Table 5-2

EXAMPLE PROBLEM DATA SUMMARY

LINK DATA

Hazen-Williams Coefficient: 130

No. of Candidate Diameters/Link: 4

Salvage Value Ratio: .1

Economic Life: 30 yr

Maintenance Cost: \$4/in/mile/yr

DIAMETER	CAPITAL COST/FT
6	10.2
8	14.8
10	19.7
12	24.9
14	30.4
16	36.1
18	42.0
20	48.2

PUMP DATA

No. of Parallel Pumps: 3

Economic Life: 15 yr

Salvage Value Ratio: .10

Pump-Motor Efficiency: .75

Electricity Cost: \$.04/kw-hr

Utilization Factor: .114

Maintenance Cost: \$4/hp/yr

STORAGE DATA

Maximum Height: 50 ft

Capital Cost: \$2000/ft

Economic Life: 30 yr

OPTIMIZATION PARAMETERS

Initial Step Size: $\alpha^1 = 25$ GPM

Minimum Step Size: $\alpha_{min} = 6$ GPM

Step Size Reduction Factor: $\beta = .6$

Ratio of Minimum Flow Change to Step Size: .2

NODAL DATA

Minimum Nodal Head: 90 ft

Table 5-3

SUMMARY RESULTS OF MINIMUM COST LAYOUT DESIGNS

Network Layout	No. Flow Iterations	Initial Cost ($)	Optimal Cost ($)	Link Cost ($)	Storage Cost ($)	Pump Cost ($)	Storage Height (ft)	Pump Head Lift (ft)
Core Tree	1	48,499	45,823	25,641	4,688	15,494	32.3	39.5
Single Loop (Link 7)	11	51,698	49,568	29,344	4,730	15,494	32.6	39.5
Single Loop (Link 8)	13	53,974	49,915	28,733	5,275	15,907	36.3	40.6
Fully Looped	7	58,100	53,496	31,949	5,385	16,162	37.1	41.3

Table 5-4

MINIMUM COST LINK DESIGN CORE TREE LAYOUT

Link No.	Total Length (ft)	Segment 1		Segment 2	
		Diameter	Length	Diameter	Length
1	3000	16	3000		
2	2500	8	2500		
3	1000	12	470	14	530
4	1500	8	1293	10	207
5	3000	6	3000		
6	3500	16	3500		
9	100	18	100		

Table 5-5

MINIMUM COST LINK DESIGN NORMAL LOADING ONLY
FULLY LOOPED LAYOUT

Link No.	Total Length (ft)	Segment 1		Segment 2	
		Diameter	Length	Diameter	Length
1	3000	16	3000		
2	2500	6	409	8	2091
3	1000	14	1000		
4	1500	8	100	10	1400
5	3000	6	3000		
6	3500	14	2728	16	772
7	4500	6	4500		
8	5000	6	5000		
9	100	18	100		

The results of Table 5-3 clearly illustrate the conclusions of Theorem I on the inherent economy of the core tree. Not restricted by loop balancing requirements, the core tree design is able to reduce the heads at the extreme demand nodes, 3, 5, and 6, to the minimum value of 90 feet.

A comparison of the detailed link design for the core tree and fully looped layout provides some insight into the role of redundant links. Although the total link costs increased by $6,308 from the core tree to the fully looped layout, the total cost of the primary links in fact actually decreased by $609. The decrease in primary link costs resulted from the diversion of flow from the primary links to the redundant links. This flow diversion allowed the primary links on the head path to the lowest head nodes to decrease their diameters, i.e., link 2 for demand node 3 and link 6 for demand node 6. Thus, the addition of redundant links does not necessarily increase the total link costs by the full cost of the redundant links.

Each of the minimum cost optimizations assumed that there are three identical pumps operating in parallel at node 8 each providing one-third of the total flow rate at the same head lift. Since the pump capital cost function is also concave in flow rate for fixed head lift, the cost of a single high flow capacity pump

is less than any equivalent number of smaller flow capacity pumps operating in parallel. The use of parallel pumps serves to insure that pump failure will not completely degrade system performance and provides considerable flexibility in efficiently meeting varying flow demands. To assess the added cost of parallel pumping Problem P13 was solved with a single pump for both the core tree and the fully looped layouts. In both cases the total system costs for the single pump system were roughly $500 less than that of the multiple pump system.

5.5.4.3 Performance Optimization of Single Fire Demand Loading

This section examines the results of applying the solution algorithm to solving the MAXWMIN problem for the fire demand loading shown in Figure 5-3. Since the formulation for this particular problem has been discussed in considerable detail in earlier sections of this chapter, the emphasis will be placed on presentation and analysis of the results. For comparison purposes, the optimization has been performed for both the core tree and the fully looped network.

5.5.4.3.1 Budget Level Selection

Although the system can only be designed for a single budget level, to assist the decisionmaker in making the tradeoff between cost and system performance it is best to provide performance data for a range of alternative budget levels. To compute a lower bound for BMAX the MINCOST problem was solved with minimum normal and emergency loading demand heads at 90 and 46 feet respectively. The initial flow distribution used for the normal loading was the optimal flow distribution from Figure 5-15. The initial emergency loading flow distribution was derived by adding the additional fire demand flow to the initial normal flow on the shortest path from the source node to the fire demand at node 6. The resulting minimum cost for the core tree layout is $50,533 and for the fully looped layout $58,942. Based on these results, the performance optimization for the core tree layout started at BMAX = $50,000 and for the fully looped layout at BMAX = $55,000. The upper budget levels were determined during the course of the optimization procedure which is described below.

5.5.4.3.2 Optimization Procedure

The following procedure was used to insure continuity of results over the range of budget levels:

STEP 1. Initialize BMAX.

STEP 2. If budget constraint is loose, STOP. Otherwise, GO TO STEP 4.

STEP 4. Increment BMAX by $5000. Initialize flow distribution and set of candidate diameters to values from previous optimal solution. GO TO STEP 2.

Convergence to a local optimum solution for the fully looped layout was fairly rapid taking only a few iterations.

5.5.4.3.3 Normal Loading Pressure Reducing Value

In the course of applying the above procedure to the example problem unexpected but valid results in the behavior of the normal pumping head led to a small but important change in both the system configuration and the model formulation. Figure 5-16 shows the heads provided by the elevated storage, the normal pump, and the standby pump for various budget levels for the core tree layout. Starting at BMAX = $50,000 the normal pump's head lift increases in direct proportion to storage height increases. Storage height increases are driven by the maximum performance objective function. Rewriting source equation (5-10) in a slightly different form we have

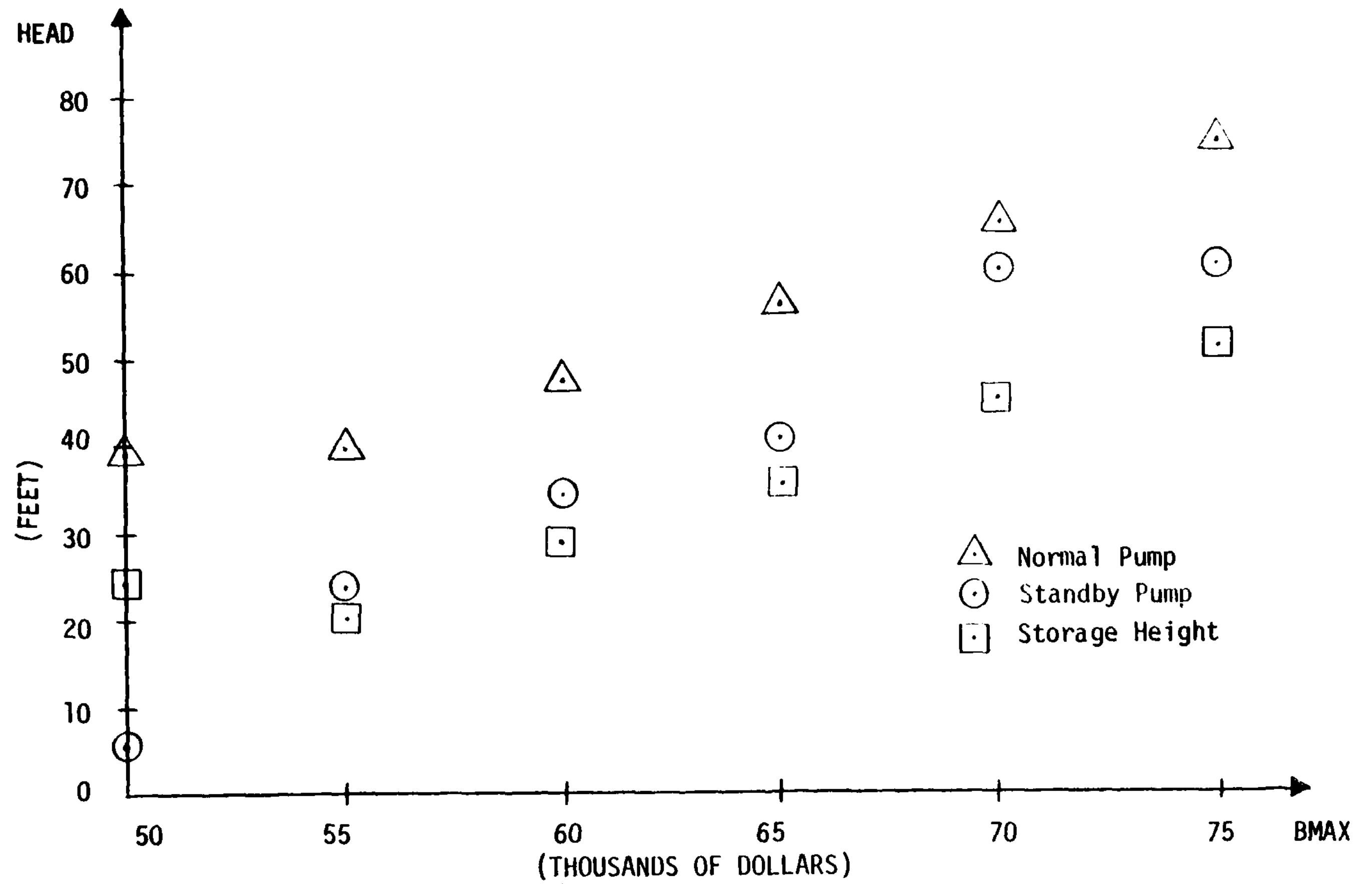


Figure 5-16

ADDED EXTERNAL ENERGY CORE TREE LAYOUT (NO SOURCE VALVE)

$$XP_1(1) = 20 + XS_1 \pm \sum_{k \in PATH_{1,8}} \Delta HF_k(1) \qquad (5\text{-}58)$$

Thus, assuming fixed link diameters and flows, increases in the height of elevated storage results in increased normal pump head lift. However, the nodal heads under the normal loading condition are not part of the objective function and need only exceed minimum levels of 90 feet. Figure 5-17, which shows a breakdown of system costs with increasing budget level for the same problem, indicates that link costs are nondecreasing and that total pump costs account for roughly 60% of the $25,000 increase in budget level. Normal pumping cost increases, which include expensive energy costs, account for roughly 80% of the $15,000 increase. The physical result is that the minimum nodal head on the normal loading condition at BMAX = $75,000 is almost 120 feet. Similar results were encountered on the performance optimizations of the fully looped layout for one and two emergency loading conditions.

As discussed in section 5.5.2.6 unremovable infeasibilities in the loop or source equations, i.e., nonzero dummy valve variables, may indicate the need for a real valve in the system. However, in this case there appears to be a need for a real valve to reduce the head provided by the elevated storage under the normal loading

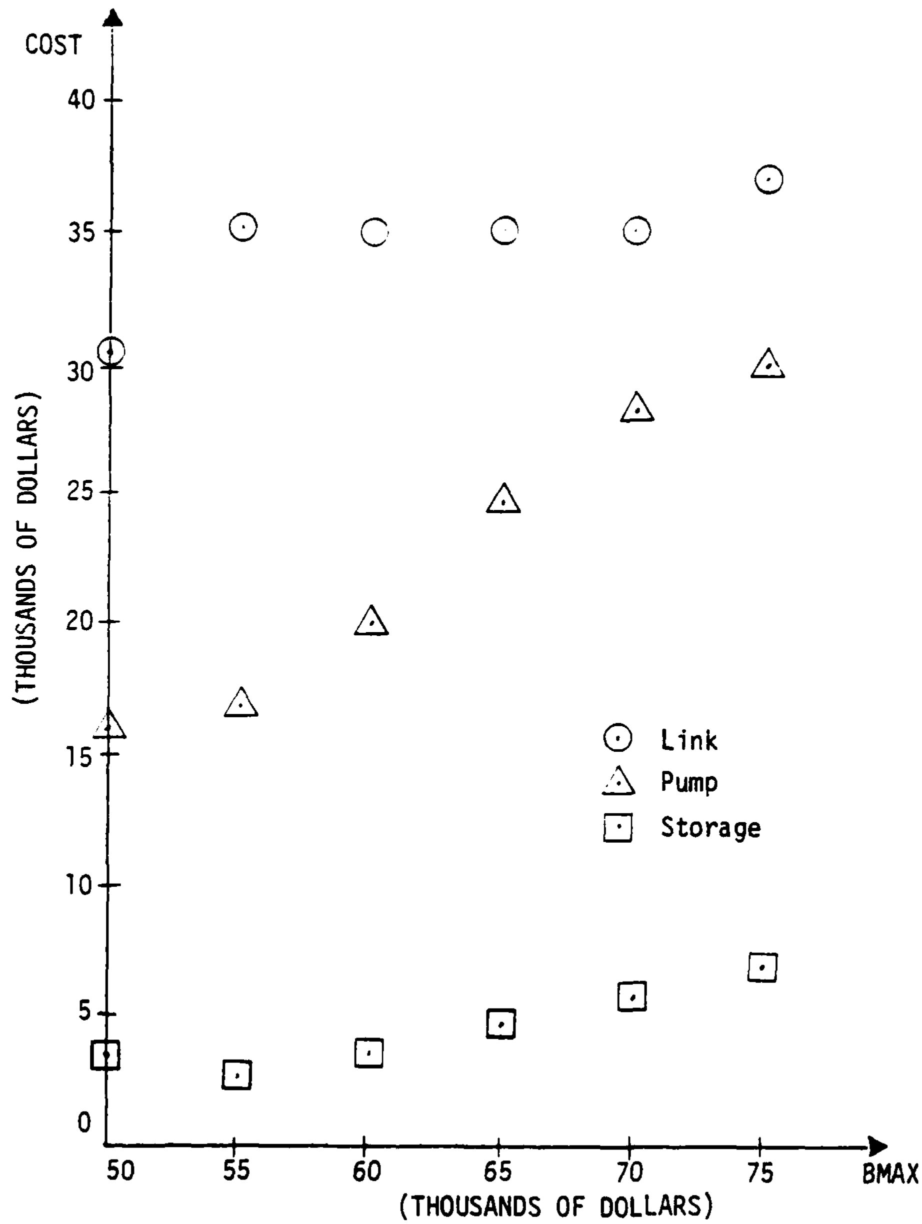


Figure 5-17

COST BREAKDOWN CORE TREE LAYOUT (NO SOURCE VALVE)

to allow the normal pump to operate at a lower head but at the same time allow the extra storage head to be available in case of emergency loading conditions. This was done by setting the penalty costs of the dummy valves on the normal loading source equation to zero and adding an upper bound equation to the model on the amount of resistance, RMAX, that the valve can provide, i.e.,

$$XP_1(1) - XS_1 \pm \sum_{k \in PATH_{12}} \Delta HF_k(1) + XV_1^+ - XV_1^- = 20 \tag{5-59}$$

and

$$XV_1^+ + XV_1^- \leqq RMAX \tag{5-60}$$

XV_1^+ corresponds to a pressure reducing valve located at the elevated storage reservoir and XV_1^- to a pressure reducing valve at the pump station. Also, any nodal pressure constraint referencing a source node with an active valve must include the valve to properly compute the nodal head. To implement the final system design a pressure reducing valve with maximum resistance given by the optimal valve resistance will be placed in the system for use under the normal loading to allow the system to balance. Figures 5-18 and 5-19 show the corresponding changes in head values and system costs for the tree layout resulting from adding the normal valve. Although

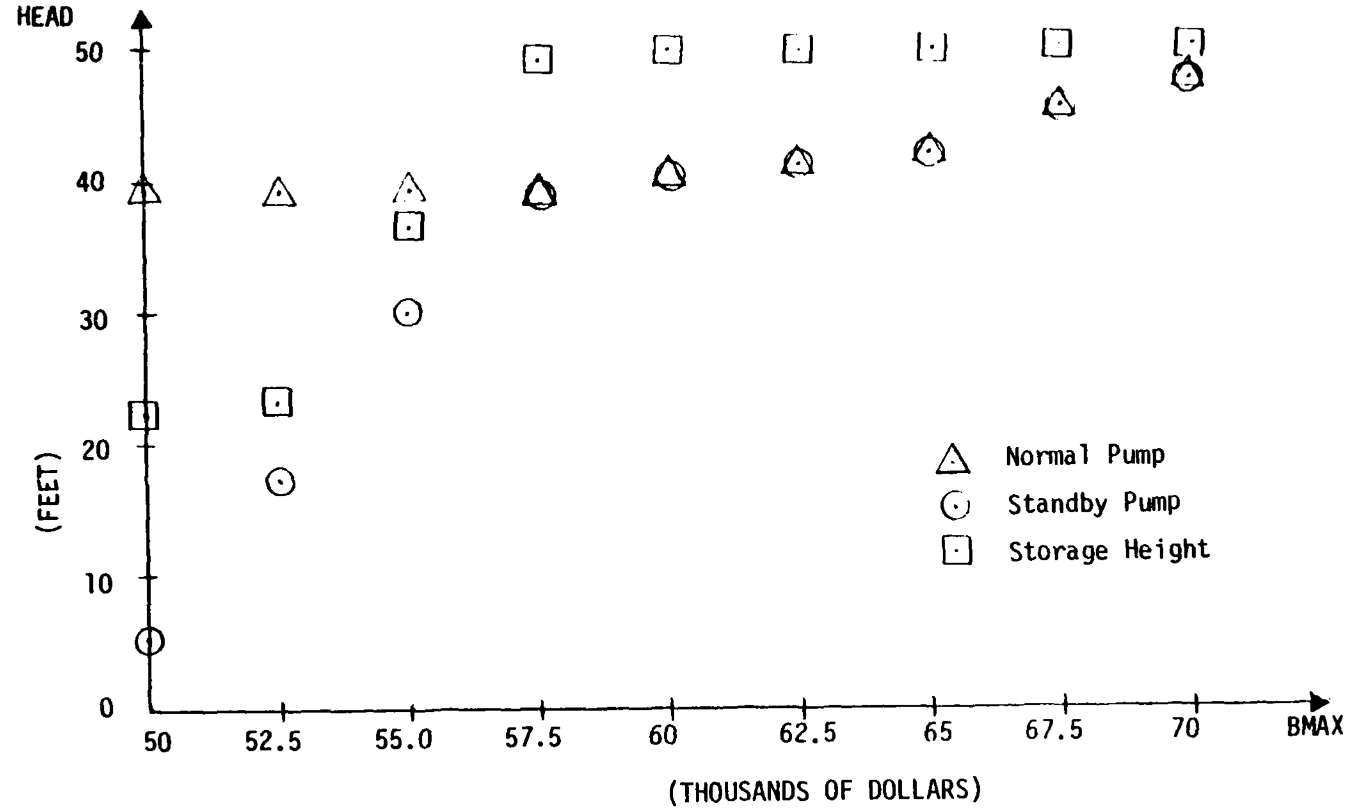


Figure 5-18

ADDED EXTERNAL ENERGY

CORE TREE LAYOUT (SOURCE VALVE)

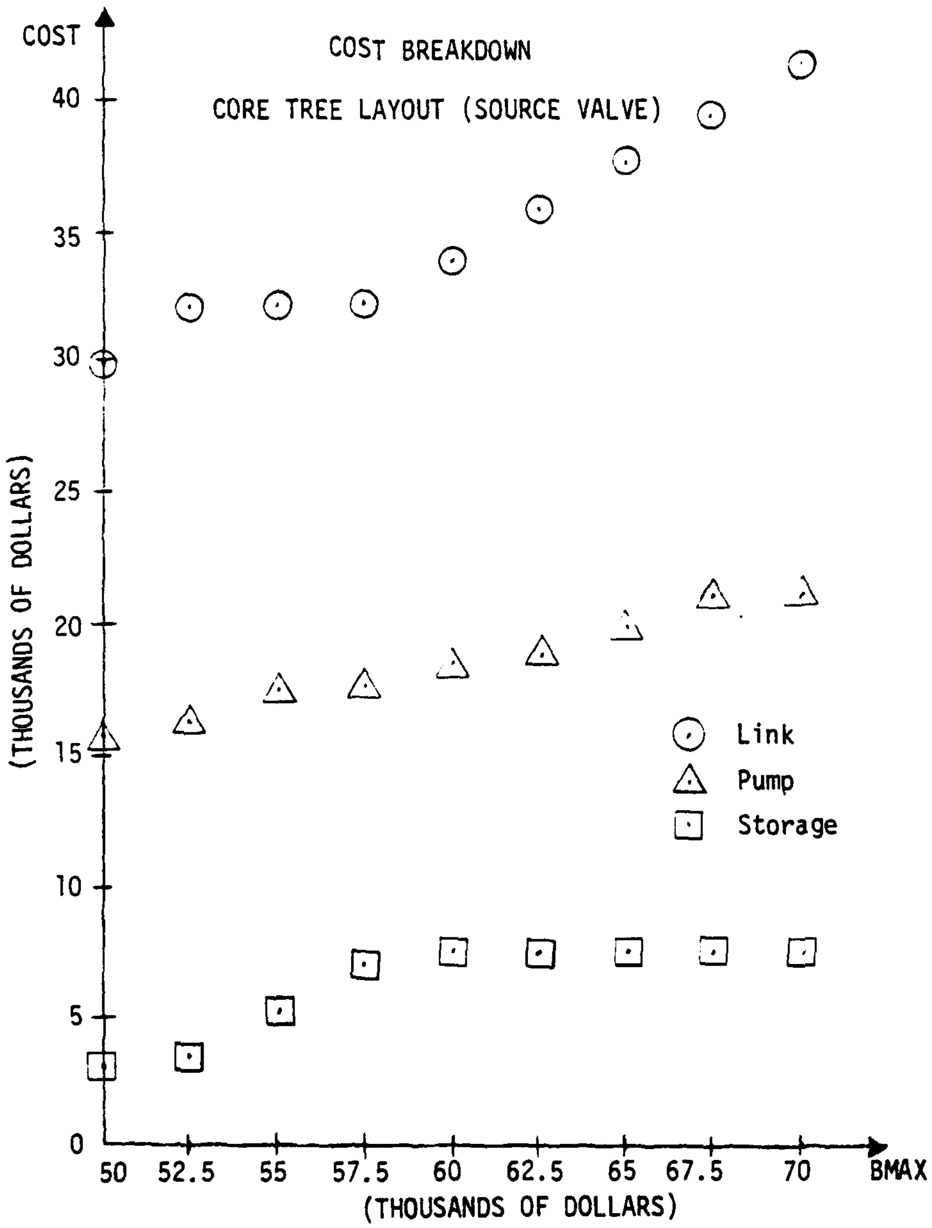


Figure 5-19

COST BREAKDOWN CORE TREE LAYOUT (SOURCE VALVE)

normal pumping head increases slightly over the budget range, this results from the constraint that the head lift of the standby pump cannot exceed the normal pumping head lift. Thus, to increase the system performance once the storage has reached its maximum height requires the normal head lift also to increase at a very high cost. All subsequent results have normal loading pressure reducing valves in the system. Because of the large reduction in costs from this change, the budget increment was reduced to $2,500 and the optimization was terminated when the minimum pressure approached normal minimum requirements of 90 feet.

5.5.4.3.4 Discussion of Results

Figure 5-20 shows the concave cost vs performance tradeoff curves for both the core tree and fully looped network layouts. Since the core tree can satisfy normal loading condition requirements at minimal cost, it has more funds than the looped layout available to allocate to maximize performance on the fire demand emergency loading condition. However, this result does not apply to the broken link emergency loading conditions.

Analysis of the performance/cost curves for both layouts reveals three distinct sections:

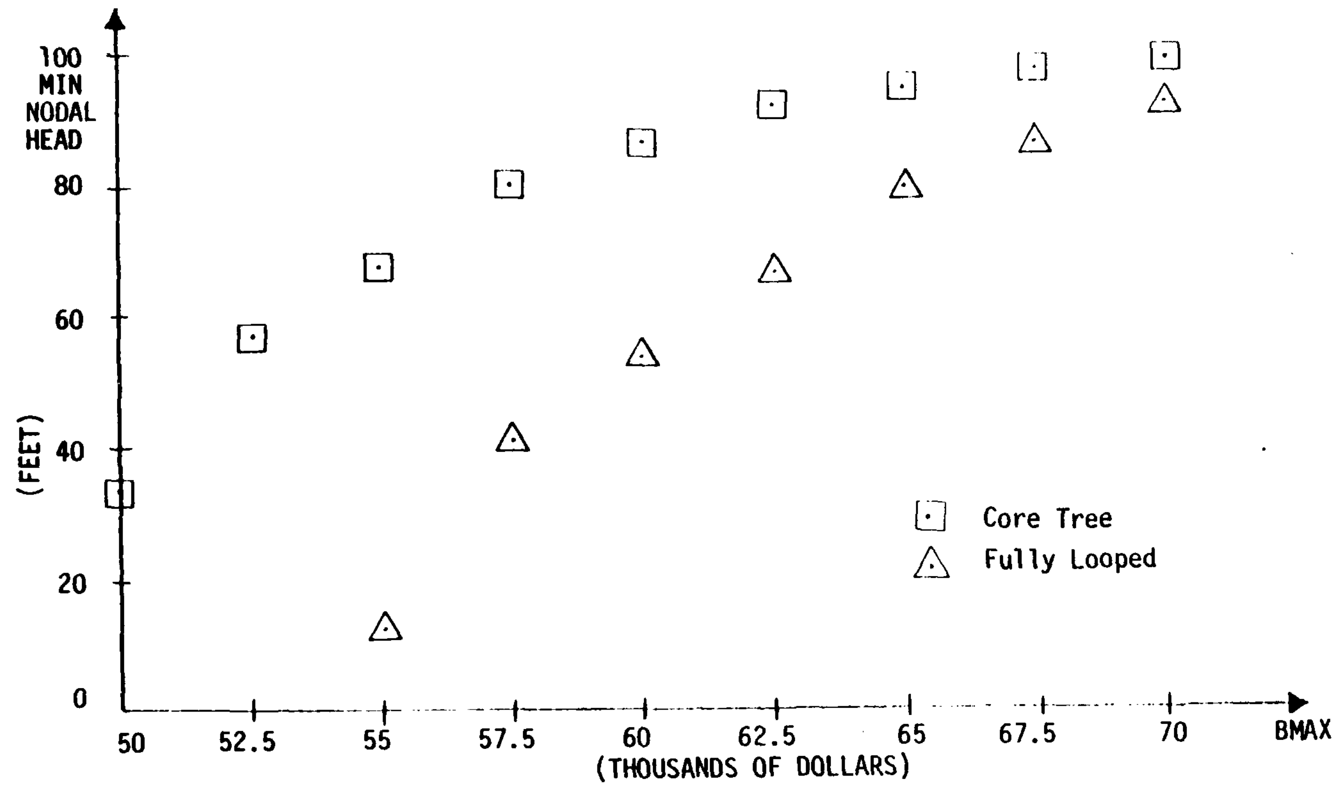


Figure 5-20

PERFORMANCE VS BUDGET LEVEL FIRE DEMAND LOADING CONDITION ONLY

1. A strictly concave section at low budget levels where small budget increases result in large performance increases.
2. A linear section in the middle where performance increases are directly proportional to budget increases.
3. A strictly concave section at the end where performance increases very slowly with budget.

The first section corresponds to rapid growth in the cost of all budget components, link, pump, and storage. The increasing system performance results both from decreasing frictional head loss as link diameters increase and from increasing external energy from pumps and storage. For storage elevation the added head is linearly proportional to the cost. For pump head lift the cost/head lift relationship is mildly concave. For link k the frictional head loss ΔHF_k is inversely proportional to the link diameter D_k

$$\Delta HF_k \sim \frac{1}{D_k^m} \tag{5-61}$$

and its diameter is directly proportional to its cost C_k

$$D_k \sim (C_k)^{1/\ell_2} \tag{5-62}$$

Substituting for D_k in (5-61) and differentiating with respect to C_k, we have

$$\frac{\partial(\Delta HF_k)}{\partial C_k} \sim \frac{-m}{\ell_2}\left(\frac{1}{(C_k)^{m/\ell_2+1}}\right) \qquad (5\text{-}63)$$

which is equal to $-3.78/(C_k)^{4.78}$ for the values $m = 4.87$ and $\ell_2 = 1.29$ used in the computation. This result indicates that the rate of reduction in frictional head loss decreases significantly with the amount, C_k, invested in link k. It explains the sharp but marginally decreasing performance improvements for small budget increases above the minimum budget level.

When the marginal return from allocating additional funds to increasing link diameters decreases sufficiently, the link cost component and the link design stabilizes. The budget increment is then completely allocated to providing increased head from pumps and storage. Since the storage costs are linear and the pump capital costs are mildly concave, the performance increase on the second section of the curve is almost directly proportional to the budget increment.

The third section of the curve begins when the storage height reaches its maximum elevation of 50 feet. Further small

performance increases require a combination of expensive normal pump head increases and larger diameter links. This results in the final strictly concave section with rapidly decreasing marginal returns.

5.5.4.4 Performance Optimization of Fire Demand and Broken Link Loading Conditions

As discussed in Chapter 4, broken link loading conditions are usually taken into account by solving the set or flow covering models. However, if failure of a specific primary link could have a catastrophic impact on the system, this loading condition can be incorporated into the detailed system design. The purpose of this section is to illustrate the model's capability to handle the broken link loading condition and multiple emergency loading conditions.

5.5.4.4.1 Broken Link Loading Condition

The broken link loading condition, failure of primary link 3, is shown in Figure 5-5. The nodal demands are average daily demands (1/2 peak hour). It is assumed that all three normal pumps are operating and that their common head lift on the emergency loading cannot exceed their head lift on the normal loading. Path constraints for this emergency loading are written in the usual manner

except that no constraint for the loading can contain link 3 and the loop associated with link 3 is deleted.

5.5.4.4.2 Discussion of Results

5.5.4.4.2.1 Equal Weights

Using the same procedure as in section 5.5.4.3, the MAXWMIN problem was solved for budget levels ranging from $62,500 to $75,000 in $2,500 increments with equal objective function weights assigned to each loading. The behavior of the total performance/cost curve in Figure 5-21 displays the same concave pattern previously noted for fire demand performance alone. However, the individual loading head curves, although monotonically increasing, do not share the same pattern. This result is not unexpected since the solution algorithm must allocate the given budget based on the overall system performance on all emergency loadings. Figure 5-22 and 5-23 display the optimal nodal and head distribution for BMAX = $70,000 for the fire demand and broken link loadings, respectively.

5.5.4.4.2.2 Unequal Weights

Figure 5-24 illustrates the sensitivity of the optimal solution to changes in emergency loading weighting coefficients for

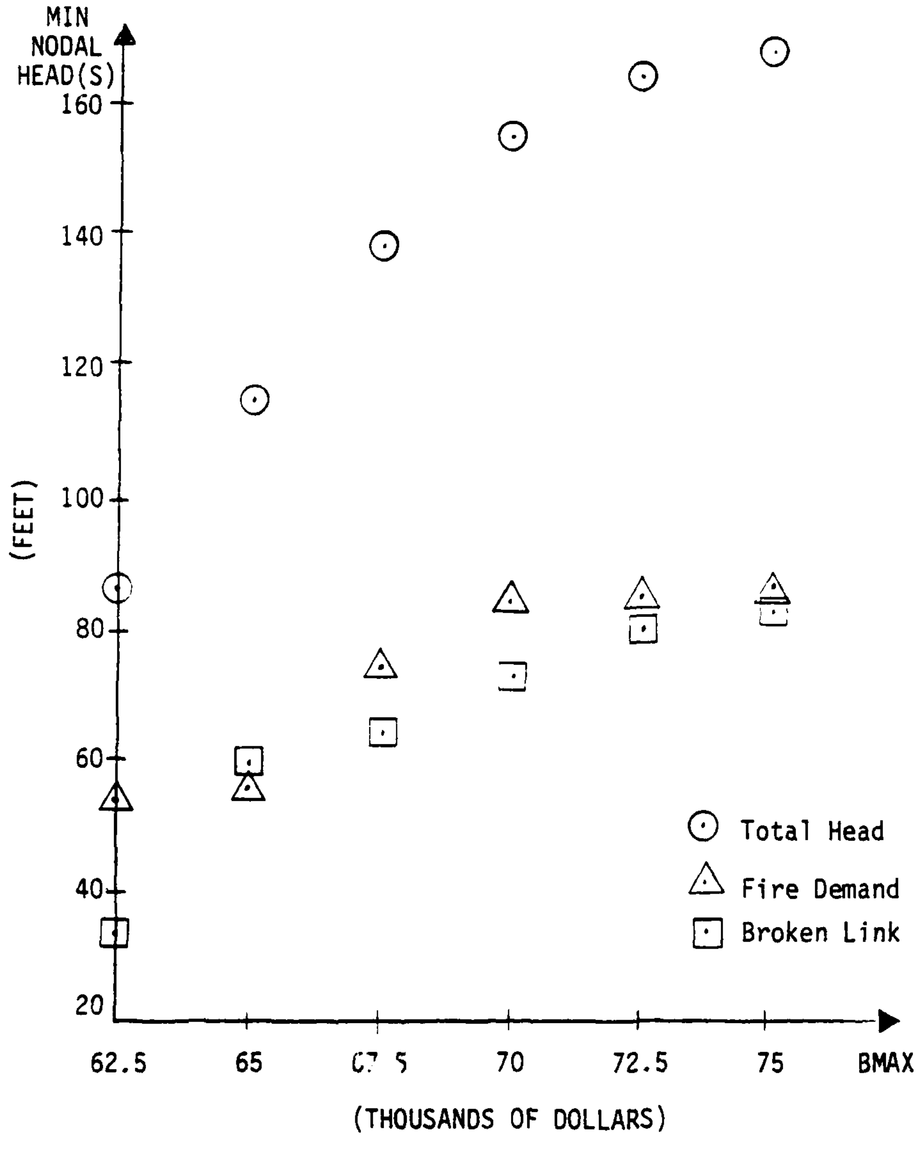


Figure 5-21

PERFORMANCE VS. BUDGET LEVEL
FIRE DEMAND AND BROKEN LINK LOADING CONDITIONS

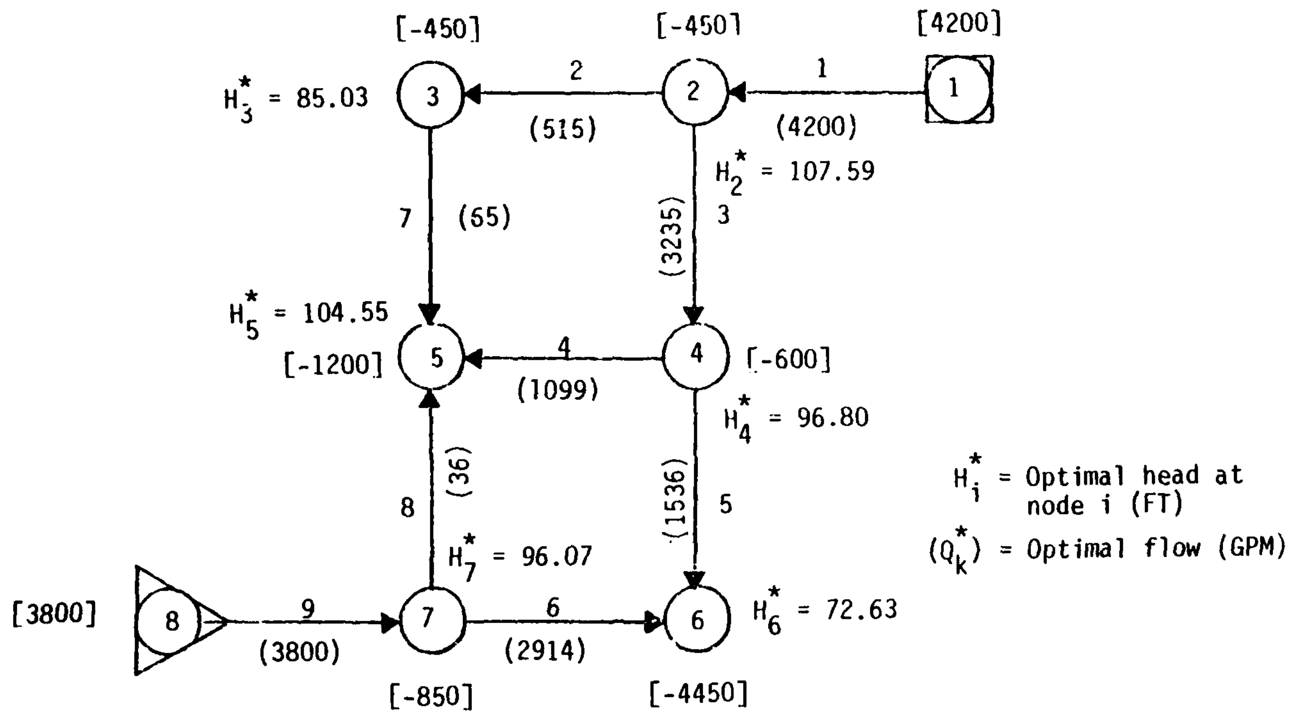


Figure 5-22

FIRE DEMAND LOADING CONDITION
OPTIMAL FLOW AND NODAL HEAD DISTRIBUTION
BMAX = $70,000

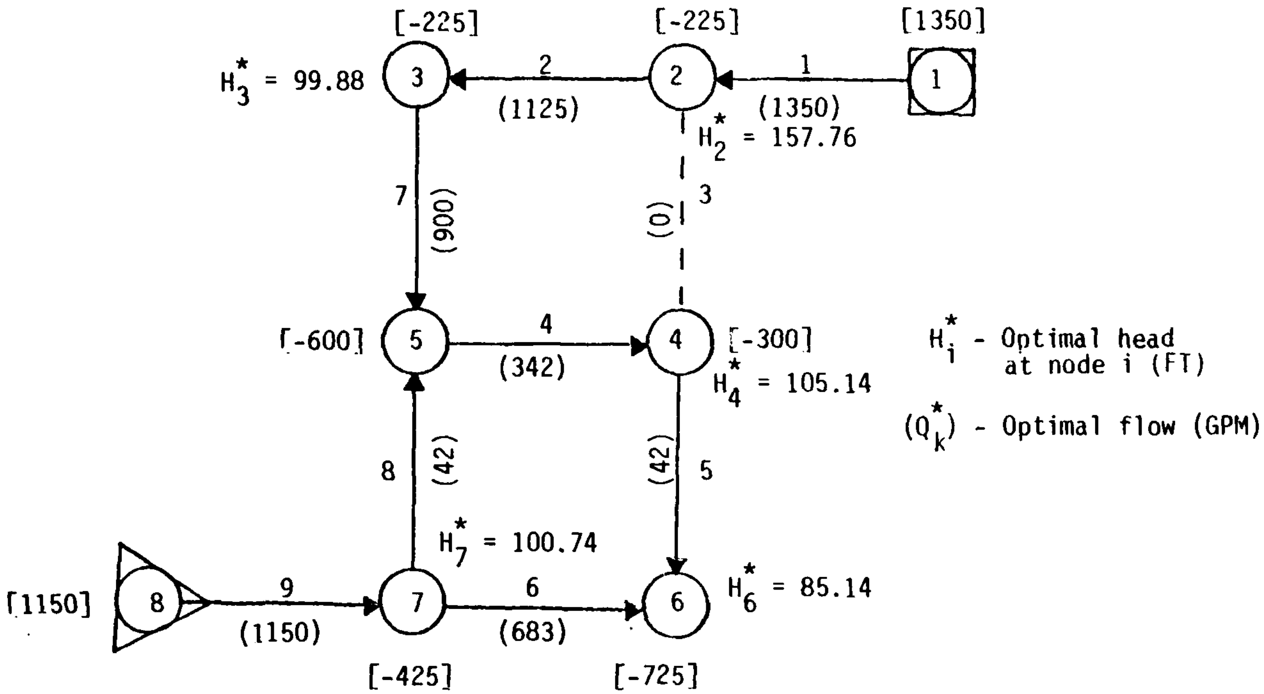


Figure 5-23

BROKEN PRIMARY LINK LOADING CONDITION
OPTIMAL FLOW AND NODAL HEAD DISTRIBUTION
BMAX = $70,000

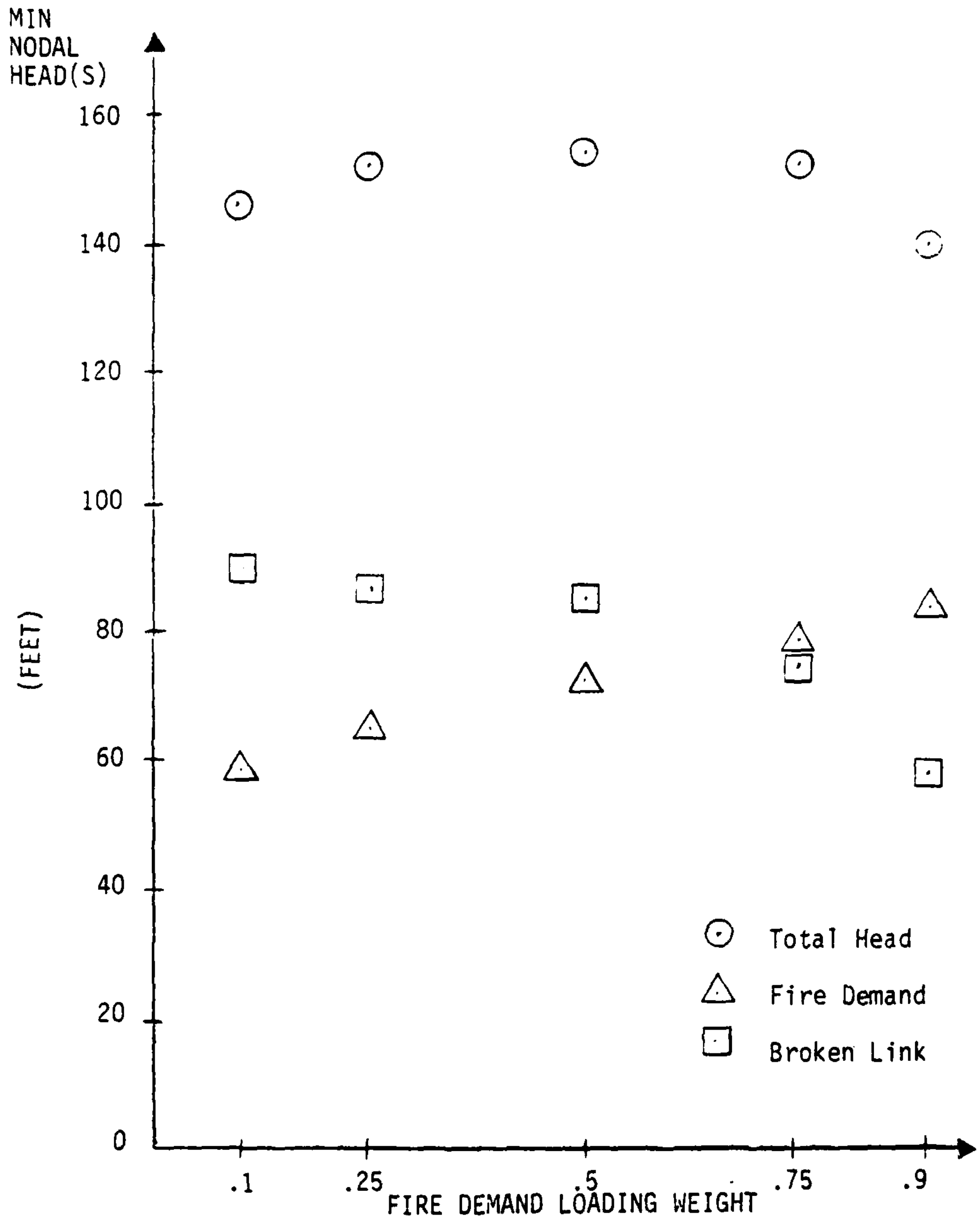


Figure 5-24

SENSITIVITY TO OBJECTIVE FUNCTION WEIGHTING COEFFICIENT CHANGES
BMAX = $70,000

BMAX = \$70,000. The horizontal axis is the weighting coefficient for the fire demand loading. The corresponding broken link weighting coefficient is found by subtracting the fire demand weighting coefficient from 1. The total objective function value for this particular problem is not especially sensitive to small changes in the weighting coefficients. As the fire demand loading weighting coefficient increases the optimal solution reallocates funds from increasing the diameters of links 2 and 7, which carry the water flow formerly transported by link 3, to increasing the head on the standby pump.

5.5.4.4.2.3 System Design Comparison

This section compares the minimum cost core tree layout with the maximum performance fully-looped system for BMAX = \$70,000. The \$24,177 cost difference between the two systems includes \$19,776 for links, \$2,577 for storage height, and \$1,824 for pumping. Of the added link costs \$12,258 was allocated to redundant links. The height of the storage reservoir increased by 17.7 feet. The \$1,824 pumping cost increase was a combination of a \$236 decrease in normal pumping cost and \$2,060 for a standby pump capable of providing 33.9 feet of head lift at a flow rate of roughly 2300 GPM. A comparison of the link designs from both optimizations (Tables 5-4 and 5-6)

Table 5-6

OPTIMAL PERFORMANCE LINK DESIGN FIRE DEMAND AND BROKEN LINK LOADINGS, BMAX = $70,000

Link No.	Total Length (ft)	Segment 1		Segment 2	
		Diameter	Length	Diameter	Length
1	3000	16	646	18	2354
2	2500	8	52	10	2448
3	1000	18	162	20	838
4	1500	12	241	14	1259
5	3000	10	98	12	2902
6	3500	16	3500		
7	4500	12	4500		
8	5000	6	4455	8	545
9	100	18	100		

reveals that the major increases in link diameter occurred in links 3, 4, and 5, all of which played a significant role in the emergency loading conditions.

5.5.5 Overall Assessment

The solution algorithm has proven itself effective for solving the MAXWMIN problem for small distribution system design problems. Using the step-by-step method for selecting the MAXWMIN initial flow distributions described in section 5.5.2.3 has been particularly helpful in accelerating convergence to a local optimum. The introduction of real valves on the source path for multiple source systems has allowed a more realistic design of the system. Nevertheless, the true test of the solution algorithm must be its ability to design realistic size systems to be treated in Chapter 6.

CHAPTER 6

APPLICATION OF METHODOLOGY

6.1 Introduction

Chapters 3, 4, and 5 developed an hierarchical system of mathematical models for complete design of a water distribution system. Emphasis was placed on laying a firm theoretical foundation for the models. Applications of the solution algorithms were limited to small example problems and principally for illustrative purposes. However, for the system of models to be truly practical, each model must be capable of satisfactorily handling the size of problem encountered during the reconnaissance phase of water distribution system design (section 2.2). This chapter applies the methodology developed in the previous chapters to a realistic distribution system design problem.

Some of the major considerations in successful application of a mathematical computer model to a real life problem include:

1. There exists real limits on the amount of computer storage available.

2. The confidence that can be placed on the results of the model is heavily dependent on the accuracy of the input data.

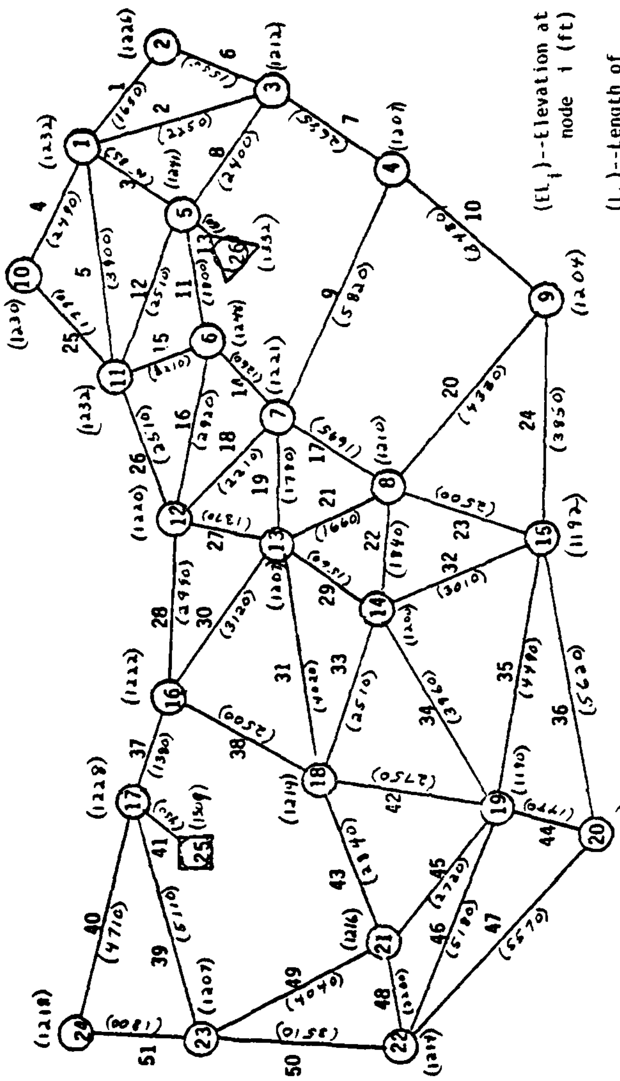
In the course of applying the hierarchical system of models to a realistic size distribution system design problem, certain difficulties arise in rigidly applying the theoretical model to the real life system. These difficulties are principally encountered in the detailed design phase. The successful resolution of this conflict between the theoretical model formulation and the practical model application form an important part of this research.

6.2 Description of System

The design methodology was applied to a real life distribution system analyzed by Alperovits and Shamir [46]. To reflect the layout problem encountered by the system designer during the reconnaissance design phase the final network layout was skeletonized, i.e., aggregation of smaller nodal demands, and additional potential links were included in the system.

6.2.1 Distribution System Topology

The network of 26 nodes and 51 potential links is shown in Figure 6-1 including link lengths and nodal elevations in feet.



(EL_i)--Elevation at node i (ft)

(L_k)--Length of link k (ft)

Figure 6-1
DISTRIBUTION SYSTEM TOPOLOGY

Nodes 1-24 are demand nodes, nodes 25 is an elevated storage reservoir and node 26 is a pumping station.

6.2.2 Pumps

Because of lack of data on the actual pumping arrangement for the system [46], the guidelines of Al-layla et al. [26] were used for the normal system pumping at node 26. Four identical pumps operating in parallel are used on the normal loading condition. Two identical standby pumps are available to replace out-of-service normal pumps. A variable speed pump designed to operate in parallel with the normal pumps is available to provide increased fire flow. Although not necessary to provide the required fire demand flow, booster fire pumps placed in series with the other pumps at the pump station at node 26 and in series with the elevated storage reservoir at node 25 may be required under the fire demand loading condition to increase pressure at the fire demand node. That is, if in the optimal solution the head-lift for a specific booster pump is non-zero, the need for a fire booster pump is indicated. Section 6.2.4 will discuss in detail the relationship between the pumps described above and the specific loading conditions under which each pump is designed to operate.

6.2.3 Elevated Storage Reservoir

The elevated storage reservoir at node 25 has a capacity of 1.68 million gallons necessary to handle normal (peak hour), fire fighting, and reserve demands. The cost of elevating the storage reservoir is $7000/ft and maximum storage elevation is 50 feet [46].

6.2.4 Loading Conditions

6.2.4.1 Normal

Figure 6-2 shows the normal (peak hour) loading conditions.

6.2.4.2 Emergency

Based on Insurance Service Office [77] and state [65, 67] and municipal [66] guidelines, two fire demand emergency loading conditions were selected.

1. Fire demand of 7500 GPM at node 9.

The flow for this demand will be supplied from the nearest source--the pumping station at node 26. Consistent with fire insurance guidelines [80], this loading condition assumes that two normal pumps are out of service and are replaced by the two identical standby pumps. An additional variable speed pump will be operating

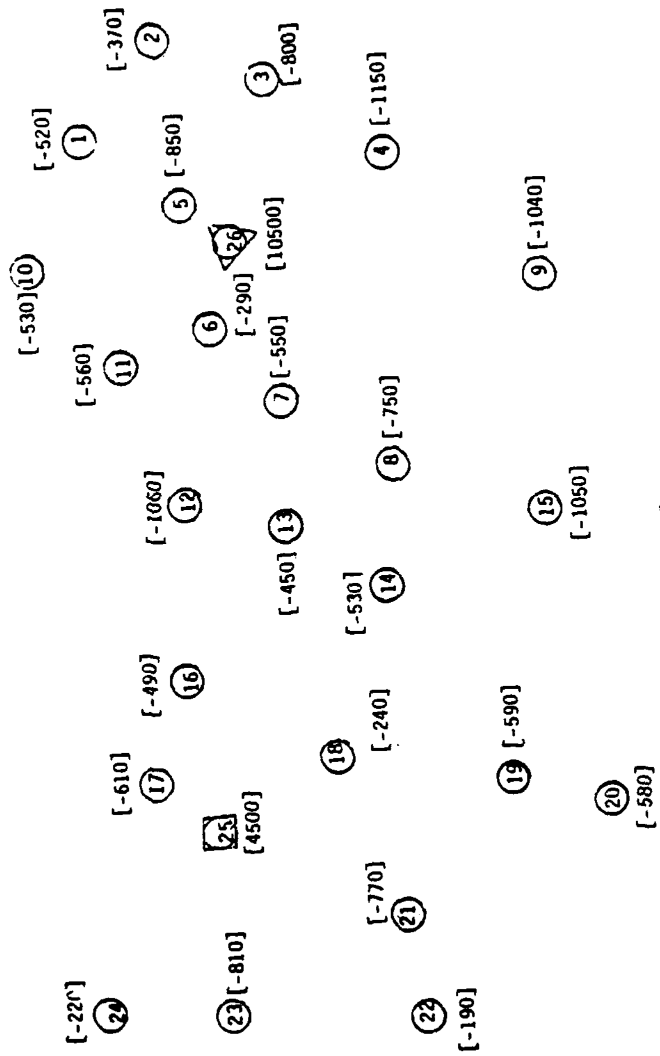


Figure 6-2
NORMAL LOADING CONDITION

in parallel with the other 4 pumps providing the additional 7500 GPM fire demand flow.

2. Fire demand of 3000 GPM at node 22.

The flow for this fire demand will be supplied from the nearest source--the elevated reservoir at node 25. Because of the remoteness of this fire demand and the relatively small normal demand in this area, two booster pumps--one in series with the elevated storage reservoir and the other in series with the other pumps at the pumping station--have been added to the network configuration to allow the system to add additional pressure to the fire demand node.

Consistent with standard practice [80] both of the above fire demands are assumed to occur simultaneously with the normal loading condition but not simultaneously with one another.

6.3 Selection of Tree Layout

6.3.1 Introduction

The first level model in the hierarchical system selects the layout of the minimal cost tree, i.e., the core tree. Applying the Matrix Tree Theorem for Graphs (section 3.3.1), there are more than 6.5×10^{10} possible spanning tree layouts making enumeration and optimization of all possible tree layouts impractical. This section

applies the shortest path tree and nonlinear minimum cost flow models to selecting the layout along with the intuitively appealing minimal spanning tree model. It concludes with a comparison of the two candidate models.

6.3.2 Shortest Path Tree Model

6.3.2.1 Assignment of Demand Nodes to Sources

To use the shortest path tree model for a multiple source system we must first assign demand nodes to their primary sources. Using the normal loading external flows (Figure 6-2) and the link lengths (Figure 6-1), application of the linear minimum cost flow problem (Problem P4) assigns demand nodes 1-15 to source node 26 and demand nodes 16-24 to source node 25.

6.3.2.2 Application of Model

Since the links are assumed to have unlimited flow capacities, the optimal solution of the minimum cost flow problem of the previous section transports water from the source to the demand nodes it supplies along the shortest path between them. Thus, the links with nonzero flow in the minimum cost flow solution are also

the links in the shortest path tree for each source which is shown in Figure 6-3.

To form the core tree for the system we must select a primary link to connect the separate spanning trees. Although the choice is somewhat arbitrary, two good candidates are the shortest link between the two trees, link 33, and the link completing the shortest path between the two sources, link 28. Althouth link 33 was chosen based on cost considerations, because in a distribution system with balancing storage water will be flowing from node 26 into the elevated storage reservoir at node 25 during periods of low demand, link 28 is a good alternate choice.

6.3.2.3 Minimum Cost Design

Using only a single pump at node 26, the minimum cost for the shortest path core tree layout (Figure 6-4) was found to be \$134,707 including \$95,859 for links, \$28,649 for pumping (15.4 feet head lift), and \$10,199 for storage (20.0 feet elevation). Since this system has no reliability in case of link failure, pump outage, or fire demand in excess of normal demand, its cost represents a baseline for assessing the cost of increasing system reliability.

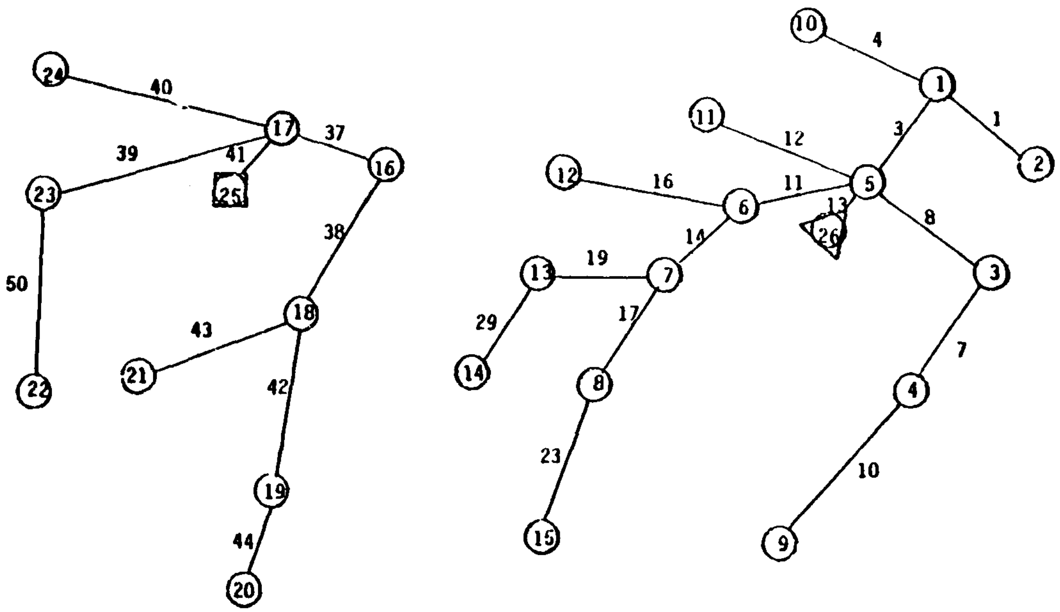


Figure 6-3

SHORTEST PATH TREE FOR EACH SOURCE

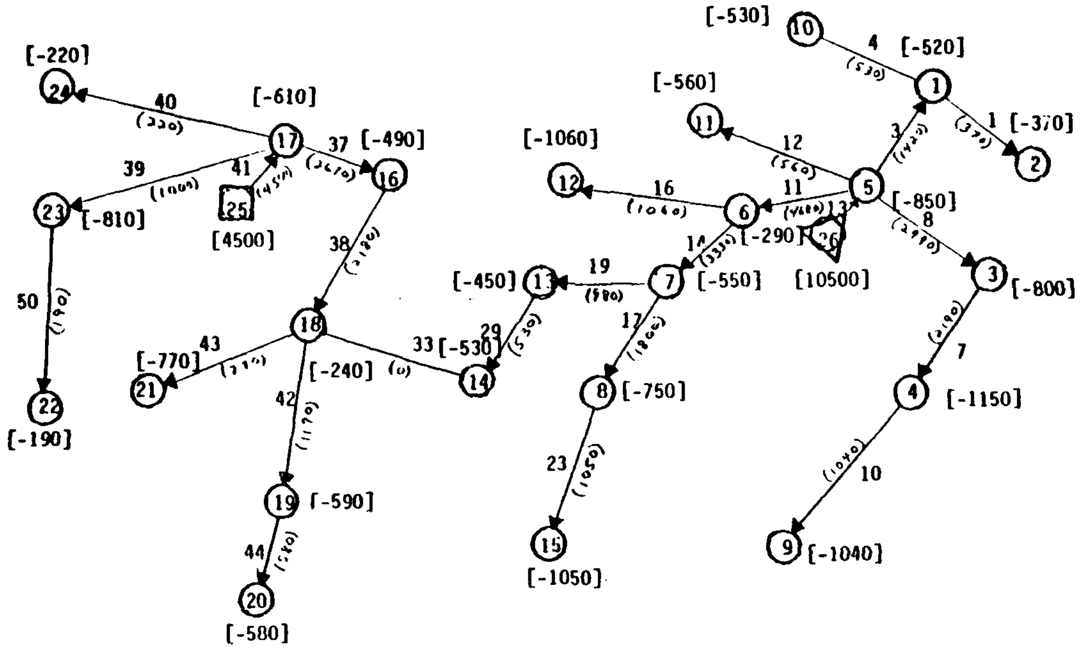


Figure 6-4

SHORTEST PATH TREE LAYOUT AND FLOW DISTRIBUTION

6.3.3 Nonlinear Minimum Cost Flow Model

6.3.3.1 Application of Model

Mylander's linear programming code LPREVISE [95] was modified to use the λ-method of separable programming to solve the nonlinear minimum cost flow model. The resulting program with 128 rows, 408 structural columns, and 1448 nonzero elements (density 2.11 percent) took 459 linear programming iterations and 284 seconds of CPU time on the University of Texas CDC 6400/6600 computer system. The high CPU time is attributable to implementation of the restricted basis entry criterion. The resulting tree layout is shown in Figure 6-5.

6.3.3.2 Minimum Cost Design

Again using only a single pump at node 26, the minimum cost design for the resulting network layout was found. The total cost of this system is $129,679 including $89,859 for links, $28,787 for pumping (15.5 feet for head lift), and $11,033 for storage (21.7 feet elevation). The cost reduction of $5,028 from the shortest path tree layout is principally due to the $6,000 reduction is link costs which the nonlinear flow model is expressly designed to minimize.

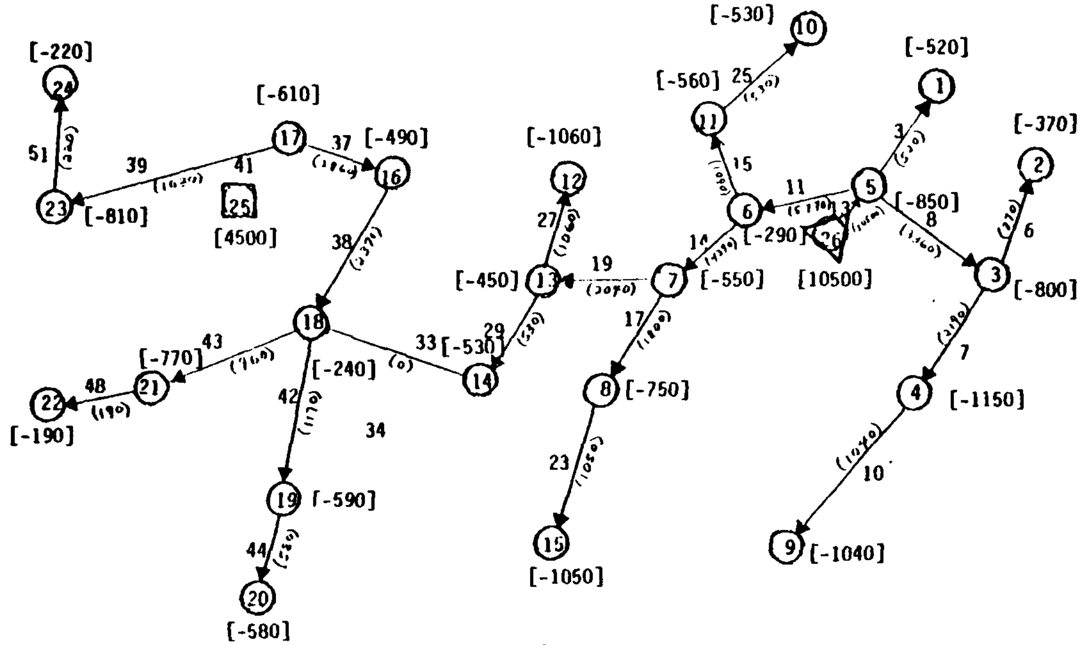


Figure 6-5

NONLINEAR MINIMUM COST FLOW TREE LAYOUT AND FLOW DISTRIBUTION

6.3.4 Minimal Spanning Tree Model

The concept behind this intuitively appealing model is to minimize the sum of link costs by installing a minimum length tree layout. For our problem the minimal spanning tree layout is shown in Figure 6-6. The minimum cost for this layout is $156,464 includ-ing $112,037 for links, $28,775 for pumping (15.5 feet head lift) and $15,652 for storage (30.8 feet elevation). This is roughly a 20% increase in cost over the other two models principally due to the need to install larger diameter links to accommodate higher link flows and to elevate the storage reservoir another 10 feet. In addition to its increased cost, because of the high link flows and extended structure, the minimal spanning tree is considerably more vulnerable to primary link failure than the other tree layouts.

6.3.5 Analysis of Results

6.3.5.1 Tree Structure

A comparison of the shortest path tree layout (Figure 6-4) and the nonlinear minimum cost flow tree layout (Figure 6-5) reveals similar tree structures especially along the links carrying large quantities of flow leaving each of the sources, i.e., links 3, 8, 7, 8, 11, and 14 for node 26 and links 37, 38, and 39 for node 25.

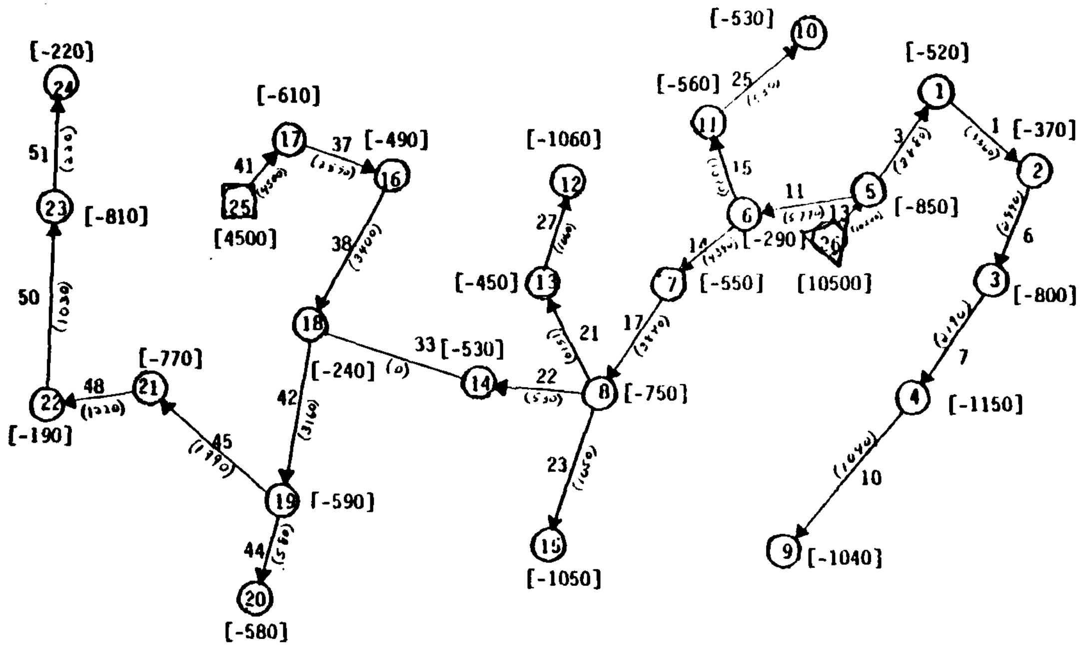


Figure 6-6

MINIMAL SPANNING TREE LAYOUT AND FLOW DISTRIBUTION

In other sections the trees complement each other, e.g., links 1 and 4, and 6 and 25. As expected, the shortest path tree shows a tendency to branch directly to demand nodes with slightly more links leaving well positioned nodes 5 and 17. This branching tendency leads to less vulnerability in case of primary link failure as evidenced by lower link flows on major primary links 11, 14, 19, 37, and 38.

6.3.5.2 System Cost

In section 5.3.2.6.2 the capital costs of the system were converted to equivalent uniform annual costs (EUAC) to allow the capital and operating costs to be combined in a single budget. Since the operating costs of both tree layouts are almost identical, it appears more appropriate to directly compare the initial capital costs of each layout, i.e., the value of the bond issued to finance the capital costs, to accurately assess the impact of using the different models. The cost breakdown in Table 6-1 shows a link capital cost savings of $83,506 and overall capital savings of $71,809 resulting from the nonlinear flow tree layout. This result provides a significant counterexample to Bhave's assertion [49] of the general optimality of the shortest path tree. This cost reduction can be attributed to the fact that the nonlinear

Table 6-1

COMPARISON OF THE TREE LAYOUT COSTS

SYSTEM ELEMENT	NONLINEAR FLOW TREE			SHORTEST PATH TREE		
	EUAC		PRESENT	EUAC		PRESENT
	CAPITAL	OPERATING	CAPITAL	CAPITAL	OPERATING	CAPITAL
LINK	89413	446	1,252,563	95,374	485	1,336,069
PUMP	613	28,174	8,587	612	28,037	8,573
STORAGE	11033	--	154,558	10,199	--	142,875
TOTAL	101,059	28,620	1,415,708	106,185	28,522	1,487,517

minimum cost flow model takes into account not only the link length but also the link flow distribution, the actual link capital costs, and the individual link roughness coefficients.

6.3.5.3 Computational Cost

The shortest path tree model took considerably less time to set up and to solve on the computer than the nonlinear flow model. This fact is a direct reflection of the relative complexity of the two models. However, from a practical viewpoint neither model took an excessive amount of time compared to the other proposed techniques (section 3.3).

6.3.5.4 Overall Assessment

The results of Table 3-2 (section 3.3.4.4) demonstrated that evaluation of a particular layout's tree path length or nonlinear flow cost is an accurate measure of the actual cost of the tree layout. Table 6-2, which presents the shortest path, nonlinear flow, and minimal spanning trees for the three measures used to derive them, further confirms the capability of the tree path length and nonlinear flow cost criteria to discriminate between economical and expensive core tree layouts.

Table 6-2

EVALUATION OF ALTERNATIVE MEASURES OF COST

LAYOUT	TREE PATH LENGTH (ft)	NONLINEAR FLOW COST (ft-GPM)	TOTAL LINK LENGTH (ft)	COST ($)
SHORTEST PATH TREE	110,205	1,820,430	57,525	134,707
NONLINEAR FLOW TREE	115,940	1,738,940	49,660	129,679
MINIMAL SPANNING TREE	150,995	1,948,020	46,450	156,464

Based on the cost reduction achieved by using the nonlinear minimum cost flow model, the increased computational burden of the nonlinear minimum cost flow model appears worthwhile. Moreover, because of the gross simplifications implicit in the shortest path tree model, the potential for significant cost savings over the wide range of distribution system design problems from using the nonlinear minimum cost flow model is considerable.

6.4 Selection of Redundant Links

6.4.1 Introduction

The next level model in our hierarchical system is responsible for selecting the redundant links to complete the network layout. This section will apply both the set covering model (Problem P6) and the flow covering model (Problem P7) to the shortest path and minimum nonlinear cost tree layouts (Figure 6-4 and 6-5)--the outputs of the first level models. This section will conclude with a comparison of the candidate models.

6.4.2 Failure Analysis of Tree Layout

For a multiple source system, failure analysis requires two major steps:

1. Identification of redundant links capable of covering the failure of each primary link (section 4.3.3).
2. Identification of primary links on all source-to-source paths whose diameter may be increased to cover failure of another primary link on the source-to-source path (section 4.4.4).

6.4.2.1 Shortest Path Tree Layout

Table 6-3 presents a failure analysis of the shortest path tree layout. To assist in following this analysis the shortest path tree with average daily demands and the non-tree (candidate redundant links) is shown in Figure 6-7. Column 1 of the table is the failed primary link. Column 2 is the set of demand nodes cut-off from the primary source by the primary link failure. Column 3 is the set of candidate redundant links capable of covering the failure of the primary links. These are the nonzero elements in the primary link covering constraints (equations (4-6) and (4-10)). Column 4 is the minimum required flow capacity (d_i) of the redundant and primary links serving the set of nodes disconnected from their principal source by the primary link failure in the flow covering model (Problem P7). This quantity is initially set equal to the average daily flow rate to the disconnected set of nodes,

Table 6-3

FAILURE ANALYSIS OF SHORTEST PATH TREE LAYOUT

PRIMARY LINK	NODES CUTOFF FROM PRINCIPAL SOURCE	COVERING CANDIDATE REDUNDANT LINKS	MINIMUM FLOW CAPACITY OF LINKS TO CUTOFF NODES	MINIMUM LINK COVER
1	2	6	185	1
3	1,2,10	2,6,5,25	710	1
4	10	25	265	1
7	4,9	9,20,24	1095	2
8	3,4,9	2,6,9,20,24	1495	3
10	6,7,8,12,13,14,15	20,24	520	1
11*	6,7,8,12,13,14,15	9,15,20,24,26,28,30,31,34,35,36	2340	5
12	11	5,15,25,26	280	1
13*	LINK ADJACENT TO SOURCE NODE 26			
14*	7,8,13,14,15	9,18,20,24,27,20,21,34,35,36	1665	3
16	12	18,26,27,28	530	1
17	8,15	20,21,22,24,32,35,36	900	2
19*	13,14	21,22,27,30,31,32,35,36	490	1
23	15	24,32,35,36	525	1
29	14	22,32,34	265	1
33*	NO NODES	CUTOFF FROM PRINCIPAL SOURCE		
37*	16,18,19,20,21	28,30,31,34,35,36,46,47,48,49	1335	3
38*	18,19,20,21	31,34,35,36,46,47,48,49	1090	2
39	22,23	46,47,48,49,51	500	1
40	24	51	110	1
41*	LINK ADJACENT TO SOURCE NODE 25			
42	19,20	34,35,36,45,46,47	585	1
43	21	45,48,49	385	1
44	20	36,47	290	1
50	22	46,47,48,49	95	1

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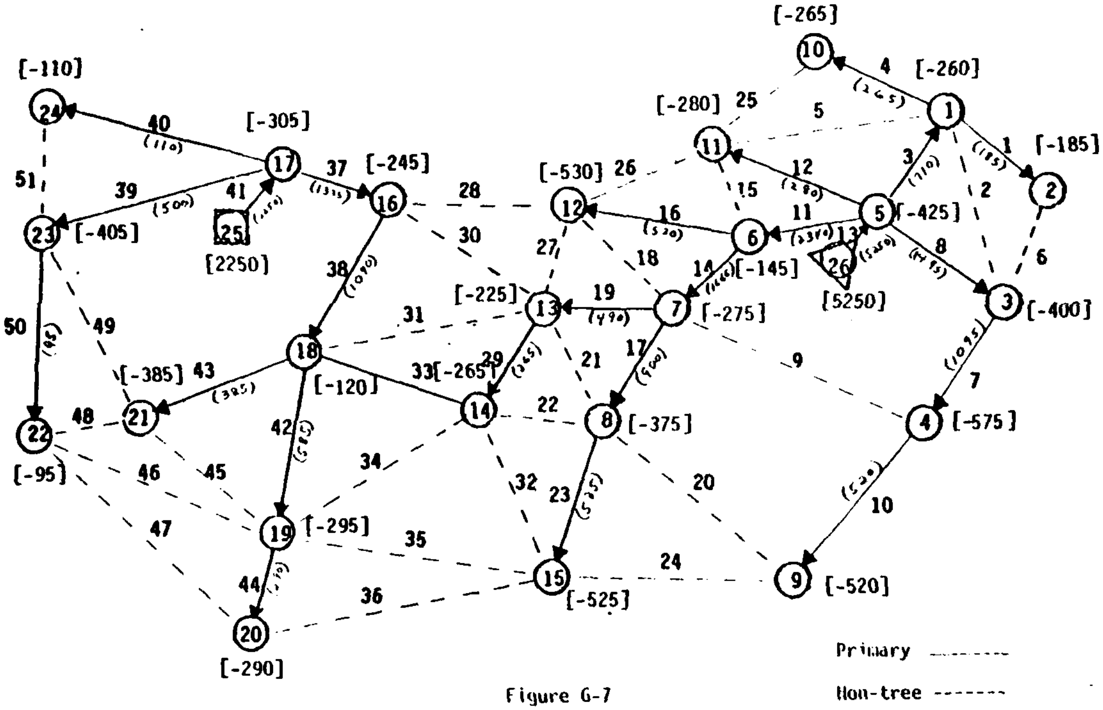


Figure G-7

SHORTEST PATH TREE LAYOUT AVERAGE DAILY LOADING CONDITION
(1/2 NORMAL DEMAND)

i.e., 1/2 the normal demand (peak hourly). The minimum flow capacity requirements for the primary links on the source-to-source path (starred in the table) were subsequently reduced by 360 GPM since the minimum diameter of the links on the source-to-source path in the MINCOST optimization (section 6.3.2.3) is 6 inches (link 33). Column 5 is the corresponding minimum link covering requirement (r_i) for the set covering problem (Problem P6). The requirements for primary links on the source-to-source path are likewise reduced by 1 to reflect the alternate supply source.

Table 6-4 presents a bottleneck link analysis of the primary links on the source-to-source path: Column 1 is the set of links on the source-to-source paths which are candidates for diameter increases. Column 2 is the link's optimal diameter in the shortest path tree's MINCOST optimization and Column 3 the accompanying empty flow capacity ($10\ D_k^2$). The entries in columns 4-9 are the average excess flow capacity for the primary links in column 1 available in case of failure of the primary links in each column

$$(QMAX_k - Q_{k_i})$$

(section 4.4.4). The link where the minimum excess capacity is achieved is the primary bottleneck for the failure of link i

Table 6-4

BOTTLENECK LINK ANALYSIS OF SHORTEST PATH TREE LAYOUT

LINK NO.	LINK DIAMETER (IN)	EMPTY FLOW CAPACITY (GPM)	AVERAGE EXCESS FLOW CAPACITY (GPM) AFTER FAILURE OF PRIMARY LINK NO. ($EQCAP_i$) 11	14	19	29	37	38
11	28	7840	x	x	x	x	5500	5500
14	16	2560	2560	x	x	x	895	895
19	10	1000	1000	x	x	x	510	510
29	8	640	640**	640**	640**	x	375**	375**
33	6	360	360*	360*	360*	360*	360*	360*
37	18	3240	1905	1905	1905	1905	x	x
38	16	2560	1470	1470	1470	1470**	2560	x

x = Failed link or link on path from disconnected source.

* = Primary bottleneck.

** = Secondary Bottleneck

(single star) and the minimum excess flow capacity is EQCAP (equation (4-14)). The secondary bottlenecks are indicated by two stars.

Since link 33 is the primary bottleneck for all link failures, we will consider incorporating the decision to increase the minimum link diameter on link 33 from 6 to 8 inches. For links 11, 14, 17, and 18 increasing link 33 to 8 inches gains 280 GPM and for links 37 and 38, 15 GPM. The cost for this increase is

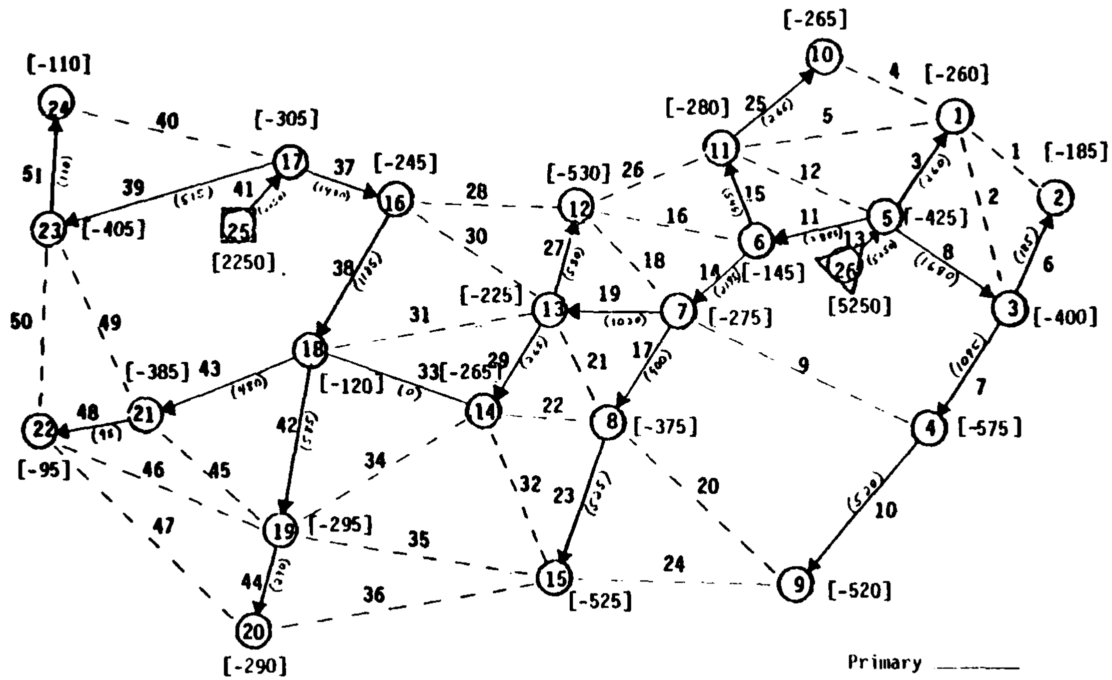
$$\ell_1 \left(8^{\ell_2} - 6^{\ell_2}\right) L_{33} \quad .$$

6.4.2.2 Nonlinear Minimum Cost Flow Tree Layout

The failure analysis for the nonlinear minimum cost flow tree layout (see Figure 6-8) is similar to the shortest path tree analysis and is presented in Table 6-5. Likewise, the accompanying bottleneck analysis is presented in Table 6-6.

6.4.3 Set Covering Model

The search enumeration 0-1 integer programming code RIP30C (Geoffrion and Nelson [96]) was used to solve both the set covering and flow covering models. The general procedure was to run RIP30C until either all possible solutions were enumerated, i.e., an optimal solution was found, or approximately 200 CPU seconds elapsed.



Primary

Non-tree

Figure 6-8

NONLINEAR MINIMUM COST FLOW TREE LAYOUT AVERAGE DAILY LOADING CONDITION (1/2 NORMAL DEMAND)

Table 6-5

FAILURE ANALYSIS OF NONLINEAR MINIMUM COST FLOW TREE LAYOUT

PRIMARY LINK	NODES CUTOFF FROM PRIMARY SOURCE	COVERING CANDIDATE REDUNDANT LINKS	MINIMUM FLOW CAPACITY OF LINKS TO CUTOFF NODES	MINIMUM LINK COVER
3	1	1,2,4,5	260	1
6	2	1	185	1
7	4,9	9,20,24	1095	2
8	2,3,4,9	1,2,9,20,24	1680	3
10	9	20,24	520	1
11*	6,7,8,10,11,12,13,14	4,5,9,12,20,24,28,30,31,34,35,36	2885	6
13*		LINK ADJACENT TO SOURCE NODE 26		
14*	7,8,12,13,14	9,16,20,24,26,28,30,31,34,35,36	2195	4
15	10,11	4,5,12,26	545	1
17	8,9	20,21,22,24,32,35,36	900	2
19*	12,13,14	16,18,21,22,26,28,30,31,32,34	1020	2
23	9	24,32,35,36	525	1
25	10	4	265	1
27	12	16,18,26, 28	530	1
29	14	22,32,34	530	1
33*	NO NODES	CUTOFF FROM PRINCIPAL SOURCE		
37*	16,18,19,20,21,22	28,30,31,34,35,36,49,50	1430	3
38*	23,24	31,34,35,36,49,50	1185	2
39	19,20	40,49,50	515	1
41*		LINK ADJACENT TO SOURCE NODE 25		
42	19,20	34,35,36,45,46,47	585	1
43	21,22	45,46,47,49,50	480	1
44	20	36,47	95	1
51	24	40	110	1

Table 6-6

BOTTLENECK LINK ANALYSIS OF NONLINEAR MINIMUM COST FLOW TREE LAYOUT

LINK NO.	LINK DIAMETER (IN)	EMPTY FLOW CAPACITY (GPM)	AVERAGE EXCESS FLOW CAPACITY (GPM) AFTER FAILURE OF PRIMARY LINK NO. ($EQCAP_i$)					
			11	14	19	29	37	38
11	30	9000	x	x	x	x	7115	7115
14	18	3240	3240	x	x	x	1045	1045
19	14	1960	1960	1960	x	x	840	840
29	8	640	640**	640**	640**	x	375**	375**
33	6	360	360*	360*	360*	360*	360*	360*
37	18	3240	1810	1810	1810	1810	x	x
38	16	2560	1375	1375	1375	1375**	2560	x

x = Failed link or link on path from disconnected source.

* = Primary bottleneck.

** = Secondary bottleneck.

Although this procedure did not always guarantee the optimal solution, in those cases where the time limit was reached, the best solution was almost always found within the first 20 seconds and the remainder of the 200 seconds spent eliminating inferior solutions. The above procedure was adopted to avoid the excessive computational cost of obtaining only a marginally better solution.

6.4.3.2 Results

The results of applying the set covering model (Problem P6) to the shortest path tree layout is depicted in the full network layout of Figure 6-9. All links were assumed to have the same minimum diameter of 6 inches. The associated equivalent uniform annual cost was $18,727.

The results of applying the set covering model to the nonlinear minimum cost flow tree layout is shown in the full network layout of Figure 6-10. The total equivalent uniform annual cost was $19,543.

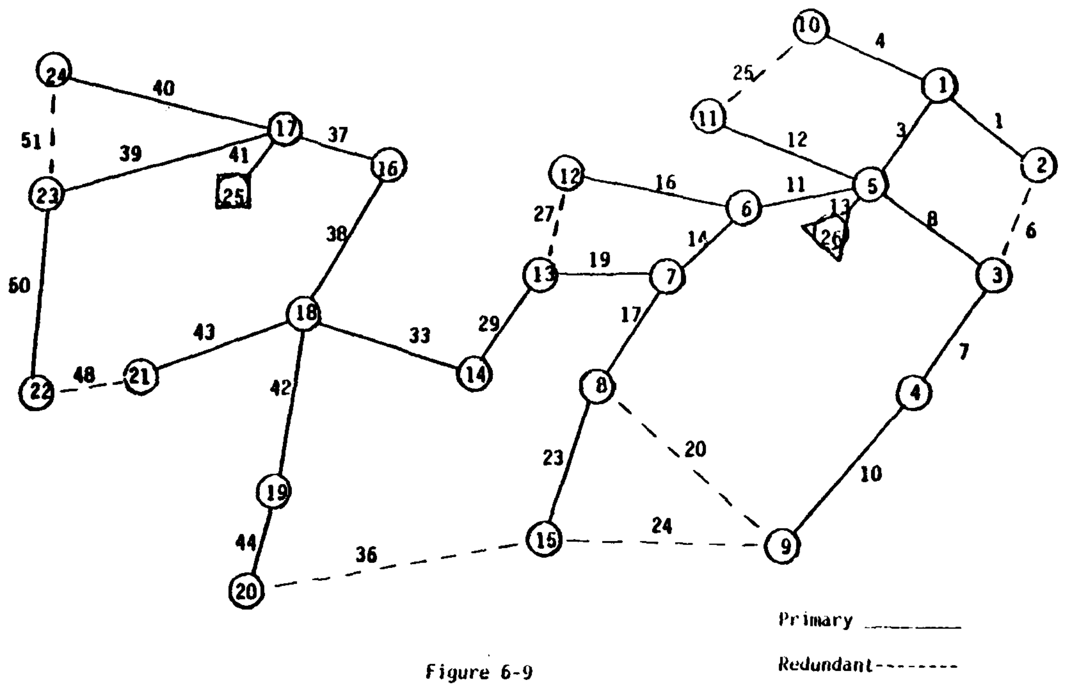


Figure 6-9

REDUNDANT LINKS SHORTEST PATH TREE LAYOUT SET COVERING PROBLEM

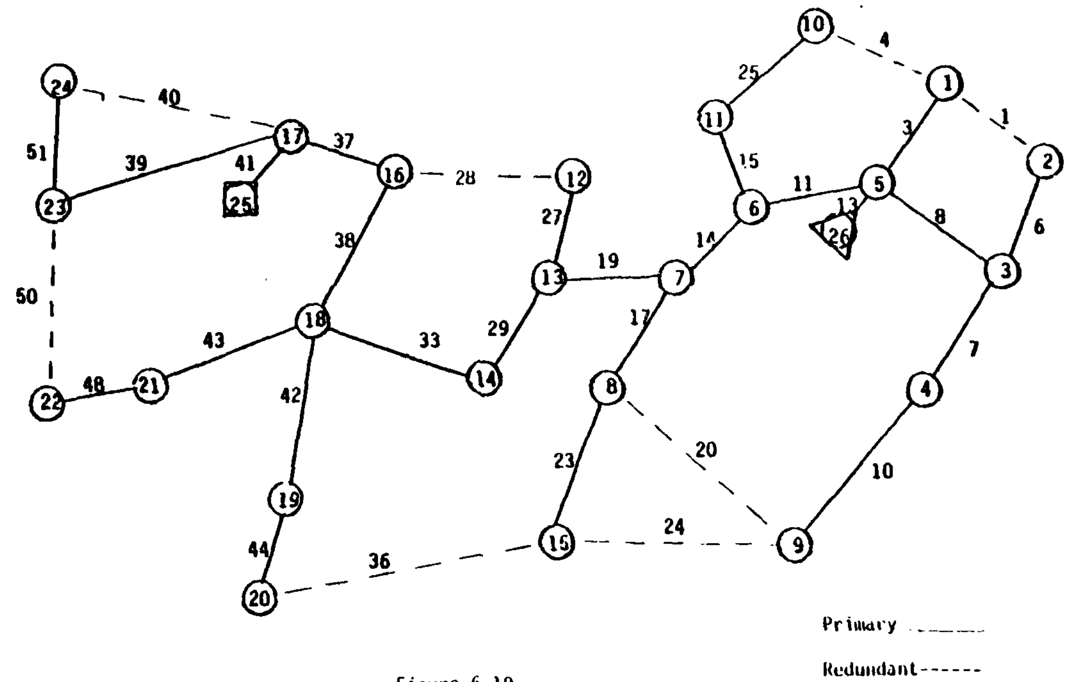


Figure 6-10

REDUNDANT LINKS NONLINEAR MINIMUM COST FLOW TREE LAYOUT SET
COVERING PROBLEM

6.4.4 Flow Covering Model

6.4.4.1 Introduction

To apply the flow covering model (Problem P7) an appropriate set of minimum candidate diameters S_k must be chosen for each link. Since most municipal systems use 6 or 8 inch minimum diameters, these were chosen as the two candidates. Since average daily flow can vary from 1/2 to 1/4 of normal (peak hour) demand, the problem was solved separately for minimum flow requirements (d_i) of 1/2, 1/3, and 1/4 normal demand.

6.4.4.2 Results

Figure 6-11 depicts the full network layout resulting from solving the flow covering problem for the shortest path tree layout with average daily flow equal to 1/2 normal flow demand. The total equivalent uniform annual cost for the redundant links is $22,572. Figures 6-12 and 6-13 show the resulting network for 1/3 and 1/4 normal flow demand which had costs of $19,612 and $14,830, respectively.

For the nonlinear minimum cost flow core tree the flow cover for 1/2 normal demand, shown in Figure 6-14, has a cost of

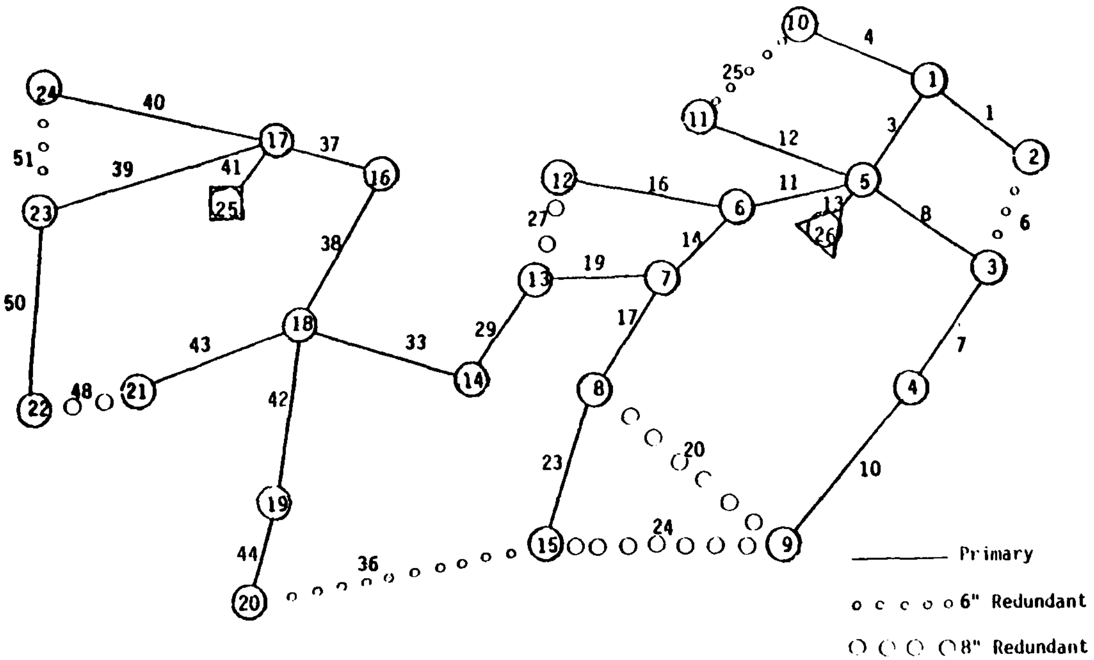


Figure 6-11

REDUNDANT LINKS SHORTEST PATH TREE LAYOUT FLOW COVERING PROBLEM
(1/2 NORMAL DEMAND)

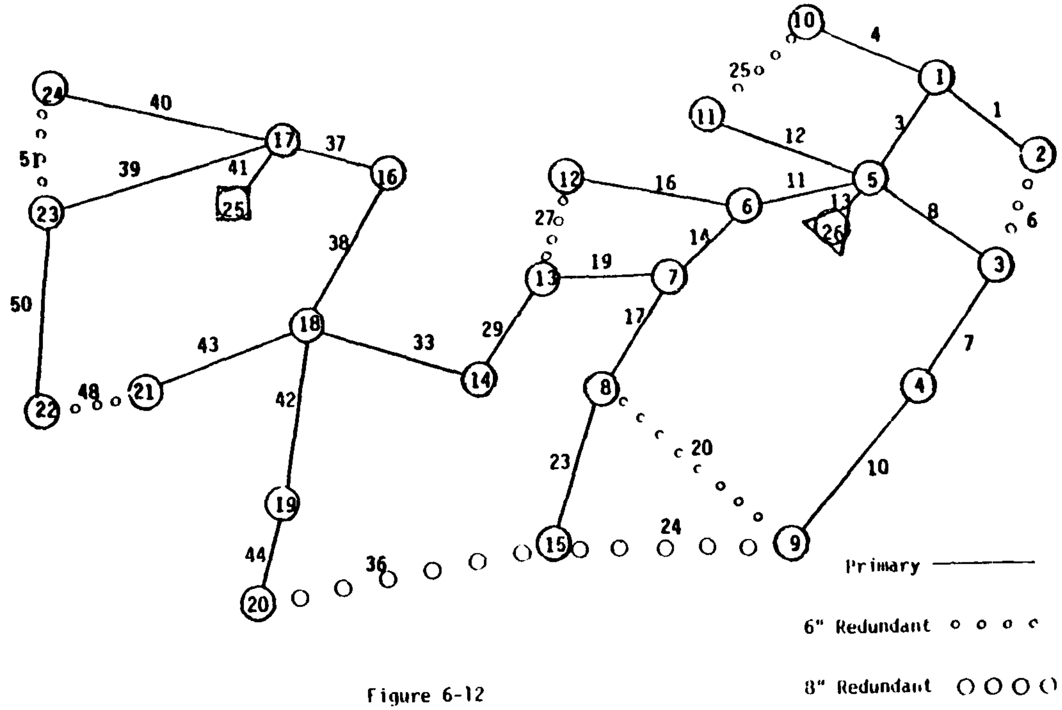


Figure 6-12

REDUNDANT LINKS SHORTEST PATH TREE LAYOUT FLOW COVERING PROBLEM
(1/3 NORMAL DEMAND)

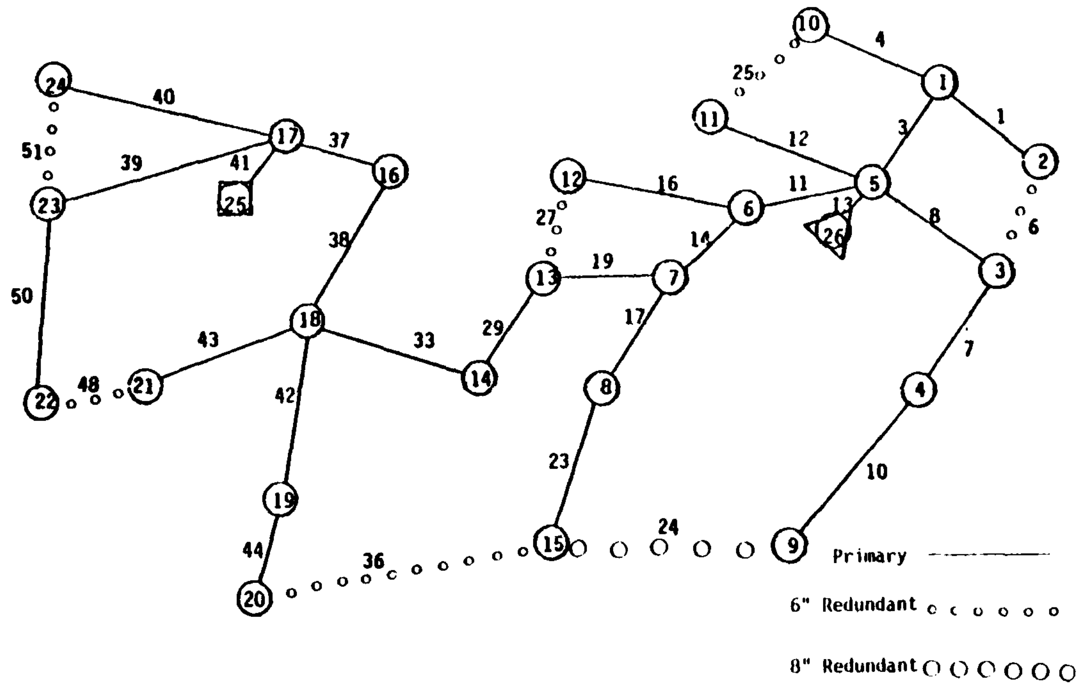


Figure 6-13

REDUNDANT LINKS SHORTEST PATH TREE LAYOUT FLOW COVERING PROBLEM
(1/4 NORMAL DEMAND)

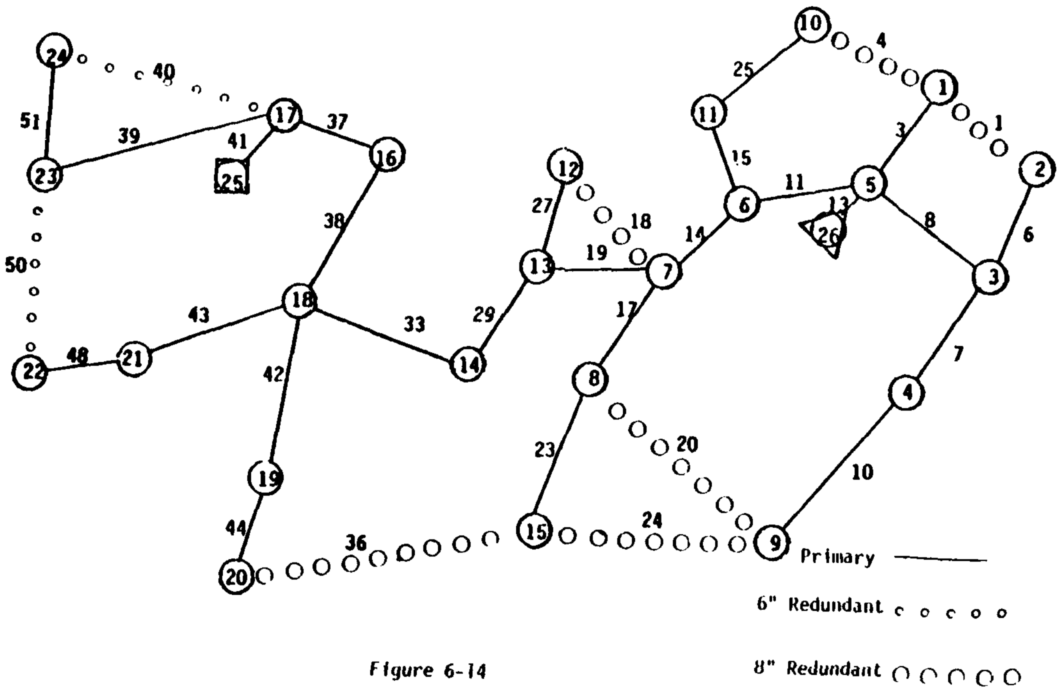


Figure 6-14

REDUNDANT LINKS NONLINEAR MINIMUM COST FLOW TREE LAYOUT FLOW COVERING PROBLEM
(1/2 NORMAL DEMAND)

\$25,694. The flow covers for 1/3 and 1/4 normal demand (not shown) have costs of \$21,602 and \$16,986 respectively.

6.4.5 Analysis of Results

6.4.5.1 Layout Structure

Analysis of the full network layouts reveals a remarkable stability in the structure of the networks. Solutions obtained using the set covering model for each tree layout contain with minor variation the same set of links as the corresponding flow covering solutions. Also, among the different flow covering solutions for each tree layout the redundant link design remains stable simply lowering diameters as the flow requirements decrease. This redundant link design stability suggests that for a given core tree layout and normal flow distribution there is a natural set of economical redundant links that best defend the system from primary link failure.

6.4.5.2 Computational Cost

Table 6-7 presents a summary of the computational experience in solving the set and flow covering problems using RIP30C. The first column under each tree layout is the total CPU time to run the

Table 6-7

RIP30C COMPUTATIONAL EXPERIENCE

	SHORTEST PATH TREE			NONLINEAR FLOW TREE		
	Total CPU (SEC)	*CPU Best* Solution (SEC)	*% Solution* Enumerated	Total CP (SEC)	*CPU Best* Solution (SEC)	*% Solution* Enumerated
SET COVERING	.70	.38	100.00	.53	.418	100.00
FLOW COVERING						
1/2 NORMAL	201.13	6.07	84.00	180.09	17.36	68.74
1/3 NORMAL	195.48	7.37	93.11	218.30	100.86	85.90
1/4 NORMAL	52.07	19.18	100.00	50.85	13.53	100.00

problem. The second column is the CPU time at which the best feasible solution was found. The third column is the percentage of feasible solutions enumerated by the algorithm at termination. If all feasible solutions have been enumerated (100%), we are guaranteed an optimal solution has been found.

As expected, the set covering problems containing approximately 20 equations and 25 decision variables were considerably easier to solve than the flow covering problems with approximately 45 equations and 50 decision variables. In general, the algorithm finds a good solution for the flow covering problem very quickly and spends the majority of its time verifying its optimality. Also, for the flow covering problem, the lower the minimum flow requirements the faster the problem is solved.

6.4.5.3 Overall Assessment

Thus, in general the set covering problem (Problem P6) because of its size is significantly easier to solve computationally than the flow covering problem (Problem P7). Although it does not provide the detailed information on the best diameters to install on the redundant links, its selection of redundant links seems to agree well with the results of comparable flow covering problems.

In light of these results it appears that a two-step procedure using both models can be used to reduce the overall computational burden and also provide detailed design information. The first step involves solving the set covering problem using all candidate redundant links to screen out undesirable links. In the second step a set of candidate diameters is selected for each of the optimal redundant links from the first step and the appropriate reduced flow covering problem is solved for the minimum link diameters. The screening process of the first step significantly reduces the number of decision variables for the flow covering problem while still assuring a good set of redundant links from which to select. Applying the above two-step procedure to the shortest path tree layout problem with flow covering at one-half normal demand resulted in a total combined CPU time of .75 seconds (.70 for the set covering problem and .05 for the reduced flow covering model) versus more than 200 CPU seconds using the full flow covering model.

6.5 Detailed System Design

6.5.1 Introduction

The detailed system design was performed for the fully looped network shown in Figure 6-11. However, before examining the details

of the design, we will discuss the difficulties encountered in applying the solution algorithm to a realistic size problem and the steps taken to make the algorithm practical for its intended application. Next, we will use the MINCOST optimization problem to assist us in selecting initial flow distributions and budget levels for the MAXWMIN optimization problem. Next, we will present the results of computational tests of Shamir and Alperovits' gradient [46] (Equation 5-51), Quindry et al.'s [94] (Equation 5-56) gradient with interaction terms, and the conjugate gradient with Beale restarts [97]. Finally, we will apply the modified solution algorithm to the MAXWMIN performance problem, discuss implementation of the resulting design, and discuss alternative applications of the detailed design model.

6.5.2 Model Modifications

Anticipating time and storage problems associated with solving a realistic size problem, several changes (most of which have been discussed in Chapter 5) had already been made to the solution algorithm.

1. Reduction in the number of candidate diameters in each link to 3 (at any iteration) (section 5.5.2.4).

2. Limiting the number of minimum head constraints and exchanging slack constraints for violated constraints (section 5.5.2.2).
3. Restricting upward expansion of the set of candidate diameters once a feasible MAXWMIN solution is obtained (section 5.5.4.3).
4. Coupling a Hardy Cross network balancer with the initial optimal flow solution to accelerate reaching an initial feasible solution (section 5.5.2.3).
5. Installing a compact pointer system to reference links in pressure equations.
6. Reducing the size of the linear program matrix by incorporating the positive loop/source dummy values (XV_i^+) as part of the initial basic feasible solution.
7. Reducing the size of the linear program matrix by allowing the user to tailor the number of loops in each loading as necessary.

However, unforeseen problems developed in trying to rigidly apply the solution algorithm to a realistic size problem. The major difficulties involved were:

1. Excessive time for updating the constraint matrix and re-solving the linear program due to the large number of loop constraints.
2. Inability to find a feasible (balanced) flow distribution on all loading conditions and frequent infeasible flow distributions even after feasibility had been achieved.
3. Flow changes frequently resulting in the linear program itself having no feasible solution, i.e., unable to find a solution satisfying minimum nodal pressure, constraints. Unlike an infeasible (unbalanced) flow distribution, this type of infeasibility automatically terminates the solution algorithm.
4. Singular or almost singular constraint matrix due to identical or almost identical flow distribution on the same loop on different loading conditions.

The first three problems led to a close re-examination of the model's requirement for simultaneously balancing all loops on all loading conditions. Unlike conventional network balancing techniques (Hardy-Cross, Newton-Rhapson) where link diameters are fixed and flow changes are made until the imbalance is within a certain tolerance, the solution algorithm attempts to balance all loadings by

both changing link diameters and loop flow distributions. For a balanced solution all loops are balanced exactly, i.e., zero tolerance. Because of the large penalty associated with any loop imbalance (1×10^{10} per foot of imbalance), the loop flow changes and link diameters are extremely responsive to any imbalance. Thus, the model and solution algorithm place a high priority on balancing the network, often to the detriment of cost and performance considerations.

For a single loading condition, i.e., known nodal supplies and demands, and the availability of a sufficiently wide range of pipe diameters, there is no difficulty in finding a balancing combination of link diameters and flows. However, with multiple loading conditions having considerably different nodal supplies and demands, the existence of a feasible solution, i.e., all loops on all loading conditions balanced, is by no means guaranteed. Furthermore, with multiple conflicting flow distributions a feasible solution at one flow iteration may not be feasible after the next flow change due to the combination of a small feasible region and the solution algorithm's desire to push the flow distribution in the direction of increasing performance or reducing costs.

What is the significance of the level of imbalance to the system designer? To properly answer this question we must examine

the meaning of steady state flow and the accuracy of the data provided to the model. In the course of a day a water distribution system moves through numerous steady state flow conditions. During each steady state period, by definition, nodal demands and supplies must remain the same. Complex transient flow conditions govern the behavior of the system as it moves from one steady state flow condition to another. Technically, any loop imbalance means that the system is in a transient state, i.e., the nodal supplies and demands are changing.

A recent committee report on the status of water distribution research and applied development needs [54] noted the roughness of both future water demand estimates and data on link characteristics. Thus, considering the transiency and uncertainty of steady state flow conditions and the roughness of the input data, it appeared reasonable to consider relaxing certain loop constraints to allow the model to better reflect the accuracy of its input data and to make it more tractable for realistic size problems.

The following alternative relaxations were each incorporated into the computer model and tested on the large design problem:

1. Partial relaxation of normal loading condition loop constraints using no-penalty dummy valves with an upper bound

on the amount of imbalance. After the solution of each CCP, the Hardy Cross network subroutine balances the relaxed loops in the normal loading condition.

2. Partial relaxation of the normal loading loop constraints as in the first alternative but with no balancing of the normal loop constraints between CCP solutions.
3. Complete relaxation of the normal loading loop constraints.

In all of the above relaxations, all other pressure constraints (normal and emergency) were strictly enforced. The initial normal loading flow distribution in all cases was the optimal MINCOST flow distribution.

Although the first alternative eliminated the difficulties with infeasible linear programs, the computational burden of updating all the loop equations persisted. The second alternative provided a significant reduction in computation burden although like the first alternative the introduction of no-penalty dummy valves and constraints on maximum imbalance did increase somewhat the number of constraints and decision variables. A range of maximum loop imbalance levels of .1 to 10 feet were tested with 5 feet working best. Since the loop constraints were relaxed, the normal loading condition nodal head values computed by the model were not necessarily correct.

However, subsequent to the optimization, the Hardy Cross subroutine balanced the normal loading loops and the normal nodal pressure heads were then computed. A survey of several runs with the maximum normal loop imbalance level set at 5 feet revealed corrected normal nodal heads within .25 feet of their uncorrected values. The third alternative, complete relaxation of the normal loop constraints, achieved the greatest reduction in computational burden. However, the uncorrected head values varied sometimes by a few feet. Perhaps more important, the real impact of the complete relaxation on the optimization results in the general case can not be accurately assessed.

Based on the above testing, the second alternative--partial relaxation of the normal loop constraints--was implemented into the solution algorithms. Thus, for each normal loading loop constraint i, the penalty costs for the dummy valves XV_i^+ and XV_i^- were set to zero and a constraint of the form

$$XV_i^+ + XV_i^- \leqq MAXIMB \qquad (6\text{-}1)$$

was added where MAXIMB is the maximum imbalance permited on loop i.

Rao et al. [52] in their work on simulation of fire demand loadings in existing water distribution systems noted that the effects

of a fire demand at a particular node on nodal pressures and flow distribution were limited to the surrounding nodes and links. During initial work with the fire demand loadings (located at opposite ends of the distribution system) similar behavior was also encountered. More specifically, the fire demand loading condition at node 9 had its principal effect on the nodal pressures and link flows in loops I-VI (Figure 6-11), while the remainder of the system was unaffected. Likewise, the fire demand loading condition at node 22 had its principal effect on the nodal pressures and link flows in loops VII and VIII. This behavior led to the important conclusion that for a sufficiently large system, the principal focus during an emergency loading condition could be limited to the section of the system affected by the condition while the remainder of the system could be assumed to be operating normally. In our design problem per standard fire insurance guidelines [80] both fire demands occur during the period of normal (peak hourly) demand. Thus, for each emergency loading condition the distribution system was partitioned to focus on the section of the system affected by the emergency loading condition, i.e., loops I-VI for the fire demand at node 9 and loops VII and VIII for the fire demand at node 22. The flow distribution on the loops in the remainder of the system is fixed at the MINCOST optimal normal flow distribution. Taking advantage of this

aspect of water distribution behavior allows the system designer to realistically analyze larger distribution systems and more emergency loading conditions. Furthermore, the matrix singularity noted in the fourth problem was removed since the emergency loading condition loops, which were unaffected by the fire demand and needlessly duplicated the corresponding normal loading condition loops, were eliminated.

6.5.3 Minimum Cost Optimization

6.5.3.1 Introduction

This section presents the results of using the MINCOST problem (Problem P13) to prepare for the MAXWMIN optimization (Problem P12) and to investigate the effectiveness of alternate formulas for computing the search direction. A summary of the relevant problem data for the MINCOST and MAXWMIN optimization is presented in Table 6-8.

6.5.3.2 Budget Level Selection

To properly assess the cost of adding redundant links to the shortest path tree layout a MINCOST optimization of the full network layout (Figure 6-11) with a single pump under the normal loading

Table 6-8

SUMMARY OF PROBLEM DATA

LINK DATA

Hazen Williams Coefficient: 130
No. of Candidate Diameters/Link: 3
Salvage Value Ratio: .1
Economic Life: 30 YR
Maintenance Cost: $4/IN/MILE/YR
Minimum Normal Hydraulic Gradient: .001
Maximum Normal Hydraulic Gradient: .025

DIAMETER (IN)	CAPITAL COST ($/FT)
6	10.2
8	14.8
10	19.7
12	24.9
14	30.4
16	36.1
18	42.0
20	48.2
22	54.5
24	60.9
26	67.6
28	74.3
30	81.2

Maximum Height: 50 FT
Capital Cost: $7000/FT
Economic Life: 30 YR

PUMP DATA

Node 26: 4 Normal Parallel Pumps
2 Standby Parallel Pumps
1 Variable Speed Pump
2 Parallel Booster Pumps

Node 25: 2 Parallel Booster Pumps

Economic Life: 15 YR
Salvage Value Ratio: 10
Pump-Motor Efficiency
Normal & Standby: .75
All Others: .70

Electricity Cost: $.04/KW-HR
Utilization Factor: .1255
Maintenance Cost: $4/HP/YR

OPTIMIZATION PARAMETERS

No Change from Example Problem

NODAL DATA

Minimum Nodal Head: 98 FT

VALVE DATA

Real Valves Installed at Each Source on All Loadings
Maximum Resistance: 30 FT

was performed. The total cost of the design was $152,951 with link costs of $112,423, pump costs of $29,252 and storage costs $11,276. Comparison of these costs with the minimum cost of the shortest path tree layout (Table 6-1) reveal a slight increase in external energy costs of $1,680 and an increase of $16,564 in link costs. Since as expected, all redundant links are at their minimum diameters (Figure 6-15), the net change in link costs $16,564 results from a $22,572 increase in redundant link costs with a $6,008 decrease in primary link costs. This reduction in primary link costs results from the diversion of water from the primary to the redundant links allowing primary link diameters to decrease as noted in section 5.6.4.2.

Next, to obtain a lower bound on the cost of the satisfying emergency loading conditions, the MINCOST problem (Problem P13) was solved with minimum normal nodal heads of 98 feet and minimum emergency nodal heads of 0 feet per section 5.4.2. The cost of the resulting design was $174,038 including $130,601 for link, $34,292 for pumps, and $9,145 for elevated storage. Of the $21,087 increase from the MINCOST normal loading only design, $18,178 were increased link costs, $5,040 increased pumping costs for added standby and variable speed pumps at the pump station at node 26, and $2,131 decreased storage costs. Although total pumping cost increased due to emergency

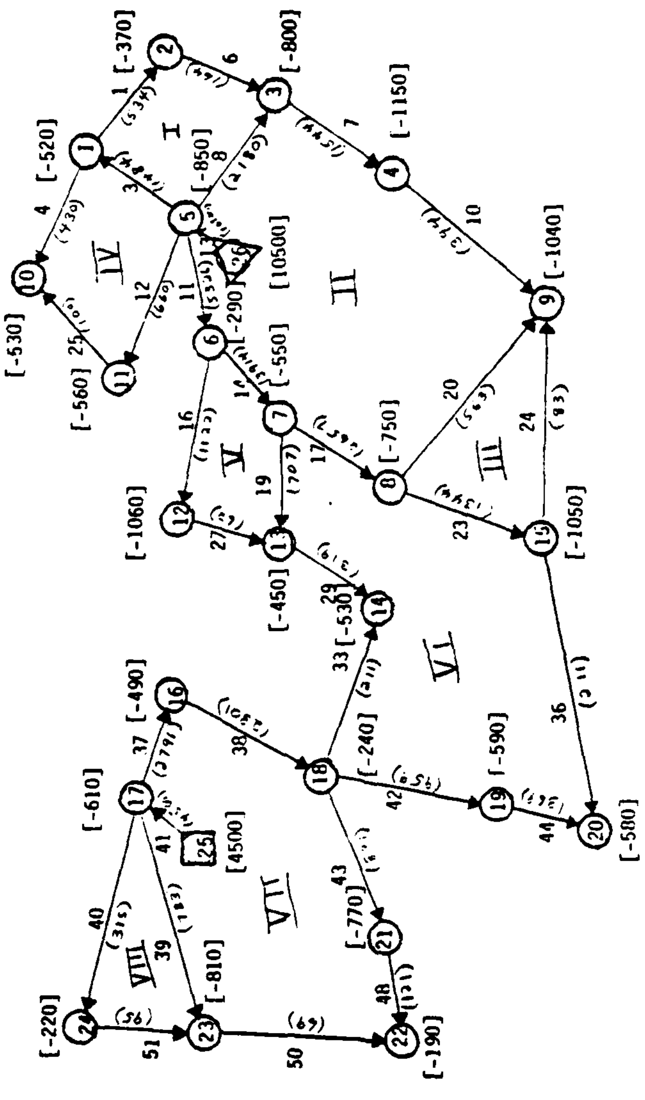


Figure 6-15

FULLY LOOPED LAYOUT NORMAL LOADING ONLY MINIMUM COST FLOW DISTRIBUTION

pumping, the total external energy (normal pumps and storage) required by the system under the normal loading decreased slightly due to the larger link diameters. Thus, $175,000 was selected as the initial budget level for the performance optimization.

6.5.3.3 Gradient Testing

To properly compare the search directions generated by Shamir's [46] negative gradient without interaction, Quindry et al.'s [94] negative gradient with interaction, and the conjugate gradient with Beale restarts [97] proposed by the author, the MINCOST optimization problem for the single normal loading condition was solved using the three different methods starting at ten widely differing initial flow distributions. Table 6-9 shows the different starting points referenced to an initial flow distribution with 100 GPM flow in each redundant link (starting point 1). Since the computation time required to calculate any of the gradients is insignificant compared to the overall solution time, our main concern was the goodness of the search direction generated by each gradient. Thus, each problem was run for 25 flow iterations. Table 6-10 shows the value of the minimum cost solution for each gradient for each starting point and the associated CPU time. Of the ten runs, the negative gradient with interaction was best on 5 runs, the negative gradient without

Table 6-9

GRADIENT TESTING STARTING POINTS

STARTING POINT	INITIAL LOOP FLOW CHANGES (GPM) I	II	III	IV	V	VI	VII	VIII
1	C	0	0	0	0	0	0	0
2	+50	+300	-150	-50	+600	-450	+450	-300
3	-50	+600	+150	-450	-300	+150	-150	+300
4	+150	-450	-50	+50	+300	+600	-300	-150
5	-150	+450	+300	-300	+50	-50	-450	+600
6	+300	-150	-450	+300	+450	-150	+50	-50
7	-300	+50	+600	-150	-50	+450	+150	+150
8	+450	-300	+450	+600	+150	+300	-50	+50
9	-450	-50	-300	+150	-450	+50	+600	+450
10	+600	+150	+50	+450	-150	-300	+300	-450

Table 6-10

RESULTS OF GRADIENT TESTING

STARTING POINT	NEGATIVE GRADIENT NO INTERACTION		NEGATIVE GRADIENT INTERACTION		CONJUGATE GRADIENT	
	OPTIMAL COST ($)	CPU TIME CPU TIME (SEC)	OPTIMAL COST ($)	CPU TIME CPU TIME (SEC)	OPTIMAL COST ($)	CPU TIME CPU TIME (SEC)
1	152,039	273	151,947	245	153,161	282
2	224,129*	164	160,048	322	252,153*	152
3	155,737	340	157,553	308	155,604	308
4	151,227	292	150,946	301	162,455	309
5	157,775	200	162,287	194	210,757*	313
6	154,755	296	152,291	278	155,785	279
7	154,166	280	154,388	240	159,218	255
8	155,696	292	156,138	285	160,318	272
9	155,397	244	154,929	227	248,401*	228
10	157,528	340	159,100	313	164,986	295

* = Unbalanced Solution.

interaction was best on 4 runs, and the conjugate gradient best on 1 run. However, excluding the run where the algorithm was unable to find a balanced flow distribution, the negative gradient had the lowest average minimum cost of $154,924 and standard deviation $2,207 compared to $155,509 and $3,688 for the negative gradient with interaction and $158,789 and $4,187 for the conjugate gradient. Examining the interaction term of the gradient, the second term in equation (5-56), we found that it was usually an order of magnitude less than the negative gradient without interaction. Thus, there appears to be little difference between the goodness of the search directions generated by the negative gradient with or without interaction except that the negative gradient without interaction appears to be somewhat more consistent. The conjugate gradient is definitely inferior to the other two gradients. Given the general irregular shape of the optimal response surface as illustrated in the three-dimensional Figure 3-2, the failure of more sophisticated techniques to generate better search directions is not completely unexpected.

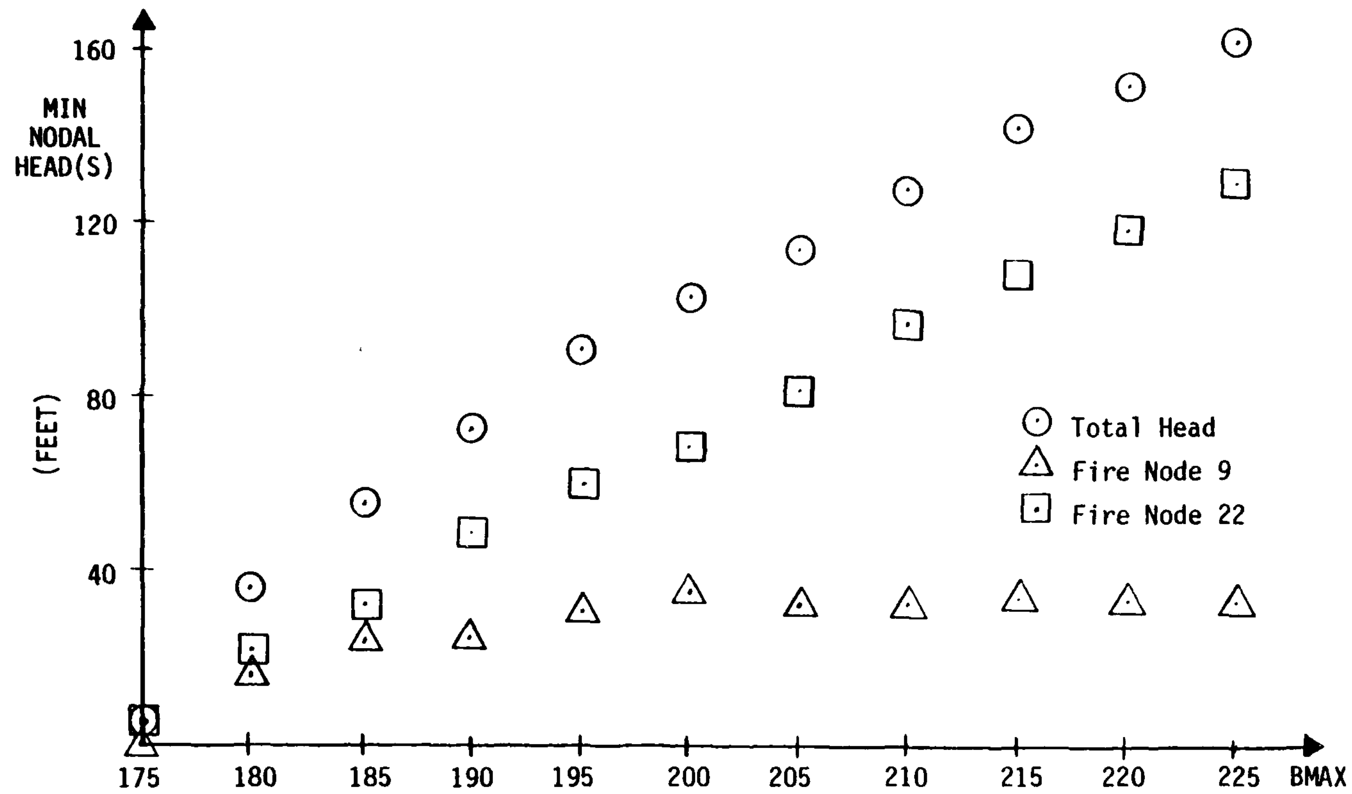
An interesting by-product of the gradient testing was the confirmation of the importance of selecting a good initial flow distribution. Because of the poor flow distribution, four runs resulted in an unbalanced flow distribution even after 25 iterations. Also, the lowest average optimal solution for all gradients occurred

starting from the base flow distribution (starting point 1) which has minimal amounts of flow in each redundant link and the balance in the core tree links.

6.5.4 Application of Model

The performance optimization was done using the same procedure as in the example problem of Chapter 5. Starting from the initial optimal flow distribution of the MINCOST problem with all three loading conditions and a budget level BMAX of $175,000 the budget was incremented in units of $5,000 up to $225,000. At that point the linearity of the performance versus budget curve was evident. In general, convergence of the solution algorithm was fairly rapid, generally taking less than 15 CCP optimizations and 200 seconds CPU time on the CDC 170/750A. Similar rapid convergence had also occurred for the small example problem (section 5.5.4). In light of the fact that the MINCOST solution is a local optimum solution the rapid convergence of the MAXWMIN problem starting with the optimal MINCOST flow distribution is not surprising.

Figure 6-16 illustrates the system performance versus budget level for equally weighted emergency loading objective function coefficients. The overall system performance displays concave behavior for small budget increments becoming linear around BMAX = $195,000.



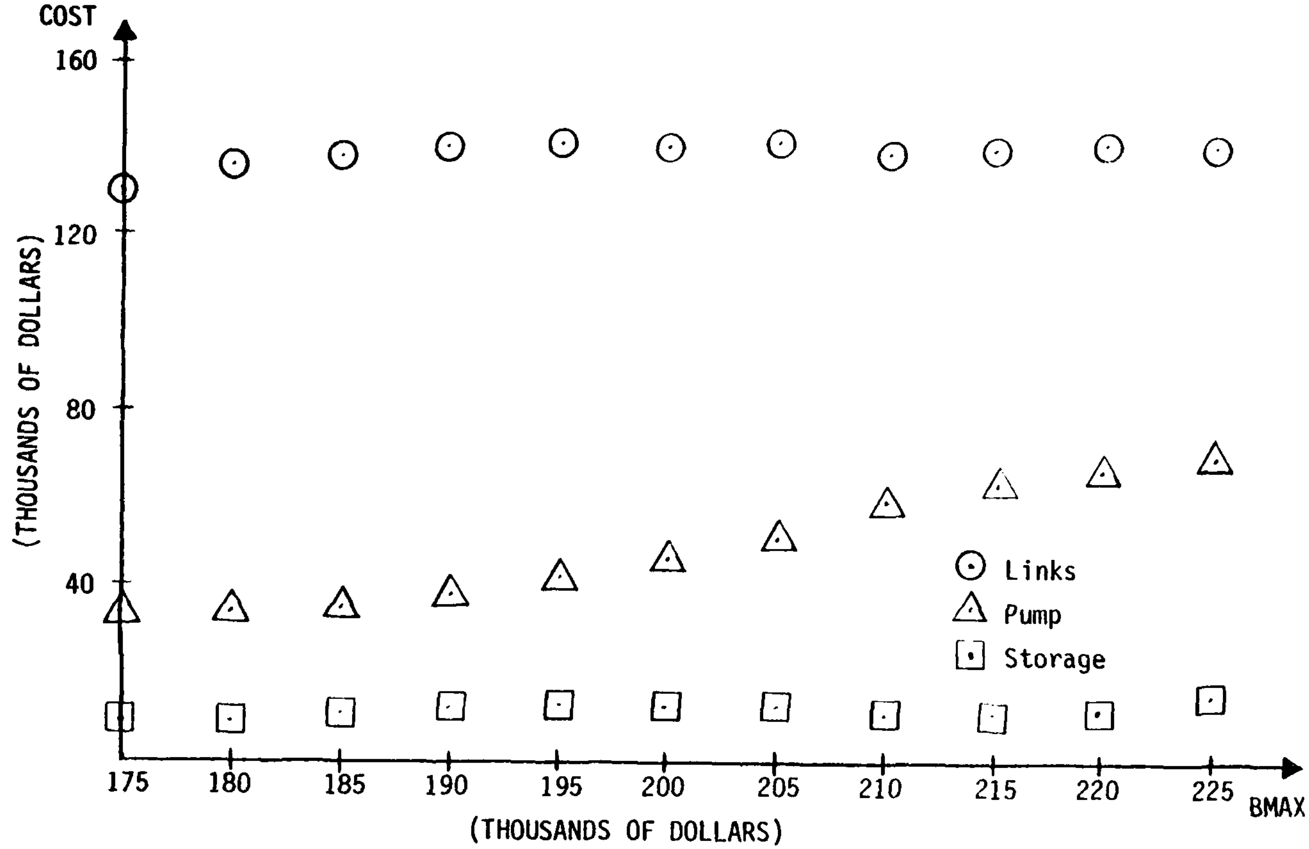
(THOUSANDS OF DOLLARS)

PERFORMANCE VS. BUDGET LEVEL

Figure 6 - 16

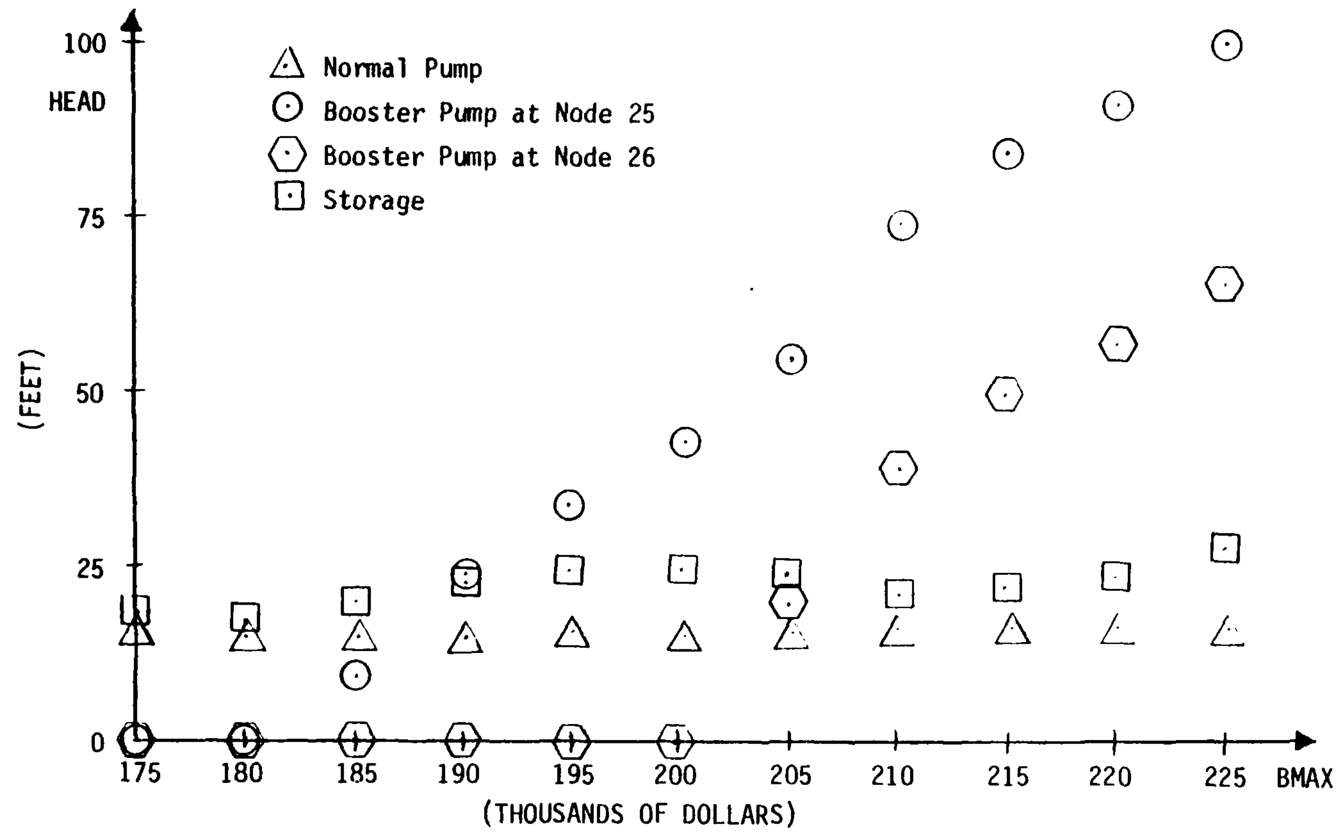
Unlike the example distribution system which had its performance abruptly limited by a combination of maximum storage height and the tremendous cost of increasing the normal pumping head, the presence of the booster fire pumps allows performance on the node 22 fire demand loading to increase with the budget. However, because of the extremely high fire demand flow rate for the node 9 fire demand (7500 GPM fire demand, plus 10,500 GPM normal), no provisions were made to boost this large 18,000 GPM flow. Further increases in the performance on the node 9 fire demand loading condition require costly increases in the normal pumping head lift. Thus, unless the node 9 loading condition objective function weighting coefficient is heavily weighted, the solution algorithm will continue to allocate funds to the less expensive, higher payoff alternative of increasing pressure at node 22.

Figure 6-17 depicts a breakdown of the three major cost components at each budget level. In general, all components increase steadily until $195,000. At that level, performance increases from increasing link diameters becomes minimal, link costs stabilize, and the optimal solution allocates added budget increments almost entirely to external energy from the booster pumps. Figure 6-18 shows the external energy added by pumps and elevated storage versus budget level. The head lift of the normal pump remains constant because of



SYSTEM COST BREAKDOWN

Figure 6 - 17



100
HEAD
75
50
(FEET)
25
0
Normal Pump
Booster Pump at Node 25
Booster Pump at Node 26
Storage
175
180
185
190
195
200
205
210
215
220
225
BMAX
(THOUSANDS OF DOLLARS)
ADDED EXTERNAL ENERGY

Figure 6 - 18

the high energy cost associated with its head lift. The two fire demand booster pumps enter the system design as the budget level increases.

For BMAX = \$185,000, the objective function weighting coefficients were varied from .1 to .9. Figure 6-19 displays the performance of the system and Table 6-11, the cost breakdown and added external energy for the selected weighted coefficients. As the weighting coefficients for the node 9 fire demand is increased the budget is reallocated from the booster fire demand pumps to increasing the normal, standby, and variable speed pump head lifts and the link diameters on the long path to the fire at node 9.

6.5.5 Design Implementation

This section discusses the implementation of the system design for the optimal solution for BMAX = \$195,000 and analyzes the cost of reliability for this system. Table 6-12 shows the optimal link design and Table 6-13 summarizes the detail pumping design for the system.

A comparison of the cost components of the minimum cost shortest path tree layout with the cost components of the \$195,000 fully looped system provides insight into the cost of increasing system reliability. The majority of the \$60,293 increase, 75.1 percent

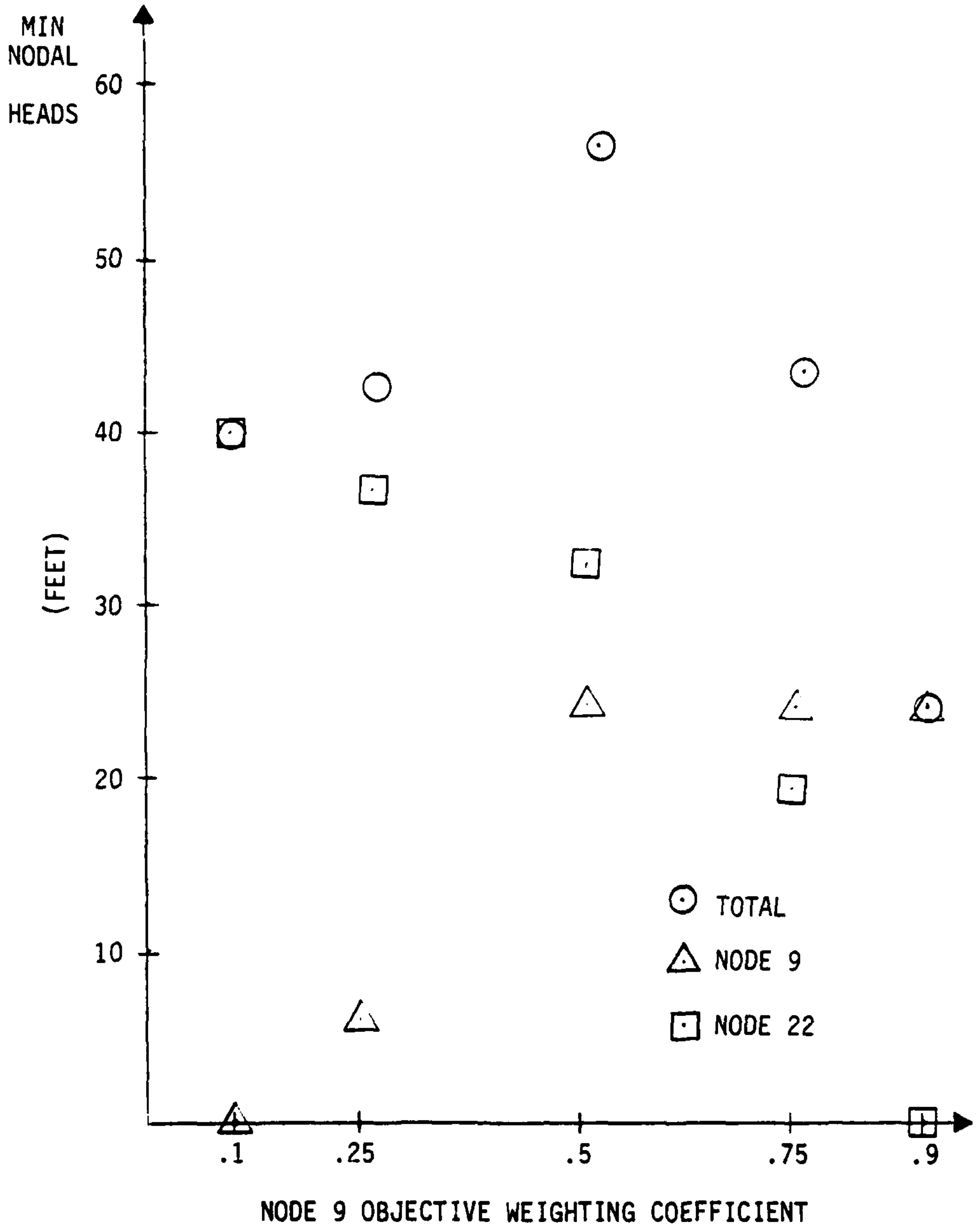


Figure 6-19

SENSITIVITY TO OBJECTIVE FUNCTION WEIGHTING COEFFICIENT CHANGES
BMAX = $185,000

Table 6-11

COMPARISON DATA FOR VARIABLE WEIGHTING COEFFICIENTS BMAX = $185,000

COEFFICIENT WEIGHT		COST ($)			HEADS (FT)			
FIRE NODE 9	FIRE NODE 22	LINKS	PUMP	STORAGE	NORMAL STANDBY & VARIABLE SPEED PUMPS	BOOSTER PUMP AT STORAGE	BOOSTER PUMP AT STATION	STORAGE
.1	.9	135,177	39,815	10,008	15.7	3.3	19.1	19.7
.25	.75	136,094	38,797	10,109	15.7	0	15.7	19.9
.5	.5	137,951	37,755	9,294	15.7	0	13.6	18.3
.75	.25	139,606	36,229	9,165	16.2	0	0	18.0
.9	.1	137,259	38,603	9,138	17.3	0	0	18.0

Table 6-12

OPTIMAL LINK DESIGN BMAX = $195,000

LINK NO.	TOTAL LENGTH (FT)	SEGMENT 1		SEGMENT 2	
		DIAMETER	LENGTH	DIAMETER	LENGTH
1	1650	6	78		
3	1535	12	1535		
4	2490	6	1423	8	1067
7	2685	20	2685		
8	2400	20	2400		
10	3480	12	3480		
11	1800	28	1800		
12	2510	8	582	10	1928
13	60	30	60		
14	1260	16	701	18	559
16	2920	10	2355	12	565
17	1695	22	1695		
19	1780	6	1093	8	687
23	2500	16	2034	18	466
29	1560	6	1560		
33	2510	6	2105	8	405
37	1380	22	1380		
38	2500	20	2500		
39	5110	8	5066	10	44
40	4710	8	1746	10	2964
41	450	24	450		
42	2750	8	2408	10	342
43	2840	14	2840		
44	1440	6	1440		
50	3510	6	3510		
6	1550	6	1550		
20	4330	10	119	12	4211
24	3850	12	3850		
25	1790	6	1790		
27	2510	8	2510		
36	5620	6	5620		
48	2200	14	2200		
51	1800	6	1800		

Table 6-13

DETAILED PUMP DESIGN BMAX = $195,000

PUMP LOCATION	TYPE	NO. OF PARALLEL PUMPS	MAXIMUM FLOW (GPM)	MAXIMUM HEAD (ft)	MAXIMUM HORSE POWER (HP)	PUMP-MOTOR EFFICIENCY
Pump Station	Normal	4	2,625	15.7	193	.75
Pump Station	Standby	2	2,625	15.7	207	.70
Pump Station	Variable Speed	1	7,500	15.7	589	.70
Storage Reservoir	Fire Booster	2	3,750	34.1	644	.70

($45,253) is due to increases in link costs. Of this $45,253, 49.9 percent ($22,572) can be attributed to installing redundant links at minimum diameter to handle emergency broken link loading conditions. The balance ($22,681) is associated with upgrading both primary and redundant links to handle the expected fire demand emergency loading conditions. The $15,040 increase in external energy costs results from an increase of $12,795 in pumping costs and an increase of $2,245 in storage costs. Of the $12,795 increase in pumping costs 89.8 percent ($11,653) is due to the cost of emergency pumping ($2,324 for the two standby pumps, $2,895 for the variable speed pumps, and $6,434 for the storage fire demand booster pumps. The balance ($1,142) is principally due to the increased capital cost of using four smaller flow capacity pumps instead of a single large flow capacity pump.

The detailed design model provides valuable insight into the best way to allocate limited funds to handle the expected emergency fire demand loading conditions. Basically, the optimization results show that the best way to design reliability into the system is to initially install oversize links in certain critical parts of the system. As more funds become available, the installation of booster pumps at the two sources becomes a good investment. It should be emphasized that the model will not design the system by itself but

is a tool to assist the system designer. The system designer must apply his engineering judgment to properly select loading conditions, pumping arrangements, placement of valves, etc., to perform the complete design.

6.5.6 Alternative Model Applications

Because the principal emphasis has been on the design of a new system, little has been said about the use of the detailed design model for expansion or replacement of components on existing systems. To describe the existing parts of the system, which will remain unchanged, link diameters and storage heights may be fixed and known capacities placed on existing pumps. The cost of existing components is set to zero in the budget constraint.

Another application of the detailed design model is to develop optimal operational responses for emergency loading conditions for a fully defined system. With elimination of the budget constraints, the decision variables become the proper operation of existing pumps and valves in order to maximize system performance. With the large reduction in decision variables associated with operation of an existing system, this model could be used in real time control. Using inputs from field sensors the current flow distribution is easily estimated.

The optimal operation of valves and pumps could then be computed to maximize system performance within existing capabilities.

CHAPTER 7

RESULTS, CONCLUSIONS, AND RECOMMENDATIONS FOR FUTURE RESEARCH

7.1 Introduction

The purpose of this chapter is to briefly review the major results of this research, to summarize the conclusions derived from these results, and to discuss recommendations for future research.

7.2 Results

This research has produced five major results:

1. Development of a comprehensive methodology for the design of water distribution systems that explicitly incorporates reliability and performance into the design of the system.

2. Development and implementation of two alternative models to enable the water distribution system designer to rapidly generate and evaluate alternative low cost network layouts.

3. Development and implementation of two complementary mathematical optimization models that enable the water distribution

system designer to incorporate a specific level of broken link performance into the system at minimal cost.

4. Development and implementation of a detailed design model that enables the water distribution system designer to allocate the available funds to achieve maximum performance on the expected emergency loading conditions.

7.3 Conclusions

The results of this research represent a significant step forward in developing an analytical methodology for the design of reliable water distribution systems. Previous research had almost wholly concentrated on the less difficult problem of minimizing the cost of water distribution design for normal system operation. This research has directly addressed the more difficult problem of how to best incorporate performance under expected emergency loading conditions within the available budget.

7.4 Recommendations for Future Research

The research areas described below are natural extensions of the work described in this dissertation:

1. Adaptation of the MAXWMIN detailed design model to analogous distribution systems. Closed conduit distribution systems transporting gas and solids are good candidates. Especially applicable to this model would be the design of hydraulic systems for military aircraft. Aircraft operating in a wartime environment are exposed to unusual stresses that can cause failure of the aircraft hydraulic system, e.g., loss of pressure, which is critical to maintaining control of the aircraft.

2. More efficient techniques for solving the multiple weighted set covering model (Problem P6) of Chapter 4. Because of the structural similarity between Problem P6, the multiple weighted set covering problem, and two other 0-1 models for which efficient solution techniques have been developed, i.e., the weighted set covering problem and the multiple set covering problem, it appears worthwhile to investigate modifying these techniques to enable more efficient solution for larger distribution system application.

3. Developing generally applicable guidelines for setting the objective function weights w_ℓ for the MAXWMIN problem. The

results of the detailed design problems of Chapters 5 and 6 strongly suggest that the choice of w_ℓ can significantly affect the resulting optimal design. However, because of the lack of data on the relative frequency of occurrence of various emergency loading conditions, it is difficult to provide detailed guidelines to the system designer on the appropriate choice of w_ℓ .

4. Development of a hybrid MAXWMIN optimization model that allows more flexibility in specifying emergency loading conditions. Instead of assuming that all external flows are fixed, external flows on emergency loading conditions would become decision variables which for noncritical nodes would be bounded below and for critical nodes, e.g., fire demand and source nodes, would be incorporated into a hybrid flow/pressure performance objective function. Such a model would allow tradeoffs between flow and pressure requirements.

APPENDIX A

HARDY CROSS LOOP METHOD

This appendix describes the Hardy Cross loop balancing method which was incorporated in the detailed design solution algorithm described in Chapter 5. A formal statement of the method followed by an application of the method to a simple two-loop distribution system is presented. The statement of the method assumes that the Hazen-Williams frictional head loss equation is used.

Formal Statement of Method

STEP 1. Initialize link flows Q_k to satisfy nodal conservation of flow equations (1-8).

STEP 2. Set i, the loop number, equal to 1, and MAXIMB the maximum loop imbalance, to zero.

STEP 3. Compute the sum of the head losses,

$$\sum_{k \in LOOP_i} \Delta HF_k \quad ,$$

taking into account the direction of flow. If

$$\left| \sum_{k \varepsilon LOOP_i} \Delta HF_k \right| > MAXIMB$$

Let

$$MAXIMB = \left| \sum_{k \varepsilon LOOP_i} \Delta HF_k \right|$$

STEP 4. Compute

$$\sum_{k \varepsilon LOOP_i} \left| \frac{\Delta HF_k}{Q_k} \right|$$

STEP 5. Compute the loop flow change,

$$\Delta Q_i = \frac{- \sum_{k \varepsilon LOOP_i} \Delta HF_k}{\sum_{k \varepsilon LOOP_i} \left| \frac{\Delta HF_k}{Q_k} \right|} .$$

STEP 6. Change link flows on loop i, i.e.,

$$Q_k = Q_k + \Delta Q_i \qquad k \varepsilon LOOP_i .$$

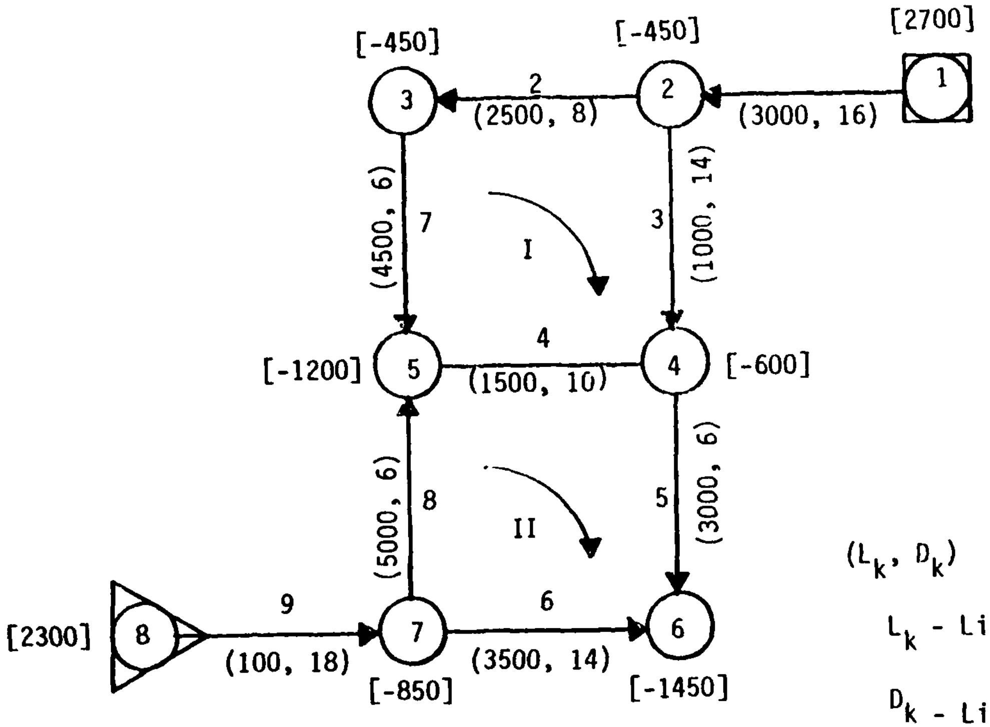
STEP 7. Let $i = i + 1$. If $i < NLOOP$ GO TO STEP 3.

STEP 8. If $MAXIMB < \varepsilon$, the maximum permissible head imbalance, STOP. Otherwise, GO TO STEP 2.

It should be noted that several variations of the original Hardy Cross method [4] have been introduced to accelerate convergence. For example, the above algorithm changes the individual link flows as soon as the loop flow changes (ΔQ_i) are generated (STEP 5 and 6) whereas Cross' original method [4] does not make link flow changes until all loop flow changes were generated.

Example Application of Method

Figure A-1 shows the example distribution system including external flows, link lengths, and link diameters. The Hazen-Williams equation (1-5) with the roughness coefficient equal to 130 was used to compute frictional head losses. Termination occurred at iteration 11 when $MAXIMB < \varepsilon = .5$ feet for both loops. Table A-1 and A-2 summarize the results of applying the method for loops I and II respectively. Figure A-2 shows the initial and final flow distributions.



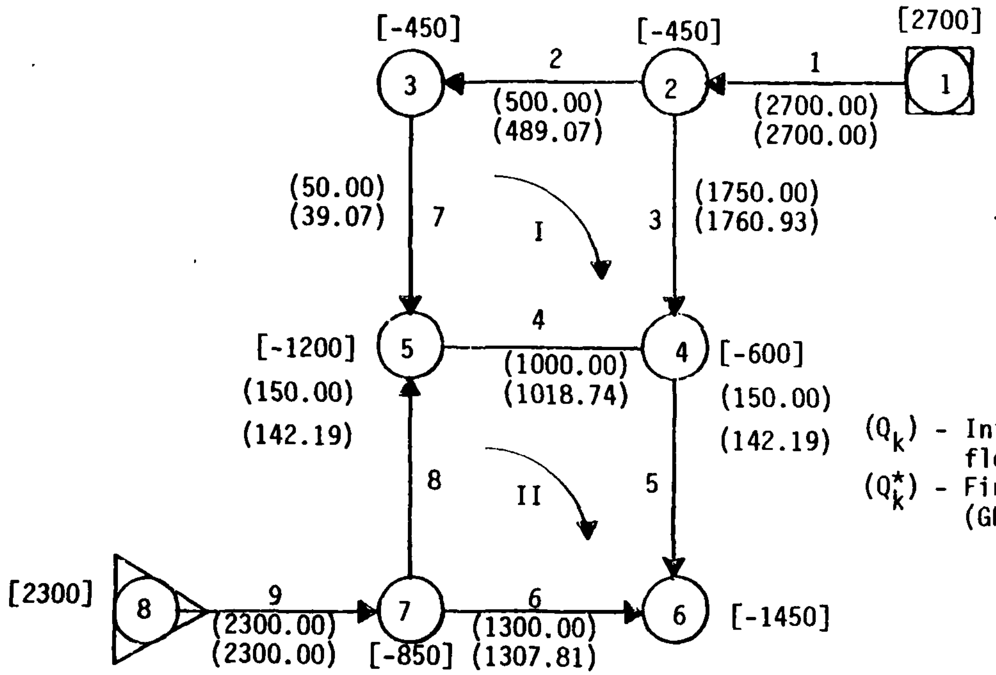
(L_k, D_k)

L_k - Link length (FT)

D_k - Link diameter (IN)

Figure A-1

DISTRIBUTION SYSTEM TOPOLOGY



(Q_k) - Initial link flow (GPM)

(Q_k^*) - Final link flow (GPM)

Figure A-2

INITIAL AND FINAL FLOW DISTRIBUTIONS

Table A-1

LOOP I

IT.	Q_k				$\Sigma \Delta HF_i$				$\Sigma \Delta HF_k$	$\Sigma \left\lvert \frac{\Delta HF_k}{Q_k} \right\rvert$	ΔQ_I
No.	2	3	4	7	2	3	4	7			
1	-500.00	1750.00	1000.00	-50.00	-12.29	3.38	9.27	-1.30	-1.34	.063	21.41
2	-478.59	1771.41	1030.89	-28.59	-11.70	3.46	9.81	-.46	1.11	.052	-21.34
3	-499.93	1750.07	1004.49	-49.93	-12.68	3.38	9.35	-1.30	-1.25	.063	19.96
4	-479.97	1770.03	1026.01	-29.97	-11.76	3.46	9.72	-.51	.91	.053	-17.23
5	-497.20	1752.80	1009.78	-47.20	-12.56	3.39	9.44	-1.17	-.90	.061	14.65
6	-482.55	1767.45	1021.59	-32.55	-11.88	3.45	9.64	-.59	.62	.054	-11.65
7	-494.06	1755.94	1014.05	-44.06	-12.41	3.41	9.51	-1.03	-.52	.060	8.75
8	-485.02	1764.68	1018.22	-35.32	-12.00	3.44	9.58	-.68	.34	.055	-5.96
9	-491.28	1758.72	1016.95	-41.28	-12.28	3.42	9.56	-.91	-.22	.058	3.70
10	-487.58	1762.42	1016.42	-36.58	-12.11	3.43	9.55	-.77	.10	.057	-1.77
11	-489.35	1760.65	1018.46	-39.35	-12.19	3.42	9.59	-.84	-.02	.058	.28
12	-489.07	1760.93	1018.74	-39.07	-12.18	3.42	9.59	-.82	.01		

Table A-2

LOOP II

IT.	Q_k				ΔHF_k				$\Sigma\Delta HF_k$	$\Sigma\left\|\frac{\Delta HF_k}{Q_k}\right\|$	ΔQ_{II}
No.	4	5	6	8	4	5	6	8			
1	-1021.41	150.00	-1300.00	150.00	-9.64	6.63	-6.83	11.08	1.24	.131	-9.48
2	-1009.55	140.52	-1309.48	140.52	-9.43	5.89	-6.92	9.82	-.64	.126	5.06
3	-1024.45	145.58	-1304.42	145.58	-9.69	6.29	-6.87	10.48	.21	.134	-1.56
4	-1008.78	144.02	-1305.98	144.02	-9.43	6.17	-6.89	10.28	.13	.129	-1.00
5	-1024.43	143.02	-1306.98	143.02	-9.69	6.08	-6.90	10.13	-.36	.128	2.84
6	-1010.08	145.86	-1304.14	145.86	-9.44	6.31	-6.87	10.52	.52	.130	-3.97
7	-1022.79	141.89	-1308.11	141.89	-9.66	6.00	-6.91	9.99	-.58	.127	4.57
8	-1012.26	146.46	-1303.54	146.46	-9.47	6.36	-6.86	10.60	.61	.130	-4.69
9	-1020.65	141.77	-1308.23	141.77	-9.63	5.99	-6.91	9.98	-.57	.127	4.49
10	-1014.39	146.26	-1303.74	146.26	-9.52	6.34	-6.87	10.58	.53	.130	-4.07
11	-1018.74	142.19	-1307.81	142.19	-9.59	6.02	-6.91	10.03	-.45		

APPENDIX B

SEPARABLE PROGRAMMING

This appendix describes the λ-method of approximation for separable programming [55] and its specific application in solving the nonlinear minimum-cost flow problem for selecting the core tree links of Chapter 3 (Problem P5).

Separable programming handles optimization problems of the form:

$$\text{Minimize} \quad \sum_{j=1}^{M} f_j (x_j) \tag{B-1}$$

subject to:

$$\sum_{j=1}^{M} g_{ij} (x_j) \leqq 0 \tag{B-2}$$

$$i = 1, \ldots, N$$

where f_j and g_{ij} are known.

Separable problems arise frequently in practice, particularly for time dependent optimization. The model also arises when optimizing over distinct geographical regions.

Instead of solving the problem directly an appropriate piecewise-linear approximation is made in order that linear programming can be utilized. In practice, two types of approximations, called the δ-method and the λ-method, are often used. Because the λ-method was used in the research, this appendix will describe its implementation.

Consider the problem of finding the core tree for Figure B-1 using the formulation of Problem P5.

$$\text{Minimize} \quad 3000\, Q_1^{.5} + 2500\, Q_2^{.5} + 1000\, Q_3^{.5} + 3500\, Q_{6B}^{.5}$$

$$+ 4500\, Q_{7A}^{.5} + 4500\, Q_{7B}^{.5} + 5000\, Q_{8A}^{.5} + 500\, Q_{8B}^{.5}$$

subject to:

$$Q_2 + Q_3 - Q_1 = -450$$

$$Q_{7A} - Q_2 - Q_{7B} = -450$$

$$Q_{4A} + Q_{5A} - Q_3 - Q_{4B} - Q_{5B} = -600$$

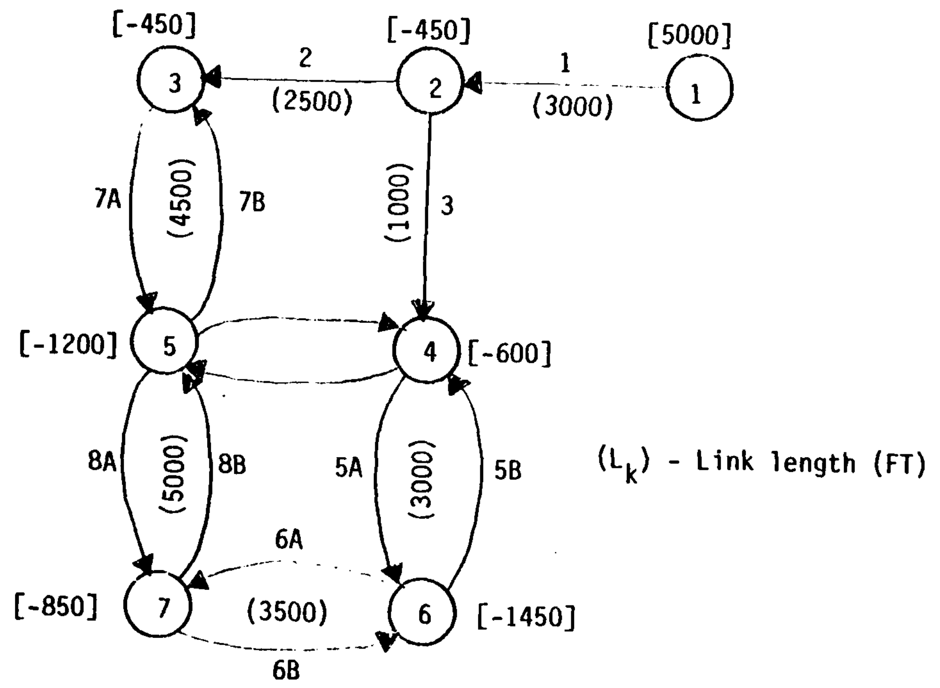


Figure B-1

TWO LOOP DISTRIBUTION SYSTEM

$$Q_{4B} + Q_{7B} + Q_{8A} - Q_{4A} - Q_{7A} - Q_{8B} = -1200$$

$$Q_{5B} + Q_{6A} - Q_{5A} - Q_{6B} = -1450$$

$$Q_{6B} + Q_{8B} - Q_{6A} - Q_{8A} = -850$$

$$Q_1, Q_2, Q_3, Q_{4A}, Q_{4B}, Q_{5A}, Q_{5B}\ Q_{6A}, Q_{6B}, Q_{7A}, Q_{7B},$$

$$Q_{8A}, Q_{8B} \geqq 0$$

The problem is formulated with directed arcs to allow direct conversion to a linear programming format. Only single variables are required for links 1, 2, and 3 since flow entering node 2 must travel to adjacent nodes and will not return.

To form the approximation problem each nonlinear term in the objective function is approximated by a piecewise-linear curve as pictured for $f_2\ (Q_2)$ in Figure B-2. The dashed approximation curve for each of the $f_k\ (Q_k)$ is determined by linear approximation between breakpoints λ_{ik} . Three segments have been used to approximate f_2 from its minimum ($Q_2 = 0$) to its maximum value ($Q_2 = 4550$). The Q_2 values of 0, 900, 2500, and 4550 have been selected as breakpoints for $f_2^a\ (Q_2)$, the approximation to f_2 .

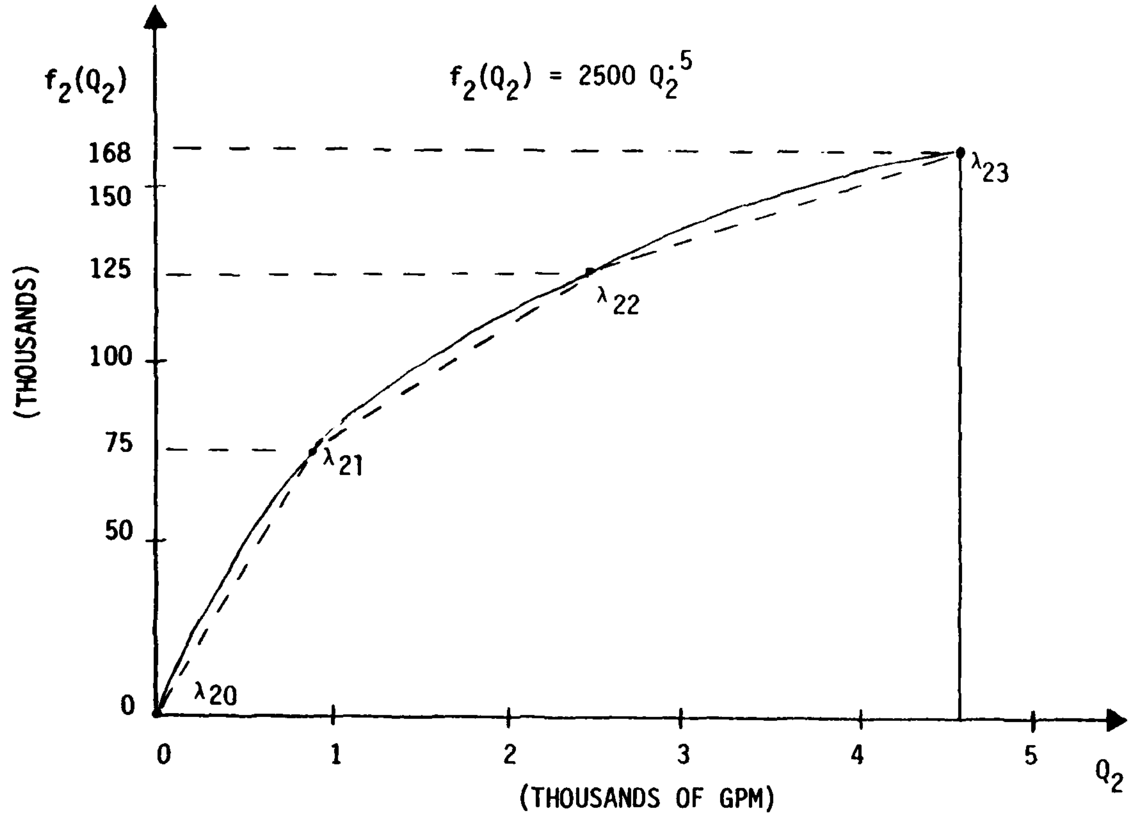


Figure B-2

CONCAVE NONLINEAR FLOW COST FUNCTION

For example, if $900 \leq Q_2 \leq 2500$, then f_2^a is given by weighing the functional values at $Q_2 = 900$ and $Q_2 = 2500$; that is as

$$f_2^a\ (Q_2) = 75000\ \lambda_{21} + 125000\ \lambda_{22}$$

where the nonnegative variables λ_{21} and λ_{22} express Q_2 as a weighted combination of 900 and 2500; thus,

$$Q_2 = 900\ \lambda_{21} + 2500\ \lambda_{22}$$

$$\lambda_{21} + \lambda_{22} = 1$$

For instance, evaluating the approximation at $Q_2 = 1600$ gives

$$f_2^a\ (1600) = (75000)\ (\tfrac{9}{16}) + (125{,}000)\ (\tfrac{7}{16}) = 96{,}825$$

$$1600 = 900\ (\tfrac{9}{16}) + (2500)\ \frac{7}{16}$$

The overall approximation curve $f_2^a\ (Q_2)$ for $f_2\ (Q_2)$ is expressed as:

$$f_2^a\ (Q_2) = 0\ \lambda_{20} + 7500\ \lambda_{21} + 125000\ \lambda_{22}$$

$$+ 168634\ \lambda_{23}$$

where $Q_2 = 0\,\lambda_{20} + 900\,\lambda_{21} + 2500\,\lambda_{22} + 4550\,\lambda_{23}$

$$\lambda_{20} + \lambda_{21} + \lambda_{22} + \lambda_{23} = 1$$

$$\lambda_{2j} \geqq 0 \qquad j = 0, 1, 2, 3$$

with the provision that the λ_{2j} variables satisfy the following restriction:

ADJACENCY CONDITION: At most two λ_{2j} weights are positive. If two weights are positive, then they are adjacent, i.e., of the form $\lambda_{2,j}$ and $\lambda_{2,j+1}$. A similar restriction applies to each approximation.

In a similar manner, piecewise-linear approximations may be derived for the other 12 nonlinear functions and substituted into the example nonlinear flow problem resulting in a linear program in the λ_{ij} decision variables. For each nonlinear function $f_i\,(Q_i)$ approximated an equation of the form

$$\sum_{j=1}^{M} \lambda_{ij} = 1$$

must be added.

The adjacency conditions on the λ_{ij} are automatically satisfied for minimizing a convex or maximizing a concave function. However in this case, minimizing a concave function, something must be done to insure that the linear program doesn't select too many or nonadjacent λ's . The simplex method is modified in the following manner to insure that the adjacency condition holds.

RESTRICTED BASIS ENTRY RULE:

Use the standard simplex criterion for selecting λ_{ik} to enter the basis but do not introduce a λ_{ik} variable into the basis unless there is only one λ_{ik} variable in the basis and it is of the form $\lambda_{i,k-1}$ or $\lambda_{i,k+1}$, i.e., is adjacent to λ_{ik} .

Using this rule, the optimal solution may contain a nonbasic variable λ_{ik} that would ordinarily be introduced into the basis by the simplex method (since its reduced cost is negative), but is not introduced because of the restricted-entry criterion. If the simplex method would choose a variable to enter the basis that is unacceptable by the restricted basis entry rule, then the next best variable according to the most negative reduced cost is chosen instead. However, the solution determined by the restricted basis entry rule in the general case can be shown to be a local optimum to the approximation problem derived from the original problem [55].

Once the approximation problem has been solved a better solution can be obtained by introducing more breakpoints. Usually more breakpoints will be added near the optimal solution given by the original approximation.

APPENDIX C

PROPERTIES OF OPTIMAL LINK DESIGN

Alperovits and Shamir [46] state without proof that it can be shown that in the optimal solution for the MINCOST problem (Problem P13) that each link will contain at most two segments with their diameters adjacent on the candidate diameter list for that link. Quindry, Brill, Liebman, and Robinson [94] by changing link costs in Alperovits and Shamir's [46] two-loop example problem claim to have found a counterexample to the adjacency condition. The following theorem spells out sufficient conditions for which Alperovits and Shamir's statement is true.

THEOREM II

Given that CL_{kj} is a strictly convex function of diameter then for Problem P13 the following is true for the local optimal solution or for any intermediate optimal linear program solution:

1. Each link k will have at most two segments of nonzero length, i.e., $XL^*_{kj} > 0$.

2. The diameters of these two segments are adjacent on the link's condidate diameter list S_k .

PROOF: First let us assume that Problem P13 has a single loading. Assume that we have the optimal solution to Problem P13 (or any intermediate optimal LP solution) and the associated optimal head losses on each link k for each loading, ΔHF_k^* . Then consider the following subproblem of selecting the segment lengths for each link in order to minimize total link costs:

PROBLEM P14

$$\text{Minimize} \quad \sum_{k=1}^{NLINK} \sum_{j \varepsilon S_k} CL_{kj} \; XL_{kj} \tag{C-1}$$

$$\text{subject to} \quad \sum_{j \varepsilon S_k} J_{kj}^* \; XL_{kj} = \Delta HF_k^* \tag{C-2}$$

$$k = 1, \ldots, NLINK$$

$$\sum_{j \varepsilon S_k} XL_{kj} = L_k \tag{C-3}$$

$$k = 1, \ldots, NLINK$$

$$XL_{kj} \geq 0$$

$$k = 1, \ldots, LINK$$

$$j \in S_k$$

where $J^*_{kj} = \dfrac{10.471\ (Q^*_k)^n}{(HW_k)^n\ (D_{kj})^m}$ and Q^*_k is the optimal link flow.

Problem P14 involves selecting the optimal mix of candidate diameters to obtain the required link head losses. Problem P14 may be separated into NLINK independent subproblems, one for each link k as follows:

PROBLEM P15

Minimize $$\sum_{j \in S_k} CL_{kj}\ XL_{kj} \tag{C-4}$$

subject to $$\sum_{j \in S_k} J^*_{kj}\ XL_{kj} = \Delta HF^*_k \tag{C-5}$$

$$\sum_{j \in S_k} XL_{kj} = L_k \tag{C-6}$$

$$XL_{kj} \geq 0 \quad j \in S_k$$

The optimal objective value for Problem P14 (the sum of the optimal objective values for the NLINK subproblems of Problem P15) must equal the link cost component of the optimal solution to Problem P13, the MINCOST problem.

Consider replacing the $|S_k|$ link segments with a single equivalent link of diameter D_k^* that provides the same frictional loss on link k where D_k^* is a convex combination of the set of candidate diameters, i.e.,

$$D_k^* = \sum_{j \,\varepsilon\, S_k} \lambda''_{kj} \; D_{kj} \qquad \text{(C-7)}$$

$$\sum_{j \,\varepsilon\, S_k} \lambda''_{kj} = 1 \qquad \text{(C-8)}$$

$$\lambda''_{kj} \geqq 0 \qquad j \,\varepsilon\, S_k$$

If the link with diameter D_k^* is to provide a head loss of ΔHF_k^*, then

$$D_k^* = \left[\frac{10.471 \; (Q_k^*)^n \; L_k}{(HW_k)^n \quad \Delta HF_k^*} \right]^{\frac{1}{m}} \qquad \text{(C-9)}$$

Dividing the objective function (C-6) and the link length constraint (C-6) by L_k , letting

$$\lambda''_{kj} = \frac{XL_{kj}}{L_k} \quad ,$$

and replacing constraint (C-5) with (C-7) in Problem P15 results in the following equivalent problem:

PROBLEM P16

Minimize $$\sum_{j \varepsilon S_k} CL_{kj} \ \lambda''_{kj} \tag{C-10}$$

subject to $$\sum_{j \varepsilon S_k} \lambda''_{kj} \ D_{kj} = D^*_k \tag{C-11}$$

$$\sum_{j \varepsilon S_k} \lambda''_{kj} = 1 \tag{C-12}$$

$$\lambda''_{kj} \geq 0 \quad j \varepsilon S_k$$

If S_k is arranged in order of increasing diameter $D_{k1} < D_{k2} < \ldots < D_{k|S_k|}$, then $CL_{k1} < CL_{k2} < \ldots < CL_{k|S_k|}$.

Let $D_{k,j-1} < D_k^* < D_{k,j}$ as shown in Figure C-1. Each point on the dashed line connecting each pair or discrete candidate diameters is a convex combination of the two end points. Thus, any pair of candidate diameters such that

$$D_{k,j_1} \leqq D_k^* \leqq D_{k,j_2}$$

can generate a feasible solution for Problem P16. However, because of the strict convexity of the pipe cost function, the chord connecting the diameters adjacent to D_k^*, i.e., $D_{k,j-1}$ and $D_{k,j}$ lies below all other feasible chords and the weights, $\lambda''_{k,j-1}$ and $\lambda''_{k,j}$, found by solving equations (C-11) and (C-12) with all other weights set to zero is optimal for Problem P16.

For multiple loading conditions the diameter of the single equivalent link for loading ℓ would be

$$D_k^*(\ell) = \left[\frac{10,471[Q_k^*(\ell)]^n L_k}{(HW_k)^n \; \Delta HF^*(\ell)}\right]^{\frac{1}{m}} \qquad \text{(C-13)}$$

The equivalent diameter for link k must be identical for all loading conditions or the weighting coefficients in Problem P16 would be

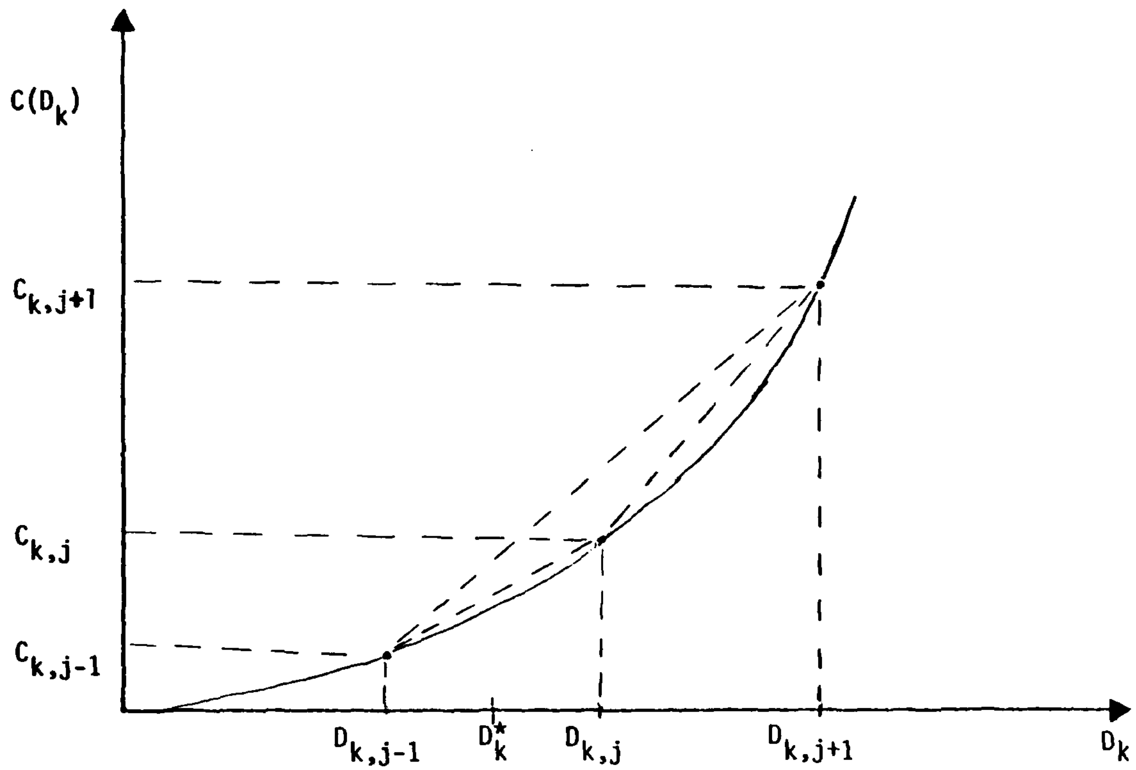


Figure C-1

CONVEX PIPE COST FUNCTION

a function of loading condition and the objective function would not apply. Q.E.D.

If the pipe cost function were strictly proportional to diameter, i.e., convex but not strictly convex, there would be alternate optimal solutions generated by all pairs of candidate diameters such that

$$D_{k,j_1} \leq D_k^* \leq D_{k,j_2} \quad .$$

Also, if the pipe cost function were concave, which might occur if different types of pipes are required for different diameter sizes, restricted basis entry rules (see Appendix B) would be required for the optimum solution to satisfy the results of Theorem II. Although Problem P13, the MINCOST problem, is used in the theorem, it is clear that the result is equally applicable for Problem P12, the MAXWMIN problem.

Application to Continuous Diameter Solutions

As noted in Chapter 1, several minimum cost optimization models ([30], [33], [35], [40], [43], [44], [45], [48]) make the link diameter a continuous decision variable. Lam [39] and Alperovits and Shamir [46] correctly note that because of the requirement

to round optimal continuous pipe diameters to the nearest commercially available size the value of the minimum cost solution will most likely increase and the rounded solution may not even be feasible. Watanatada [40] used a trial and error method to round the diameters. One possible way of solving the rounding problem would be to formulate an unconstrained integer programming problem where the decision variables would be the set of discrete diameters and the objective function would contain the costs plus the sum of the infeasibilities weighted by a penalty factor. However, it appears that this approach may be worse than the original minimum cost problem.

From a practical standpoint an optimal continuous diameter solution is not even feasible since links are only available in discrete sizes. Furthermore, with continuous diameters, the link costs are underestimated anyway. Relaxing the unrealistic requirement to have a single diameter per link, we can use Problem P14 or equivalently Problem P16 to find the optimal link diameter mix given Q_k^* and ΔHF_k^* or equivalently D_k^* and let S_k be the set of all commercially available diameters.

Especially for multiple loading conditions, for the set of commercially available pipe diameters Problem P14 or equivalently Problem P16 may not have a feasible solution. For NLOAD loading conditions we can use the following quadratic programming problem:

PROBLEM P17

$$\text{Minimize} \quad \sum_{k=1}^{NLINK} \sum_{j \in S_k} CL_{kj} \; XL_{kj} \; +$$

$$\sum_{\ell=1}^{NLOAD} \sum_{k=1}^{NLINK} PEN_{k\ell} \left(\sum_{j \in S_k} J^*_{kj\ell} \; XL_{kj} - \Delta HF^*_k (\ell) \right)^2 \qquad (C\text{-}14)$$

subject to

$$\sum_{j \in S_k} XL_{kj} = L_k \qquad (C\text{-}15)$$

$$k = 1, \ldots, NLINK$$

$$XL_{kj} \geq 0$$

$$k = 1, \ldots, NLINK$$

$$j \in S_k$$

where $PEN_{k\ell}$ is a positive penalty function weight and

$$J^*_{kj\ell} = \frac{10.471\left(Q^*_k\ (\ell)\right)^n}{(HW_k)^n\ (D_{kj})^m}$$

Problem P17 is also separable giving us for each link k the following problem:

PROBLEM P18

$$\text{Minimize} \quad \sum_{j \varepsilon S_k} CL_{kj}\ XL_{kj} \ +$$

$$\sum_{\ell=1}^{NLOAD} PENK_{k\ell} \left(\sum_{j \varepsilon S_k} J^*_{kj\ell}\ XL_{kj} - \Delta HF_k^{\ *}(\ell) \right)^2 \tag{C-16}$$

subject to

$$\sum_{j \varepsilon S_k} XL_{kj} = L_k \tag{C-17}$$

$$k = 1, \ldots, NLINK$$

$$XL_{kj} \geq 0$$

$$j \varepsilon S_k$$

Problem P18 is analogous to a constrained regression problem and may be solved by a variety of solution algorithms for quadratic programs [55].

APPENDIX D

USER'S MANUAL/SOURCE PROGRAM LISTING

Introduction

The detailed design computer program was written in FORTRAN and implemented on the University of Texas CDC 6400/6600 computer system. The existing program requires approximately 220K words of memory. This appendix contains a user's manual for the program, which includes a general program description, a detailed description of the program input, and the actual input and output for a simple problem, and a listing of the source program.

User's Manual

General Program Description

The computer program for the detailed design model consists of a single main program and 11 subroutines. The program is centralized about the controlling main program WATOP. Figure D-1 depicts the normal program flow assuming that no changes are made in the candidate diameter set S_k or the capital pump cost coefficients

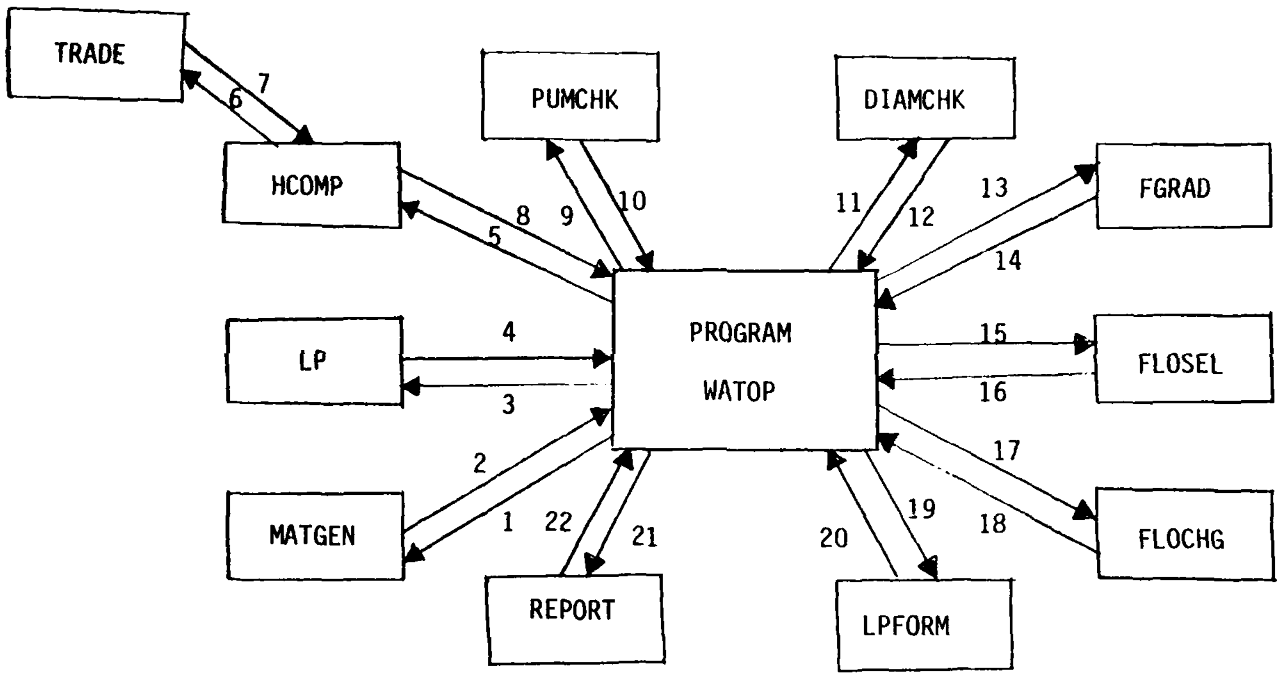


Figure D-1

COMPUTER PROGRAM FLOW

and that the problem is solved in a single iteration. The following is a description of the functions of the main program and each of the subroutines:

WATOP--the main program which is totally responsible for centralized program control.

MATGEN--the subroutine responsible for reading and echoing back the input data and generating the linear programming matrix.

LP--the subroutine responsible for solving the linear program using the primal simplex method with the standard full tableau.

HCOMP--the subroutine responsible for computing the nodal heads on each of the loadings. If a nodal head constraint is violated, HCOMP calls subroutine TRADE to exchange the violated (relaxed) head constraint for a slack (enforced) head constraint in the constraint matrix.

TRADE--the subroutine responsible for exchanging a violated (relaxed) head constraint for a slack (enforced) head constraint in the constraint matrix.

PUMCHK--the subroutine responsible for checking for convergence of the capital pump cost coefficients. If the convergence criteria are not satisfied, the coefficients of the pump capital cost in the constraint matrix are adjusted.

DIAMCHK--the subroutine responsible for checking the diameters used in the current linear program optimal solution and, if necessary, adjusting the set of link candidate diameters and changing the constraint matrix.

FGRAD--the subroutine responsible for computing the loop flow change vector.

FLOSEL--the subroutine responsible for balancing each loading condition using the Hardy Cross method.

FLOCHG--the subroutine responsible for implementing in the constraint matrix the loop flow change vectors generated by FGRAD and FLOSEL.

LPFORM--the subroutine responsible for placing the linear programming matrix back into standard form after changes by TRADE, PUMCHK, DIAMCHK and FLOCHG.

REPORT--the subroutine responsible for output of the optimal design solution and other summary program data.

Description of Input

This section presents a line by line description of the input data. The structure of the input data for the first 11 lines of input, presented below, remains constant regardless of the topology of the distribution system.

LINE NUMBERS: 1-2

FORMAT: 20A4, /20A4

VARIABLES: (C (i), i = 1, 40)

VARIABLE DEFINITIONS: These input lines are used to identify the particular problem solved. The array C is a dummy array subsequently used for the cost vector.

LINE NUMBER: 3

FORMAT: 16I5

VARIABLES: MINCOST, MAXWMIN

VARIABLE DEFINITIONS:

MINCOST--set equal to 1 to solve minimum cost optimization problem (MINCOST) and 0 otherwise.

MAXWMIN--set equal to 1 to solve maximize sum of minimum weighted emergency loading heads (MAXWMIN) and 0 otherwise.

LINE NUMBER: 4

FORMAT: 16I5

VARIABLES: MCRASH, IMAT, IFLODIS

VARIABLE DEFINITIONS:

MCRASH--set equal to 1 to restart problem from optimal flow distribution, candidate diameter set, and pump capital cost coefficient of previous optimal solution and 0 otherwise. This data has been stored on output file 8 from the previous run.

IMAT--set equal to 1 to print nonzero elements in constraint matrix, and all objective function and right hand side elements and 0 otherwise. This is a debugging option and the program terminates following return from subroutine MATGEN.

IFLODIS--set equal to 2 to balance the loading flow distribution after every flow iteration, set equal to 3 to balance flow distribution after first flow iteration only, and set equal to 0 otherwise.

LINE NUMBER: 5

FORMAT: 16I5

VARIABLES: INTER, ICG

VARIABLE DEFINITIONS:

INTER--set equal to 1 to compute loop flow change vector using interaction with other pressure equations and 0 otherwise.

ICG--set equal to 1 to compute loop flow change vector using conjugate gradient with Beale restarts.

LINE NUMBER: 6

FORMAT: 16I5

VARIABLES: NS, NJ, IDMIN, IDMAX, NEXCAV, NQ, NEMERG, NPUMP, NVL, NST, NCLASS, NSOURCE

VARIABLE DEFINITIONS:

NS--the total number of links.

NJ--the total number of nodes.

IDMIN--the minimum commercially available pipe diameter in inches.

IDMAX--the maximum commercially available pipe diameter in inches.

NEXCAV--the number of links with above average excavation costs.

NQ--the total number of loading conditions both normal and emergency.

NEMERG--the number of emergency loading conditions.

NPUMP--the number of pumps.

NVL--the number of real valves.

NST--the number of elevated storage reservoirs.

NCLASS--the number of different classes of pipe of a single diameter.

NSOURCE--the number of source nodes.

LINE NUMBER: 7

FORMAT: I5, 10F5.0

VARIABLES: NPDIAM, DPSPACE

VARIABLE DEFINITIONS:

NPDIAM--the number of candidate diameters per link

DPSPACE--the number of inches between adjacent candidate diameters.

LINE NUMBER: 8

FORMAT: F10.0, F5.0, I5, 2F5.0

VARIABLES: BMAX, IRATE, NYPIPE, SVPIPE, PIPEM

VARIABLE DEFINITIONS:

BMAX--the maximum budget level in dollars.

IRATE--the interest rate used in calculating equivalent uniform annual costs.

NYPIPE--the number of years used in computing the equivalent uniform annual costs for pipes and storage.

SVPIPE--the salvage value ratio for pipes.

PIPEM--the yearly maintenance cost for pipes in dollars/inch of diameter/mile of pipe.

LINE NUMBER: 9

FORMAT: 16F5.0

VARIABLES: (WL(j), j = NQ-NEMERG + 1, NQ)

VARIABLE DEFINITIONS:

WL(j)--the weight assigned to each emergency loading condition j . It is assumed that all normal loading conditions are placed before any emergency loading conditions. This line is deleted for a MINCOST optimization.

LINE NUMBER: 10

FORMAT: I5, 10F5.0

VARIABLES: MXHCIT, HDEVMX, LIMBAL, SIMBAL

VARIABLE DEFINITIONS:

MXHCIT--the maximum number of Hardy Cross iterations for balancing in the subroutine FLOSEL.

HDEVMX--the maximum head imbalance allowed for convergence of the Hardy Cross method in feet.

LIMBAL--the maximum loop imbalance allowed on a relaxed loop equation in feet.

SIMBAL--the maximum resistance of a valve placed between two sources.

LINE NUMBER: 11

FORMAT: I5, 10F5.0

VARIABLES: NYPUMP, SVPUMP, PUMPEFF, POWCOST, PCDIFF

VARIABLE DEFINITIONS:

NYPUMP--the number of years used in computing the equivalent uniform annual costs for pumps.

SVPUMP--the salvage value ratio for pumps.

PUMPEFF--the standard combined pump-motor efficiency. Individual pump-motor efficiency can be specified in subsequent input.

POWCOST--the cost per kilowatt hour of electricity in dollars.

PCDIFF--the maximum ratio difference between estimated and actual pump capital costs. This is the convergence criterion for the iterative linearization of the capital pump costs.

Henceforth, the specific line numbers are dependent on the system configuration. Input line numbers will be identified by their order within each class of data.

Individual Pump Data

For each pump k four input lines are necessary.

LINE NUMBER: 1

FORMAT: 2I5, 2F5.0, I5, 3F5.0

VARIABLES: k, PML(k), HPMIN(k), HPMAX(k), LPUCRIT(k), PPUMP(k), HSTART(k), PUMPF(k)

VARIABLE DEFINITIONS:

k--the pump number.

PML(k)--the link on which pump k is located.

HPMIN(k)--the minimum horsepower of pump k .

HPMAX(k)--the maximum horsepower of pump k . If HPMAX(k) is greater that 9000, there is no limit on pump horsepower.

LPUCRIT(k)--the critical loading for pump k .

PPUMP(k)--the number of identical parallel pumps which pump k is composed of.

HSTART(k)--the initial estimated head for pump k on its critical loading.

PUMPF(k)--the combined pump-motor efficiency for pump k .

LINE NUMBER: 2

FORMAT: 10 (I5, F5.0)

VARIABLES: ((PCOM(k,j), LPCON(k,j)), j = 1, ... NQ)

VARIABLE DEFINITIONS: These two input variables are used to define upper bound constraints on pump head lift between the same pump on different loadings or between different pumps on the same or different loadings.

PCON(k,j)--the number of the pump which pump k's head lift on loading j cannot exceed.

LPCON(k,j)--the particular loading of pump PCON(k,j) which pump k's head lift on loading j cannot exceed.

LINE NUMBER: 3

FORMAT: 10 (I5, F5.0)

VARIABLES: ((LPUMP(k,j), QPUMP(k,j), j = 1, NQ)

VARIABLE DEFINITIONS:

LPUMP(k,j)--set equal to the number assigned to pump k on loading j if pump k is operating and to 0 otherwise.

QPUMP(k,j)--the proportion of the flow on the link PML(k) which pump k on loading j handles.

LINE NUMBER: 4

FORMAT: 8F10.0

VARIABLES: (PUMPHR(k,j), j = 1, ..., NQ)

VARIABLES DEFINITIONS:

PUMPHR(k,j)--the number of hours that pump k operates on loading j per year.

Optimization Parameters

LINE NUMBER: 1

FORMAT: 4F5.0, 2I5

VARIABLES: PSCALE, ALPHA, DQMAX, QRATIO, MXFLOIT, MXLPIT

VARIABLE DEFINITIONS:

PSCALE--a factor used to scale the pressure constraints to reduce the condition number of the constraint matrix.

ALPHA--the initial step length for the flow change vector (GPM).

DQMAX--the optimization terminates when the current step length is less than DQMAX. (GPM)

QRATIO--the proportion of reduction in the step length if the objective value worsens from the previous flow iteration.

MXFLOIT--the maximum number of flow iterations allowed.

MXLPIT--the maximum number of linear programming iterations for each flow iteration.

Storage Data

LINE NUMBER: 1

FORMAT: 8F10.0

VARIABLES: ((STCOST(k), STMAX(k), k = 1, ..., NST)

VARIABLE DEFINITIONS:

STCOST(k)--the cost per foot for elevation of storage reservoir k (dollars).

STMAX(k)--the maximum elevation to be added to storage reservoir k (feet).

Source Data

LINE NUMBER: 1

FORMAT: 16I5

VARIABLES: ((SOURCE(j), j = 1, ..., NSOURCE)

VARIABLE DEFINITIONS:

SOURCE(j)--the node number of source j .

Node Data

For each node $i = 1, \ldots, NJ$ two input lines are necessary.

LINE NUMBER: 1

FORMAT: 1X, I5, 5X, F7.1, 10(2X, F5.1)

VARIABLES: i, ELV(i), (B(i,j), j = 1, ..., NQ)

VARIABLE DEFINITIONS:

i--the node number.

ELV(i)--the elevation of node i (feet).

B(i,j)--the external flow on node i on loading j.

LINE NUMBER: 2

FORMAT: 15X, 6F10.0

VARIABLES: (PR(i,j), j = 1, ..., NQ)

VARIABLE DEFINITIONS:

PR(i,j)--the minimum head at node i under loading j.

Link Data

For each link $i = 1, \ldots, NS$ two input lines are necessary.

LINE NUMBER: 1

FORMAT: I5, 2F10.0, 3I5

VARIABLES: PIPE(i), AL(i), HW(i), IDN(i), IDX(i), ICLASS(i)

PIPE(i)--the link number of the i-th link. Unlike nodes, links do not have to be numbered consecutively.

AL(i)--the length of the i-th link (feet).

HW(i)--the Hazen-Williams roughness coefficient of the i-th link.

IDN(i)--the initial minimum diameter (inches) in the candidate diameter set for the i-th link. If IDN(i) is negative, it is also the minimum allowable diameter on the i-th link.

IDX(i)--the initial maximum diameter (inches) in the candidate diameter set for the i-th link. If IDX(i) is negative, it is also the maximum allowable diameter on the i-th link.

ICLASS(i)--the pressure class number for the i-th link.

LINE NUMBER: 2

FORMAT: 15X, 6F10.0

VARIABLES: (Q(i,j), j = 1, ..., NS)

VARIABLE DEFINITIONS:

Q(i,j)--the initial flow on the i-th link under loading j .

Pressure Constraints

For each loading condition j an input line is required.

LINE NUMBER: 1

FORMAT: 16I5

VARIABLES: NQHEQ(j), NQSEQ(j), NQLEQ(j)

VARIABLE DEFINITIONS:

NQHEQ(j)--the number of nodal demand pressure constraints on loading j .

NQSEQ(j)--the number of source constraints on loading j .

NQLEQ(j)--the number of loop constraints on loading j .

For each pressure constraint a maximum of 5 input lines may be required.

LINE NUMBER: 1

FORMAT: 16I5

VARIABLES: ITYP, IDUP, NSTAR, NFINIS, NLOA, IPM, ISS

VARIABLE DEFINITIONS:

ITYP--set equal to 1 for nodal head constraint, to 2 for source constraint, and to 3 for loop constraint. If set equal to -1, the nodal constraint is not included in the initial set of constraints but may be exchanged. If set equal to -2 or -3,

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A METHODOLOGY FOR OPTIMAL DESIGN OF WATER DISTRIBUTION SYSTEMS.(U)

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the source or loop constraint is relaxed. If set equal to 99999, this is end of data set.

IDUP--set equal to 0 if the set of links in the pressure constraint is not duplicated in a previous loading. For a nodal constraint that is duplicated in a previous loading, set IDUP to the loading number in which the constraint is duplicated. For a duplicate source or loop constraint, set IDUP to the source or loop number which has been duplicated when counting all original loop constraints consecutively. The use of IDUP is not mandatory but can save considerable storage for large problems.

NSTAR--the starting node for the pressure constaint. For nodal and source constraints NSTAR must be a source node. For loop constraints it can be any node in the loop. In this case it is used for identification purposes only.

NFINIS--the finishing node for the pressure constraint. For nodal constraints NFINIS must be a demand node. For source constraints it must be a source node. For loop constraints NFINIS = NSTAR.

NLOA--the loading condition number.

IPM--the number of pumps in the constraint.

ISS--the number of elevated storage reservoirs in the constraint.

LINE NUMBER: 2

FORMAT: 16I5

VARIABLES: N

VARIABLE DEFINITIONS:

N--the number of links in the pressure constraint.

LINE NUMBER: 3

FORMAT: 16I5

VARIABLES: (NO(j), j = LPTR + 1, ..., LPTR + N)

VARIABLE DEFINITIONS:

NO(j)--the links in the pressure equation. Both lines 2 and 3 are deleted for duplicate constraints.

LINE NUMBER: 4

FORMAT: 16I5

VARIABLES: (IPN(i,j), j = 1, ..., IPM)

VARIABLE DEFINITIONS:

IPN(i,j)--the list of pumps in the i-th pressure constraint. This line is deleted if IPM equals 0.

LINE NUMBER: 5

FORMAT: 16I5

VARIABLES: (ISTOR(i,j), j = 1, ..., ISS)

VARIABLE DEFINITIONS:

ISTOR(i,j)--the list of elevated storage reservoirs in the i-th pressure constraint. This line is deleted if ISS equals 0.

Example Problem

The example problem is taken from section 5.5. The topology of the distribution system is shown in Figure D-2. The initial flow distribution for the normal and fire demand emergency loading conditions are shown in Figures D-3 and D-4 respectively. The input data for the problem is shown in Exhibit D-1 and the resulting optimal detailed design output data is shown in Exhibit D-2.

Computer Program Source Listing

The source listing of the program is as shown in Exhibit D-3.

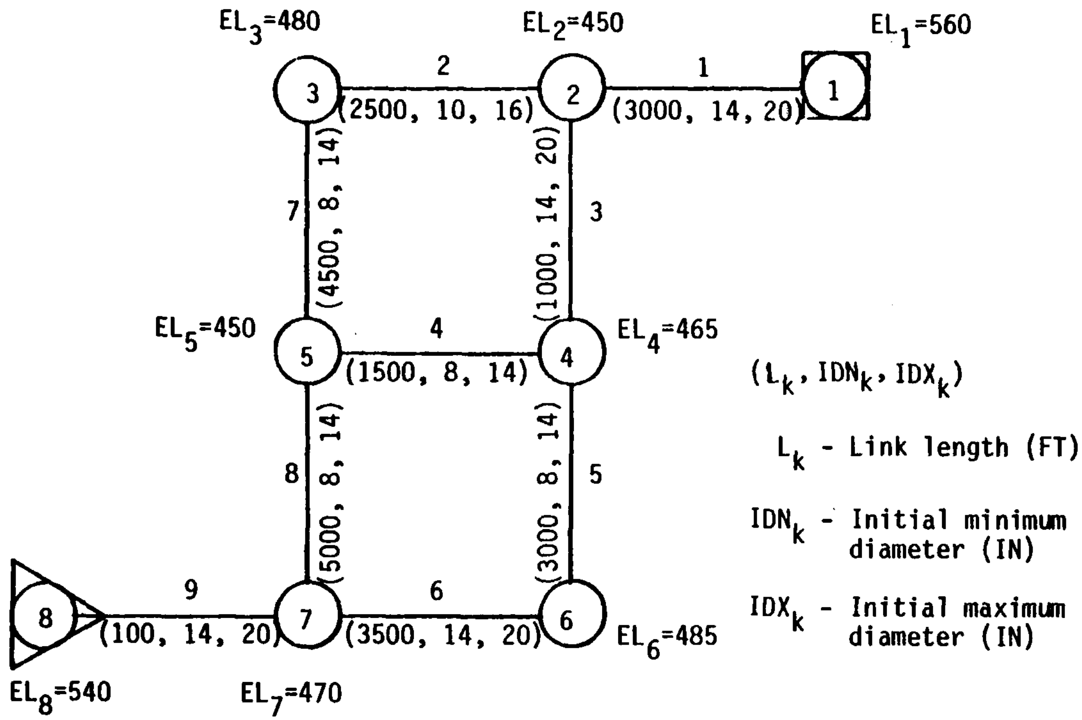

Figure D-2

EXAMPLE DISTRIBUTION SYSTEM

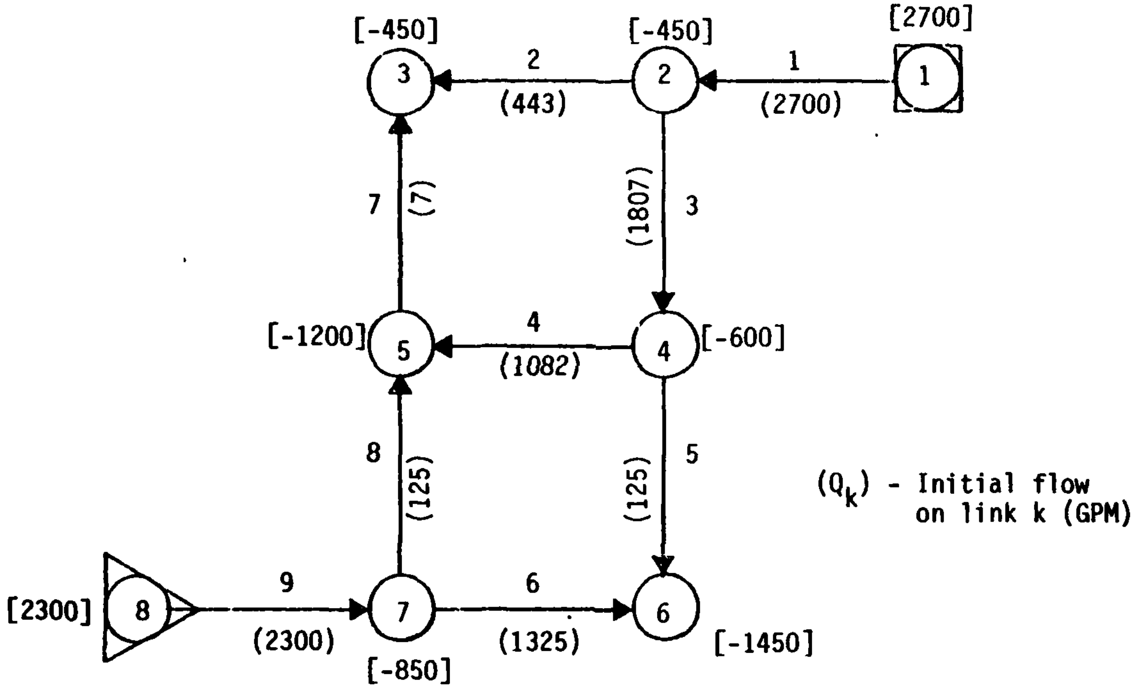

Figure D-3

NORMAL LOADING CONDITION
INITIAL FLOW DISTRIBUTION

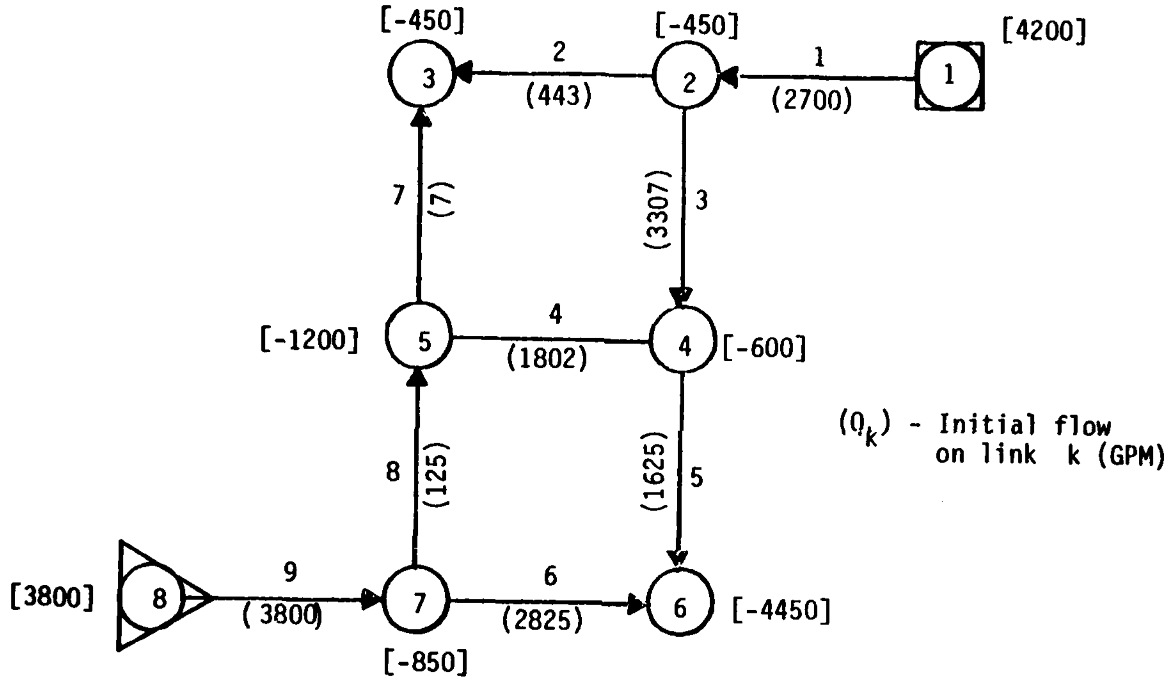

Figure D-4

FIRE DEMAND EMERGENCY LOADING CONDITION
3000 GPM AT NODE 6
INITIAL FLOW DISTRIBUTION

EXHIBIT D-1

```
EXAMPLE PROBLEM FOR DISSERTATION
FIRE DEMAND AT NODE 6(3000 GPM)
   0     1     0
   0     0     0
   1     0     0     1     0
   9     9     6    20     0     2     1     2     0     1     1     1
   4    2.
  70000.  .06    30    .1    4.
  1.     1.
  25    .1    30.
  15    .1    .75   .04    4.   .01
   1     9    0.9999.    1    3.    5.   .75
   0     0     0     0
   1.3333      0     0.
    1000.
   2     9    0.9999.    2    1.    5.   .70
   0     0     1     1
   0    0.     1.5966
        0.
 100.  25.    5.   .15    1.  .005     5   300
   2000.          50.
   1     8     9
   1       560.     -2700.        0.
                        0.        0.
   2       450.       450.      450.
                       90.       46.
   3       480.       450.      450.
                       90.       46.
   4       465.       600.      600.
                       90.       46.
   5       450.      1200.     1200.
                       90.       46.
   6       485.      1450.     4450.
                       90.       46.
   7       470.       850.      850.
                       90.       46.
   8       540.     -2300.    -3800.
                        0.        0.
   9       545.         0.    -4200.
                        0.        0.
   1      3000.       130.   14   20
                     2700.     4200.
   2      2500.       130.   10   16
                      443.      443.
   3      1000.       130.   14   20
                     1807.     3307.
   4      1500.       130.    8   14
                     1082.     1082.
   5      3000.       130.    8   14
                      125.     1625.
   6      2500.       130.   14   20
                     1325.     2825.
   7      4500.       130.    8   14
                        7.        7.
   8      5000.       130.    8   14
                      125.      125.
   9       100.       130.   14   20
                     2300.     3800.
   6     1     2
   6     1     2
   1     0     8     2     1     1     3
```

EXHIBIT D-1 (Continued)

4						
3	6	-5	-3			
-1						
1	0	8	3	1	1	0
5						
9	6	-5	-3	2		
-1						
1	0	8	4	1	1	0
3						
9	6	-5				
-1						
1	0	8	5	1	1	0
4						
9	6	-5	4			
-1						
1	0	8	6	1	1	0
2						
9	6					
-1						
1	0	8	7	1	1	0
1						
4						
-1						
1	0	9	2	2	0	1
1						
1						
-1						
1	0	9	3	2	0	1
2						
1	2					
-1						
1	0	9	4	2	0	1
2						
1	3					
-1						
1	0	9	5	2	0	1
3						
1	3	4				
-1						
1	0	8	6	2	1	1
2						
9	6					
-2						
1	0	8	7	2	1	0
1						
9						
-2						
-2	0	1	8	1	1	1
5						
1	3	4	-8	-9		
1						
-1						
2	0	9	8	2	1	1
5						
1	3	4	-8	-9		
2						
-1						
3	0	0	0	1	0	0
4						
-2	3	4	7			
3	0	0	0	1	0	0
4						

EXHIBIT D-1 (Continued)

5	-6	8	-6				
3	1	0	[illegible]	2	0	[illegible]	
3	2	0	[illegible]	2	0	[illegible]	
99999							
	0.		0.		[illegible]		0.

EXHIBIT D-2

EXAMPLE PROBLEM FOR DISSERTATION

FIRE DEMAND AT NODE 6(3000 GPM)

==

MAXIMIZE WEIGHTED SUM OF MINIMUM HEAD NODES OVER EMERGENCY LOADINGS SUBJECT TO MAXIMUM [illegible] LEVELS
NEGATIVE GRADIENT USED IN COMPUTING DIRECTION VECTOR
INTERACTION BETWEEN PATHS COMPUTED IN GRADIENT
SIGN IF LOOP TERMS IN GRADIENT COMPUTATION IGNORED
LOAD NO. 2 OBJECTIVE FUNCTION WEIGHT= 1.000 LOAD NO.

GENERAL DATA

NUMBER OF SECTIONS	9
NUMBER OF NODES	9
GREATEST DIAMETER ALLOWED (INCH)	20
SMALLEST DIAMETER ALLOWED (INCH)	6
NUMBER OF DIFFERENT FLOW DISTRIBUTIONS	2
NUMBER OF EMERGENCY LOADING CONDITIONS	1
NUMBER OF NORMAL LOADING CONDITIONS	1
NO. OF SOURCE NODES	3
NO. OF LINKS W/HIGH EXCAVATION COST	0
NUMBER OF PUMPS	2
NUMBER OF VALVES	0
NUMBER OF STORAGES	1
ANNUAL TOTAL BUDGET	70000.
INTEREST RATE	.06
PIPE LIFE IN YEARS	30

```
                    PUMPS DATA
                    ==========
-----------------------------------------------------------
    PUMP LIFE IN YEARS              15
-----------------------------------------------------------
    PUMP SALVAGE VALUE RATIO                  .10
-----------------------------------------------------------
    PUMP-MOTOR COMBINED EFFICIENCY     .75
-----------------------------------------------------------
    ELECTRICITY COST($/KW-HR)            .04
-----------------------------------------------------------
    PUMP MAINTENANCE COST($/HP/YR)          4.0
-----------------------------------------------------------
    ALLOWABLE EST/ACTUAL COST , DIFFERENCE  .01

PUMP NO.  LOCATION  HPMIN  HPMAX  C/LOAD HSTART
    1        9        0.    9999.     1  6.
    2        9        0.    9999.     2  6.
LOAD NO. 1 PUMP NO. 1 LOAD PUMP NO. 1 QPUMP   .3333 OPERATING HOURS  100 .00
PUMP COMPOSED OF 3. PUMPS IN PARALLEL
LOAD NO. 1 PUMP NO. 2 LOAD PUMP NO. 0 QPUMP  0.0000 OPERATING HOURS      .00
LOAD NO. 2 PUMP NO. 1 LOAD PUMP NO. 0 QPUMP  0.0000 OPERATING HOURS    - .00
LOAD NO. 2 PUMP NO. 2 LOAD PUMP NO. 1 QPUMP   .5966 OPERATING HOURS    -..00
MAXIMUM NO. OF FLOW ITERATIONS/NETWORK      5
MAXIMUM NO. OF LP ITERATIONS/FLOW         300
INITIAL STEP SIZE(GPM)       = 25.0
MINIMUM ALLOWABLE STEPSIZE  =  5.0
RATIO FOR MINIMUM FLO CHANGE=   .2

    ADDITIONAL STORAGE ELEVATION COST(PER UNIT ELEV)
 STORAGE           COST     MAX HEIGHT
   1         2000.0       50.0


        PIPES COST
        ==========
DIAMETER        UNIT COST(ACCORDING TO CLASS)
   1              1..
   2              2.5
   3              4.2
   4              6.0
   5              8.1
   6             10.2
   7             12.4
   8             14.8
   9             17.2
  10             19.7
```

19	45.1
20	40.2

NODES DATA
==========

NODE	ELEVATION FT.	MIN/MAX PRESSURE ALLOWED LOAD1	LOAD2	LOAD3	LOAD4	LOAD5	LOAD6	LOAD7	LOAD8	LOAD9	LOAD10
1	560.0	0.0	0.0								
2	455.0	90.0	46.0								
3	480.0	90.0	46.0								
4	465.0	90.0	46.0								
5	450.0	90.0	46.0								
6	485.0	90.0	46.0								
7	470.0	90.0	46.0								
8	540.0	0.0	0.0								
9	535.0	0.0	0.0								

CONSUMPTION DATA
(GPM)

NODE	LOAD1	LOAD2	LOAD3	LOAD4	LOAD5	LOAD6	LOAD7	LOAD8	LOAD9	LOAD10
1	-2700.00	0.00								
2	450.00	450.00								
3	450.00	450.00								
4	600.00	600.00								
5	1200.00	1200.00								
6	1450.00	4450.00								
7	850.00	850.00								
8	-2300.00	-1800.00								
9	0.00	-4200.00								

SECTIONS DATA
=============

LINK	LINK	LENGTH	-C-	RANGE OF ALLOWABLE (INCHES)		CLASS	SELECTED (INCHES)				DIS. NO. (FT.)		
1	1	3000.0	130.0	14	20	1	14.	16.	18.	20.			
2	2	2500.0	130.0	10	16	1	10.	12.	14.	16.			
3	3	1000.0	130.0	14	20	1	14.	16.	18.	20.			
4	4	1500.0	130.0	8	14	1	8.	10.	12.	14.			
5	5	3000.0	130.0	8	14	1	8.	10.	12.	14.			
6	6	3500.0	130.0	14	20	1	14.	16.	18.	20.			
7	7	4500.0	130.0	8	14	1	8.	10.	12.	14.			
8	8	5000.0	130.0	8	14	1	8.	10.	12.	14.			
9	9	100.0	130.0	14	20	1	14.	16.	18.	20.			

INITIAL FLOW DISTRIBUTION

LINK	LINK	LOAD1	LOAD2	LOAD3	LOAD4	LOAD5	LOAD6	LOAD7	LOAD8	LOAD9	LOAD10
1	1	2700.0	4200.0								

```
  2    2    561.0    563.
  3    3   1601.0   1157.0
  4    4   1092.0   1092.0
  5    5    125.0   1625.0
  6    6   1125.0   2025.0
  7    7      1.0      1.0
  8    8    125.0    125.0
  9    9   2301.0   3006.
 18   12    2    4
PUMP NO. 1 COST COEFFICIENT=   54.41 MAINTENANCE COST=   [illegible] ENERGY COST=  [illegible]
PUMP NO. 2 COST COEFFICIENT=   [illegible] MAINTENANCE COST=   [illegible] ENERGY COST=    0.00

    STRINGS OF PIPES FOR PRESSURE OR LOOP  CONSTRAINTS
    ----------------------------------------------------
  FROM TO   LOAD       NUMBER ORDER OF SECTIONS                  NUMBER ORDER OF

 NODE NODE  NUM.       CONNECTED BETWEEN THE NODES               PUMPES/VALVES     STOR
PRESSURE EQ.S.
    8    2    1          9   6  -1  -3
.                                                                -1
    8    3    1         -9  -6   5   3  -2
.                                                                -1
    8    4    1         -9  -6   5
.                                                                -1
    8    5    1          9   6  -5   4
.                                                                -1
    8    6    1         -9  -6
.                                                                -1
    8    7    1         -9
.                                                                -1
    9    2    2          1
.                                                                          -1
    9    3    2          1   2
.                                                                          -1
    9    4    2          1   3
.                                                                          -1
    9    5    2          1   3   4
.                                                                          -1
    8    6    2          1   6
.                                                                -2
    8    7    2          9
.                                                                -2
SOURCES EQ.S.
    1    8    1          1   3   4  -8  -9
.                                                                 1
.                                                                          -1
    9    8    2          1   3   4  -8  -9
.                                                                 2
.                                                                          -1
LOOP      EQ.S.
    X    X    1         -2   3   4   7
    X    X    1          5  -6   8  -4
    X    X    2         -2   3   4   7
    X    X    2          5  -6   8  -4
NOD  1)=  4 NOD  2)=  4 NOD  3)=  4 NOD  4)= -5 NOD  5)= [illegible] NOD  6)=  5 [illegible] 7)= -9 [illegible]
NOD 11)= -2 NOD 12)=  3 NOD 13)= -9 NOD 14)= -6 NOD 15)= [illegible] NOD 16)=  4 NOD 17)=  9 [illegible]
NOD 21)=  2 NOD 22)= -9 NOD 23)= -6 NOD 24)=  1 NOD 25)= [illegible] NOD 26)=  1 NOD 27)=  1 [illegible]
NOD 31)=  2 NOD 32)=  1 NOD 33)=  3 NOD 34)=  3 NOD 35)= [illegible] NOD 36)=  3 NOD 37)=  4 [illegible]
NOD 41)=  1 NOD 42)=  9 NOD 43)=  5 NOD 44)=  1 NOD 45)= [illegible] NOD 46)=  4 NOD 47)= -8 [illegible]
NOD 51)=  3 NOD 52)=  4 NOD 53)= -8 NOD 54)= -9 NOD 55)= [illegible] NOD 56)= -2 NOD 57)= [illegible]
NOD 61)=  5 NOD 62)= -6 NOD 63)=  8 NOD 64)= -6 NOD 65)= [illegible]
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```
H( 1)=  99999.00 H( 2)=      30.41 H( 3)=      10.60 H( 4)=      20.55 H( 5)=      31.07
H( 6)=       2.70 H( 7)=      20.37 H( 8)=   99999.00 H( 9)=   99999.00 H(

                                    HEADS FOR LOADING  2
H( 1)=  99999.00 H( 2)=      45.19 H( 3)=      10.01 H( 4)=      20.85 H( 5)=      30.73
H( 6)=        .00 H( 7)=      26.24 H( 8)=   99999.00 H( 9)=   99999.00 H(
PUMP NO. 1 EST COST 1041.20 ACT COST= 1044.01 CAPITAL COST=  368.57 MAINT COST= 1759.4  ENERGY COST=     HP=   539.03
PUMP NO. 2 EST COST  572.64 ACT COST=  569.16 CAPITAL COST=  569.16 MAINT COST= 1850.2  ENERGY COST=  0.00 HP=   464.57
FLOW ITERATION NO.  1 PUNCHK COMPUTATION TIME=  0.0000
NO. OF PUMP COEFFICIENT CHANGES  0
L=  1 K1=  1 K2=  1 K3=  4 ILEQ(L)= 15

   INTERMEDIATE RESULTS FOR COMPUTING GRADIENTS AND FLOW CHANGES IN LOOPS FOR NEXT MAJOR ITERATION
 LOOP DUAL          DB(LOOP)     G(INTER)     U(LOOP)      GRAD         FLOW CHANGE
   1  1.0000E+12   1.9277E-02  2.4573E-03  1.9277E+10  1.9277E+10      24.70
   2   .5188       7.0713E-02  4.7287E+09  3.6694E-02  4.7287E+09       6.66
   3 -1.0000E+12   1.9513E-02 -2.9150E-03 -1.9513E+10 -1.9513E+10     -25.00
   4  -.6165       7.7532E-02 -4.7287E+09 -4.7784E-02 -4.7287E+09      -6.66

                                FLOW DIRECTION OF LINK  7 IN LOAD NO.  2 CHANGED DIRECTION
NO. OF LOOPS CHANGING FLOW =  4
NO. OF IMAGINARY BASIC VARIABLES AFTER LPFORM  1

                                    FLOW ITERATION NO.  2

LOAD    LOOP     LOOP     LOOP     LOOP     LOOP     LOOP     LOOP     LOOP     LOOP     LOOP     LOOPLOOP
NO.     NO.1     NO.2     NO.3     NO.4     NO.5     NO.6     NO.7     NO.8     NO.9     NO.10    NO.11    NO.12
CHANGES FROM INITIAL LOOP FLOW DISTRIBUTION
$$1     24.7      6.1
$$2    -25.0     -6.1


 FEASIBLE SOLUTION REACHED AFTER     2 MINOR ITERATIONS    COST=      -84.41
$$ Z(1)=     0.00 Z(2)=     84.41 Z(

 SOLUTION REACHED AFTER     2 MINOR ITERATIONS
$$   2 (W/PENALTY)  -84.41      (W/O PENALTY)  -84.41
NO. OF IMAGINARY BASIC VARIABLES AFTER LP  0

                                    HEADS FOR LOADING  1
H( 1)=  99999.00 H( 2)=      38.52 H( 3)=      18.66 H( 4)=      27.28 H( 5)=      31.07
H( 6)=       2.24 H( 7)=      19.70 H( 8)=   99999.00 H( 9)=   99999.00 H(

                                    HEADS FOR LOADING  2
H( 1)=  99999.00 H( 2)=      47.31 H( 3)=       7.98 H( 4)=      28.74 H( 5)=      31.93
H( 6)=        .00 H( 7)=      25.14 H( 8)=   99999.00 H( 9)=   99999.00 H(
PUMP NO. 1 EST COST 1028.76 ACT COST= 1030.44 CAPITAL COST=  343.48 MAINT COST= 1721.6  ENERGY COST=1284   HP=   530.60
PUMP NO. 2 EST COST  560.92 ACT COST=  561.33 CAPITAL COST=  561.33 MAINT COST= 1018.6  ENERGY COST=  0.00 HP=   459.60
FLOW ITERATION NO.  2 PUNCHK COMPUTATION TIME=  0.0000
NO. OF PUMP COEFFICIENT CHANGES  0
L=  1 K1=  1 K2=  1 K3=  4 ILEQ(L)= 15

   INTERMEDIATE RESULTS FOR COMPUTING GRADIENTS AND FLOW CHANGES IN LOOPS FOR NEXT MAJOR ITERATION
 LOOP DUAL          DB(LOOP)     G(INTER)     U(LOOP)      GRAD         FLOW CHANGE
   1   .1363       2.8849E-02  2.9098E-03  3.8684E-03  6.7782E-03       3.64
   2   .5127       7.4011E-02  7.6210E-04  3.7945E-02  3.8717E-02      20.75
   3 -4.2725E-02   2.9217E-02 -3.3967E-03 -1.2483E-03 -4.6450E-03      -2.99
   4  -.6165       7.5120E-02 -2.3541E-04 -4.6113E-02 -4.6548E-02     -25.00
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NO. OF LOOPS CHANGING FLOW = [illegible]
NO. OF IMAGINARY BASIC VARIABLES AFTER LPFORM  [illegible]

                                   FLOW ITERATION NO.  [illegible]

LOAD    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOPLOOP
 NO.    NO.1    NO.2    NO.3    NO.4    NO.5    NO.6    NO.7    NO.8    NO.9    NO.10   NO.11   NO.12
CHANGES FROM INITIAL LOOP FLOW DISTRIBUTION
SS1     24.7    26.8
SS2    -25.0   -31.1


 FEASIBLE SOLUTION REACHED AFTER    12 MINOR ITERATIONS    COST=     -79.63
SS Z(1)=     0.00 Z(2)=     80.81 Z(
 SOLUTION REACHED AFTER   17 MINOR ITERATIONS
SS   3 (W/PENALTY)  -80.81      (W/O PENALTY)  -80.81
NO. OF IMAGINARY BASIC VARIABLES AFTER LP  0

                                   HEADS FOR LOADING  1
H( 1)= 99999.00 H( 2)=     30.18 H( 3)=     20.80 H( 4)=     28.64 H( 5)=     30.19
H( 6)=     1.95 H( 7)=     20.24 H( 8)= 99999.00 H( 9)= 99999.00 H(

                                   HEADS FOR LOADING  2
H( 1)= 99999.00 H( 2)=     55.15 H( 3)=     15.07 H( 4)=     37.15 H( 5)=     55.02
H( 6)=     0.00 H( 7)=     29.17 H( 8)= 99999.00 H( 9)= 99999.00 H(
PUMP NO. 1 EST COST 1844.33 ACT COST= 1040.42 CAPITAL COST= 366.81 MAINT COST= 1747.5[illegible] ENERGY COST=13039.[illegible] HP= [illegible]
PUMP NO. 2 EST COST  588.90 ACT COST=  566.77 CAPITAL COST=  566.77 MAINT COST= 1856.1[illegible] ENERGY COST=     0.00 HP= 461.5[illegible]
FLOW ITERATION NO.  3 PUMCHK COMPUTATION TIME=  0.0000
NO. OF PUMP COEFFICIENT CHANGES  0
L=  1 K1=  1 K2=  1 K3=  4 ILFQ(L)= 15


   INTERMEDIATE RESULTS FOR COMPUTING GRADIENTS AND FLOW CHANGES IN LOOPS FOR NEXT MAJOR ITERATION
 LOOP DUAL        DU(LOOP)     G(INTER)     G(LOOP)      GUSD       FLOW CHANGE
  1  -.8667      3.4983E-02 -3.0191E-03 -2.6853E-02 -2.9072E-02     -10.83
  2  -.4639      7.6994E-02 -5.6489E-03 -3.5717E-02 -4.1358E-02     -15.00
  3   .2425      3.1688E-02 -3.8783E-03  8.3144E-03  4.4162E-03       1.41
  4  -.5920      6.6447E-02  1.7192E-03 -3.9348E-02 -3.7620E-02     -13.64
NO. OF LOOPS CHANGING FLOW =  3
NO. OF IMAGINARY BASIC VARIABLES AFTER LPFORM  1

                                   FLOW ITERATION NO.  4

LOAD    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOPLOOP
 NO.    NO.1    NO.2    NO.3    NO.4    NO.5    NO.6    NO.7    NO.8    NO.9    NO.10   NO.11   NO.12
CHANGES FROM INITIAL LOOP FLOW DISTRIBUTION
SS1     13.9    11.8
SS2    -25.0   -44.7


 FEASIBLE SOLUTION REACHED AFTER     5 MINOR ITERATIONS    COST=     -82.21
SS Z(1)=     0.00 Z(2)=     85.49 Z(
 SOLUTION REACHED AFTER    9 MINOR ITERATIONS
SS   4 (W/PENALTY)  -85.49      (W/O PENALTY)  -85.49
NO. OF IMAGINARY BASIC VARIABLES AFTER LP  0
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                                HEADS FOR LOADING  1
HT 1)= 99999.00 HT 2)=    36.[illegible] HT 3)=    19.39 HT 4)=    26.79 HT 5)=    [illegible]
HT 6)=     2.46 HT 7)=    19.[illegible] HT 8)= 99999.00 HT 9)= 99999.00 HT

                                HEADS FOR LOADING  2
HT 1)= 99999.00 HT 2)=    50.[illegible] HT 3)=    10.51 HT 4)=    32.90 HT 5)=    [illegible]
HT 6)=      .00 HT 7)=    21.39 HT 8)= 99999.00 HT 9)= 99999.00 HT
PUMP NO. 1 EST COST 1028.69 ACT COST= 1028.69 CAPITAL COST= 1[illegible] MAINT COST= 1717.31 ENERGY COST=120[illegible] HP= [illegible]
PUMP NO. 2 EST COST  568.96 ACT COST=  568.38 CAPITAL COST= 568.38 MAINT COST= 1013.[illegible] ENERGY COST=    0.00 HP= [illegible]
FLOW ITERATION NO. 4 PUMCHK COMPUTATION TIME= 0.0000
NO. OF PUMP COEFFICIENT CHANGES  0
L= 1 K1= 1 K2= 1 K3= 4 ITEOILI= 15

   INTERMEDIATE RESULTS FOR COMPUTING GRADIENTS AND FLOW CHANGES IN LOOPS FOR NEXT MAJOR ITERATION
 LOOP DUAL         DU(LOOP)    G(INTER)    G(LOOP)      GRAD        FLOW CHANGE
  1  -1.031       3.9351E-02 -3.9279E-03 -4.0591E-02 -4.4519E-02     -12.72
  2  -.5798       7.7148E-02 -6.9876E-03 -4.4710E-02 -5.[illegible]E-02  -15.[illegible]
  3   .1723       3.9673E-02 -4.0244E-03  1.4815E-02  1.0795E-02      3.13
  4  -.5859       6.1114E-02  2.5572E-03 -3.5807E-02 -3.3249E-02     -9.65
NO. OF LOOPS CHANGING FLOW =  4
NO. OF IMAGINARY BASIC VARIABLES AFTER LPFORM  2

                                FLOW ITERATION NO.  5

LOAD     LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP LOOP
 NO.     NO.1    NO.2    NO.3    NO.4    NO.5    NO.6    NO.7    NO.8    NO.9    NO.10   NO.11   NO.12
CHANGES FROM INITIAL LOOP FLOW DISTRIBUTION
$$1       .9     -3.2
$$2    -21.9    -54.4
$$INFEASIBILITY X( 67)=    65.79 LOOP   1 LOAD NO.   1
$$ Z(1)=      3.00 Z(2)=     88.03 Z(

 SOLUTION REACHED AFTER    9 MINOR ITERATIONS
$$   5 (W/PENALTY)  6.6790E+11 (W/O PENALTY)  -68.01
NO. OF IMAGINARY BASIC VARIABLES AFTER LP  0

                                HEADS FOR LOADING  1
HT 1)= 99999.00 HT 2)=    37.49 HT 3)=    17.10 HT 4)=    25.03 HT 5)=    34.99
HT 6)=     2.28 HT 7)=    10.63 HT 8)= 99999.00 HT 9)= 99999.00 HT

                                HEADS FOR LOADING  2
HT 1)= 99999.00 HT 2)=    46.79 HT 3)=     7.63 HT 4)=    29.18 HT 5)=    37.50
HT 6)=     0.00 HT 7)=    20.24 HT 8)= 99999.00 HT 9)= 99999.00 HT
PUMP NO. 1 EST COST 1008.23 ACT COST= 1012.00 CAPITAL COST= 337.33 MAINT COST= 1674.11 ENERGY COST=124[illegible] HP= 418.5[illegible]
PUMP NO. 1 OLD COEFFICIENT=     25.85 NEW COEFFICIENT =    25.15
PUMP NO. 2 EST COST  544.88 ACT COST=  551.79 CAPITAL COST= 551.79 MAINT COST= 1768.[illegible] ENERGY COST=    0.00 HP= [illegible]
PUMP NO. 2 OLD COEFFICIENT=     14.88 NEW COEFFICIENT =    14.25
FLOW ITERATION NO. 5 PUMCHK COMPUTATION TIME= 0.0000
NO. OF PUMP COEFFICIENT CHANGES  2
NO. OF IMAGINARY BASIC VARIABLES AFTER LPFORM  0

                                FLOW ITERATION NO.  5

LOAD     LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP    LOOP LOOP
 NO.     NO.1    NO.2    NO.3    NO.4    NO.5    NO.6    NO.7    NO.8    NO.9    NO.10   NO.11   NO.12
CHANGES FROM INITIAL LOOP FLOW DISTRIBUTION
$$1       .9     -1.2
$$2    -21.9    -54.4
$$INFEASIBILITY X( 67)=    65.79 LOOP   1 LOAD NO.   1
```

```
$$ Z(1)=      0.00 Z(2)=      00.00 Z(

 SOLUTION REACHED AFTER    1 MINOR ITERATIONS
$$   5 IN/PENALTY)  6.5200E+11 (W/O PENALTY)  -00.00
NO. OF IMAGINARY BASIC VARIABLES AFTER LP  0

                                   HEADS FOR LOADING   1
HE 1)=  99999.00 HE 2)=      37.90 HE 3)=      17.07 HE 4)=      25.00 HE 5)=      34.90
HE 6)=       2.25 HE 7)=      10.50 HE 8)=  95999.00 HE 9)=  99.99.00 HE

                                   HEADS FOR LOADING   2
HE 1)=  99999.00 HE 2)=      46.79 HE 3)=       7.67 HE 4)=      29.18 HE 5)=      37.50
HE 6)=        .00 HE 7)=      24.54 HE 8)=  95999.00 HE 9)=  99.99.00 HE
PUMP NO. 1 EST COST  1011.21 ACT COST=  1011.59 CAPITAL COST=   137.16 MAINT COST= 1172.6 , ENEY COST= 2479.6 HP=  118.2
PUMP NO. 2 EST COST   550.06 ACT COST=   551.01 CAPITAL COST=   551.01 MAINT COST= 1766.8 , ENEY COST=     0.0 HP=  441.7
FLOW ITERATION NO.  5 PUMCHK COMPUTATION TIME=  0.0000
NO. OF PUMP COEFFICIENT CHANGES  0


COMPUTATION TIME TOTALS(IN SECONDS)
SUBROUTINE MATGEN      .0097
SUBROUTINE NETOP     9.0490
SUBROUTINE FLOSEL    0.0000
SUBROUTINE LP        5.1290
SUBROUTINE LPFORM    2.7940
SUBROUTINE PUMCHK     .5271
SUBROUTINE GMAD       .1419
SUBROUTINE DIAMCHG    .0360
SUBROUTINE FLOCHG     .4190
SUBROUTINE SAVEOPT   0.0000
                     $$OPTIMAL FLOW ITERATION NO.  4


        $$OPTIMAL DIAMETERS
       -------------------------------------

 $$SEC LENGTH        DIAM1  LENGTH1 DIAM2  LENGTH2   DIAM3    LENGTH3
  $$NO   FT.          IN      FT       IN   FT.       IN.      FT.
-----------------------------------------------------------------------------------------------------
$$  1      3000.00   20.00 3000.00    0.00     0.00    0.00     0.00
$$  2      2500.00   10.00 2131.84    6.00   368.16    0.00     0.00
$$  3      1000.00   14.00   73.73   20.00   926.27    0.00     0.00
$$  4      1500.00    8.00  159.08   12.00  1340.92    0.00     0.00
$$  5      3000.00   10.00  959.09   12.00  2040.91    0.00     0.00
$$  6      3500.00   14.00  513.57   20.00  2986.43    0.00     0.00
$$  7      4500.00    6.00 4500.00    0.00     0.00    0.00     0.00
$$  8      5000.00    8.00   53.77    6.00  4946.23    0.00     0.00
$$  9       100.00   10.00  100.00    0.00     0.00    0.00      .00


                    $$LINK FLOW DISTRIBUTION (GPM)
$$LINK  LOAD1     LOAD2    LOAD3    LOAD4    LOAD5    LOAD6    LOAD7    LOAD8    LOAD9    LOAD10
$$   1 2700.00000 4200.00000
$$   2  429.13592  464.00000
$$   3 1670.86408 3282.00000
$$   4 1084.01679 1181.70248
$$   5  136.84729 1580.29712
$$   6 1313.15271 2869.70248
$$   7   20.86400  -18.00000
```

```
$$  8  136.89729   80.24712
$$  9 2300.00000 3100.00100


                    LINK HEAD LOSS (FT)
LINK   LOAD1    LOAD2    LOAD3   LOAD4   LOAD5   LOAD6         LOAD7   LOAD8   LOAD9   LOAD10
$$  1    3.98986    9.54332
$$  2    8.40410    9.93833
$$  3     .80240    2.56798
$$  4    7.34341    7.56609
$$  5     .13897   12.98296
$$  6    2.80620    8.38491
$$  7     .25829     .19649
$$  8    9.27873    3.45389
$$  9     .16509     .41836
        $$TOTAL EQUIVALENT ANNUAL PIPELINE COST        44808.

  $$EQUIVALENT ANNUAL PIPELINE CAPITAL COST       44590.

  $$ANNUAL PIPELINE O&M COST                         218.

        $$TOTAL EQUIVALENT ANNUAL STORAGE COST          7265.
$$PUMP NO.   TOTAL    CAPITAL    PAINT    ENERGY   HP
$$ 1  15557.11   1628.79   1117.31  12811.12    929.33
$$ 2   2374.20    560.38   1813.82      0.00    453.45
                 $$TOTAL EQUIVALENT ANNUAL NETWORK(NO PENALTY)       79609.


                 NODES DATA
                 ==========
      NODE   NO.   FRICTION        MIN/MAX         EXISTING      DUAL
                    LOSSES    PRESSURE ALLOWED      PRESSURE     ACTIVITY

PRESSURE EQ.S.

         PUMPS ACTIVITY (FT)
         ---------------

LOAD     PUMP     PUMP     PUMP     PUMP     PUMP NO.     NO.1     NO.2     NO.3     NO.4     NO.5     NO.6

$$ 1      39.7        1

$$ 2         1     39.7

         DUMMY VALVE  ACTIVITY
         ----------------------

LOAD     SOURCE    SOURCE    SOURCE    SOURCE    SOURCE
NO.      NO.1      NO.2      NO.3      NO.4      NO.5
     ADDITIONAL STORAGE ELEVATIONS (FT)
   ---------------------------------
STORAGE NO.          1

$$ADDED ELEVATION  60.8
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EXHIBIT D-3

COMPUTER SOURCE LISTING

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      PROGRAM WATOP (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE8,TAPE11,
     1TAPE12)
      TRACE STATEMENT NUMBERS
C
C**********************************************************************
C     QRATIO     = IF (QG/QMAX).LT.QRATIO THEN TERMINATE
C     BMAX   = MAXIMUM ANNUAL TOTAL BUDGET(CAPITAL&OPERATING)
C     DQMAX  = VALUE OF FLOW CHANGE BELOW WHICH ITERATIONS ARE STOPPED
C     GRMAX  = MAXIMUM GRADIENT ALLOWED
C     GRMIN  = MINIMUM GRADIENT ALLOWED
C      HPMAX   = LARGEST PUMP HORSEPOWER ALLOWED
C     IDMIN  = MINIMUM DIAMETER ALLOWED
C     IDMAX  = MAXIMUM DIAMETER ALLOWED
C     IRATE  =INTEREST RATE USED IN PRESENT WORTH COMPUTATIONS
C     MXLPIT  =MAXIMUM NO. OF LP ITERATIONS PER FLOW ITERATION
C     MXFLOIT =MAXIMUM NO. OF FLOW ITERATIONS PER NETWORK OPTIMIZATION
C     MXNETIT =MAXIMUM NO. OF NETWORKS OPTIMIZED PER COMPUTER RUN
C     NOVARS     = NUMBER OF DECISION VARIABLES IN THE INITIAL LP
C                =INITIAL NO. OF SEGMENTS IN PIPES+ NO. OF PUMPS,
C                VALVES AND RESERVOIRS + OBJECTIVE FUNCTION VARIABLES
C     NMROWS      = NUMBER OF CONSTRAINTS IN THE LP = NHEQ+NSEQ+NLEQ+NS
C     NMSLACK   = NO. OF NEGATIVE SLACK VARIABLES IN PRESSURE CONSTRAINT
C     NPEQ    =NHEQ+NSEQ+NLEQ TOTAL NUMBER OF PRESSURE CONSTRAINTS
C     NEXCAV = NUMBER OF LINKS WITH EXTRA EXCAVATION COSTS
C     NMCOLS        NUMBER OF VARIABLES  IN THE LP (INCLUDING SLACKS AND
C                ARTIFICIAL VARIABLES)= NOVARS+NMROWS+NMSLACK
C     NCLASS  = NUMBER OF DIFFERENT PIPE CLASSES (WALL THICKNESSES)
C     NEMERG   =NUMBER OF EMERGENCY LOADING CONDITIONS
C     NJ     = NUMBER OF NODES
C     NLOOP  = TOTAL NUMBER OF LOOPS UNDER ALL LOADING CONDITIONS
C     NNORM  = NUMBER OF NORMAL LOADING CONDITIONS
C     NPBZ =NUMBER OF BUDGET CONSTRAINTS
C      NPEQ   =NHEQ+NSEQ+NLEQ -TOTAL NO. OF PRESSURE EQUATIONS
C     NLEQ   = NUMBER OF LOOP CONSTRAINTS
C     NSEQ   = NUMBER OF CONSTRAINTS BETWEEN FIXED HEAD NODES
C     NHEQ   = NUMBER OF PRESSURE CONSTRAINTS AT NODES
C     NPUMP  = NUMBER OF PUMPS
C     NQ     = NUMBER OF LOADINGS
C     NRL    = NUMBER OF REDUNDANT LINKS IN THE SYSTEM
C     NS     = NUMBER OF SECTIONS (PIPES)
C     NST    = NUMBER OF STORAGE RESERVOIRS WHOSE ELEVATION IS TO BE
C                DESIGNED
C     NT     = NUMBER OF LOOPS PLUS PATHS IN WHICH THE FLOW IS ALLOWED
C                CHANGE
C     NVL    = NUMBER OF VALVES
C     NYPIPE   =USEFUL ECONOMIC LIFETIME FOR PIPELINE IN YEARS
C     NYPUMP   =USEFUL ECONOMIC LIFETIME FOR PIPELINE IN YEARS
C     PIPEM  = PIPELINE O&M COST/INCH OF DIAMETER/MILE/YEAR
C     POWCOST  = COST OF ELECTRICITY IN $/KW-HR
C     PUMPEFF  = PUMP-MOTOR COMBINED EFFICIENCY
C     PUMPM    = MAINTENANCE COST OF PUMPS/HORSEPOWER/YEAR
C     SVPIPE   = RATIO OF PIPE SALVAGE VALUE TO INITIAL VALUE
C     ALPHA     = INITIAL STEP SIZE FOR FLOW CHANGES
C**********     MATRICES AND THEIR DIMENSIONS     ********************
C     AAL(3)        = TEMPORARY OPTIMAL DIAMETERS OF A LINK
C     ALL(NL,5)     = LENGTHS OF THE OPTIMAL SEGMENTS
C     AL(NS)        = LENGTH OF THE LINK
C     B(NMROWS)          = R.H.S. VECTOR FOR THE LP
C     BCCON(NMCOLS) = SEPARATE CAPITAL COST  COEFFICIENTS
C     BOCON(NMCOLS) = SEPARATE OPERATING COST   COEFF'S
C     BCON(NMCOLS) = COMBINED CAPITAL&OPERATING COST ARRAY
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C     HW(NS)          = HAZEN-WILLIAMS COEFFICIENTS
C     CONS(NJ,NQ)  = CONSUMPTIONS AT NODES
C     D(NS,MAX. NO. OF DIAMETERS PER LINK) = DIAMETERS
C     DOP(3)          = OPTIMAL DIAMETERS OF A LINK
C     DQ(NLOOP)   = FLOW CHANGES IN THE LOOPS
C     ELV(NJ)         = NODE ELEVATIONS
C     FF(NT)          = USED IN COMPUTING THE GRADIENT
C     GB(NT)     = GRADIENT COMPONENTS W.R.T. BUDGET CONSTRAINTS
C     GPP(NMROWS) = USED IN COMPUTING PERFORMANCE GRADIENT
C     GZ(NT)    = GRADIENT COMPONENTS W.R.T. PERFORMANCE OBJ. FUNCTION
C     HCORR(NPEQ)  = HEAD CORRECTION FOR RHS DUE TO MINIMAL PUMP SIZE
C     HMAX(NPUMP,NQ) = MAXIMUM HEAD FOR EACH PUMP/LOADING COMBINATION
C     HMIN(NPUMP,NQ) = MINIMUM HEAD  FOR EACH PUMP/LOADING COMBINATIOON
C     HPMIN(NPUMP)    = MINIMUM HORSEPOWER CAPACITY REQUIRED FOR PUMP
C     HF(NS,NQ)    = HEAD LOSS IN LINK UNDER EACH LOADING
C     HFQ(NS,NQ)     = THE RATIO HF/Q, USED IN COMPUTING THE GRADIENT
C     HMIN(NPUMP,NQ)= MINIMUM HEAD FOR EACH PUMP/EMERG. LOADING
C     IAM( , , ,)     = USED IN GRADIENT COMPUTATIONS
C                       GRADIENTS OF THE OBJECTIVE FUNCTION
C     IBC(NMROWS)    = THE BASIS OF THE LP
C     ICLASS(NS)     = CLASS OF THE SECTION
C      ICOMLIN( , , )= USED IN GRADIENT COMPUTATIONS
C     IDN(NS)         = MIN DIAMETER ALLOWED FOR A PARTICULAR LINK
C     IDX(NS)         = MAX DIAMETER ALLOWED FOR A PARTICULAR LINK
C     IEQSTAT(NPEQ) = STATUS OF PRESSURE EQUATION(ACTIVE/INACTIVE)
C     IEQRL(NPEQ)     = ARRAY OF REDUNDANT LINKS ASSOCIATED WITH A
C                     PARTICULAR PRESSURE EQUATION
C     IPLEQ(NS,10)   = ARRAY OF PRESSURE EQUATION NUMBERS ASSOCIATED
C                     WITH A PARTICULAR REDUNDANT LINK
C     IPLSTATUS(NS,MXNETIT) = STATUS OF EACH LINK IN EACH NETWORK OPTIMI
C     NSTART(NPEQ) = STORES START NODE FOR PRESSURE CONSTRAINT COMPUTATI
C     PIZ(NMROWS) = DUAL VARIABLES W.R.T. PERFORMANCE FUNCTION
C                       CONSTRAINT IS FORMULATED
C     NLOAD(NPEQ)        = NO. OF LOADS FOR EACH CONSTRAINT
C     NLINK(NPEQ)         = NO. OF SECTIONS IN A CONSTRAINT
C     IPMP(NPEQ)         = USED TO HOLD THE NUMBERS OF PUMPS AND VALVES
C                       IN THE CONSTRAINT
C     IPN(MA,MAX. NO. OF PUMPS AND VALVES IN ANY CONSTRAINT) =
C                       LIST OF PUMP AND VALVE NUMBERS IN THE CONSTRAINTS
C     IST(NO. OF PRESSURE CONSTRAINTS+LOOPS+BETWEEN NODES) = NO. OF
C                       RESERVOIRS IN THE CONSTRAINT
C     ISTOR(NPEQ,4) = NO. OF RESERVOIRS IN CONSTRAINT
C     ITYPE(NMROWS) = EQUATION TYPE 0-HEAD MAX 1-HEAD MIN 2-SOURCE
C           3-LOOP 4-LENGTH 5-BUDGET 6-STORAGE 7-PUMP
C     IPIV(NMROWS)     = WORK VECTOR
C     LINCOL(NS)      = VECTOR CONTAINS STARTING COL. NO. FOR LENGTH
C                     DECISION VARIABLE
C     LOADCOL(NQ)  = FIRST COLUMN ASSOCIATED WITH EACH LOADING
C     NCOL(NQ)    = NO. OF COLUMNS ASSOCIATED WITH EACH LOADING
C     NCOM(NPEQ,NLOOP)  = NO. OF COMMON LINKS BETWEEN EQUATIONS
C     NDIAM(NS)          = USED TO STORE THE NUMBER OF SELECTED DIAMETERS
C                       FOR EACH LINK
C     NO(NPEQ,MAX NO. OF LINKS IN PR CONSTRAINT) =USED TO STORE THE
C                       CONLINKUTIVE SECTIONS OF THE CONSTRAINT
C     NFINISH(NPEQ) = STORES END NODE FOR PRESSURE CONSTRAINT COMPUTATIO
C     NOPEQ(NQ)    = NO. OF PRESSURE EQUATIONS IN LOADING
C     NQLEQ(NQ)    = NO. OF LOOP EQUATIONS IN LOADING
C     NQSEQ(NQ)    = NO. OF SOURCE EQUATIONS IN LOADING
C     NQHEQ(NQ)    = NO. OF HEAD EQUATIONS IN LOADING
C     PIB(NMROWS) = DUAL VARIABLES W.R.T. BUDGET CONSTRAINTS
C     PIZ(NMROWS) = DUAL VARIABLES W.R.T. PERFORMANCE FUNCTION
C     PML(NPUMP)  = LOCATIONS OF THE PUMPS
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C      PUMPHR(NPUMP,NQ)    = NO. OF OPERATING HOURS OF PUMP FOR EACH LOADI
C      PVL(NVL)    = LOCATION OF REAL VALVES ON LINKS
C      PR(NJ,NQ)         = MIN/MAX PRESSURE  AT NODES UNDER EACH LOADING
C      Q(NS,NQ)      = LINK FLOWS FOR EACH LOADING
C      QD(NLOOP)       = ACCUMULATED FLOW CHANGES IN LOOPS
C      QRS(NQ)        = FLOWS IN A LINK (DIMENSION NQ)
C      C(NMCOLS+1)          = OBJECTIVE FUNCTION
C      STCOST(NST) = COSTS FOR STORAGE VARIABLES
C      STMAX(NST)    = MAXIMUM STORAGE HEIGHT FOR VARIABLE HEAD SOURCE
C      TAB(NO. OF SELECTED DIAMETERS, NO. OF CLASSES) =
C                    USED TO STORE PIPE COST DATA
C      WL(NHEQ)    = WEIGHT OF EACH HEAD CONSTRAINT IN THE OBJECTIVE FUNCT
C                    CONDITION IN OBJECTIVE FUNCTION
C      X(NMCOLS)          = STORES VALUE OF DECISION VARIABLES
C      Y(NMROWS)    = STORES VALUES IN COMPUTING DUAL VARIABLES
C      YB(NMROWS)    = WORK VECTOR FOR PIB CALCULATION
C*****************************************************************
C
      LIST
      LIST
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),C(45,3)
      COMMON /EQ/ IHEQ(3),ISEQ(3),ILEQ(3),NOHEQ(3),NOLEQ(3),NOSEQ(3)
      COMMON /LINK/ AL(45),EXCAVF(45),HW(45),ICLASS(45),LINCOL(45),NDIAM
     1(45),TAB(30,1),IDN(45),IDX(45)
      COMMON /MIND/ MIND(45)
      COMMON /MAXD/ MAXD(45)
      COMMON /PIPE/ PIPE(45)
      COMMON /BASIC/ IBV(325),IPIV(125)
      COMMON /BUF12/ PIZ(125),HF(45,3),X(325)
      COMMON /FLOA/ DQ(45),QD(45),ALFA(3)
      COMMON /PUMPA/ HPMIN(5),HPMAX(5),HMIN(5,3),HMAX(5,3),LPUMP(5,3),LP
     1UCRIT(3),NOPUMP(3),PML(5),PUCOEF(5),PUMPHR(5,3),PVL(1)
      COMMON /ZLOAD/ ZLOAD(3)
      COMMON /ZPEN/ ZPEN(3)
      COMMON /GRAD/ INTER,ICG,IBFGS,GZMCOST,GZMPER,ALPHA,IALP,ICRIT
      COMMON /PREQ/ NHEQ,NSEQ,NLEQ,NPEQ
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NQ,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /OPTION/ IFLODIS,MAXMMIN,MCRASH,MINCOST
      COMMON /MOUT/ MOUT,MIN
      COMMON /IMATGEN/ IMATGEN
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      COMMON /CTIME/ TMATT,TNETT,TFLOS,TLPT,TLPFT,TPUMT,TGRAT,TDIAT,TSAV
     1T,TFLOT
      COMMON /FLOV/ ZFLOOP,ITFLOOP,ITFLO
      COMMON /NTIME/ NDIACHG,NPUMCHK,NFLOCHG,NROWPIV
      COMMON /Z/ Z
      COMMON /MATRIX/ NMROWS,NMCOLS,NMSLACK,NOVARS,NBUROW,MXLPIT
      COMMON /NRHSCHG/ NRHSCHG
      COMMON /IEX/ IEX
      COMMON /IPUMP/ IPUMP
      DATA MIN,MOUT/5,6/
      DIMENSION ZOLD(3)
C
C***** INITIALIZE VARIABLES
C
      Z=0.
      ZLAST=1.E30
      IFLOSEL=0
      NPUMCHK=0
      NDIACHG=0
      NRHSCHG=0
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```
      ITFLOOP=1
      TFLOS=0.
      TLPT=0.
      TPUMT=0.
      TSAVT=0.
      TGRAT=0.
      TDIAT=0.
      TFLOT=0.
      TLPFT=0.
      ZFLOOP=1.E15
      ITFLO=1
      ILP=0
      ILPFORM=0
C
C***** READ IN PROBLEM DATA
C
      CALL SECOND (STATIME)
      CALL MATGEN
      CALL SECOND (ENDTIME)
      TMATT=ENDTIME-STATIME
      IF (IMATGEN.EQ.1) GO TO 260
      DO 10 J=1,NMCOLS
         IBV(J)=0
   10 CONTINUE
      IF (MAXWMIN.EQ.1) IBV(LINCOL(NS)+NDIAM(NS))=-1
      DO 20 I=1,NMROWS
         IBV(IBC(I))=I
         IPIV(I)=0
   20 CONTINUE
      CALL SECOND (STIME)
      DO 30 I=1,NLEQ
         DD(I)=0.
   30 CONTINUE
      DO 40 I=1,NO
         ZLOAD(I)=0.
         ALFA(I)=ALPHA
         ZOLD(I)=1.E30
   40 CONTINUE
      IF (IFLODIS.LT.3) GO TO 50
      READ (MIN,280,END=50) (DD(I),I=1,NLEQ)
      CALL FLOCHG
      IF (IFLODIS.EQ.2) GO TO 70
   50 WRITE (MOUT,290)
      DO 60 I=1,NS
         WRITE (MOUT,300) I,(Q(I,L),L=1,ND)
   60 CONTINUE
C
C***** PLACE MATRIX IN STANDARD FORM
C
   70 CALL SECOND (STATIME)
      CALL LPFORM
      CALL SECOND (ENDTIME)
      CTIME=ENDTIME-STATIME
      TLPFT=TLPFT+CTIME
C
C     $     WRITE(MOUT,260)ITFLO,CTIME,NROWPIV
C     $260 FORMAT(* COMPUTATION TIME FOR FLOW ITERATION NO.*,I3,
C     $    1 *FOR LPFORM =*,F8.4,/,* NO. OF ROW PIVOTS =*,I5)
C
      IF (ILPFORM.EQ.1) GO TO 200
      WRITE (MOUT,310) ITFLO
      IF (NLEQ.EQ.0) GO TO 90
```

```
      WRITE (MOUT,320)
      WRITE (MOUT,330)
      K1=1
      DO 80 I=1,NO
         IF (NQLEQ(I).EQ.0) GO TO 80
         K2=K1+NQLEQ(I)-1
         WRITE (MOUT,340) I,(QD(L),L=K1,K2)
         K1=K2+1
   80 CONTINUE
C
C***** PERFORM LP OPTIMIZATION
C
   90 CALL SECOND (STATIME)
      CALL LP
      CALL SECOND (ENDTIME)
      CTIME=ENDTIME-STATIME
      TLPT=TLPT+CTIME
C
C     $     IF(IEX.EQ.1)WRITE(MOUT,230)((J,X(J)),J=1,NMCOLS+1)
C     $230 FORMAT(7(* X(*,I3,*)=*,F11.2))
C
      IF (ITFLO.GT.1) GO TO 100
      WRITE (MOUT,350) ITFLO,CTIME
  100 IF (IEX.EQ.1) GO TO 110
C
C***** CHECK NODAL HEADS
C
      NPHSCHG=0
C
C     $     DO 110 J=1,NO
C     $     IF(NQHEQ(J).GT.0) CALL HCOMP(J)
C     $110 CONTINUE
C
      IF (NPUMP.EQ.0) GO TO 110
C
C***** CHECK&ADJUST SLOPE OF CAPITAL PUMP COST COEFFICIENT
C
      CALL SECOND (STATIME)
      CALL PUMCHK
      CALL SECOND (ENDTIME)
      CTIME=ENDTIME-STATIME
      TPUMT=TPUMT+CTIME
      WRITE (MOUT,360) ITFLO,CTIME,NPUMCHK
C
C***** CHECK & ADJUST CANDIDATE DIAMETERS IF NECESSARY
C
      IF (IPUMP.EQ.1.AND.NPUMCHK.GT.0) GO TO 70
      IF (IPUMP.EQ.1.AND.NPUMCHK.EQ.0) GO TO 140
  110 CALL SECOND (STATIME)
      CALL DIAMCHK
      CALL SECOND (ENDTIME)
      CTIME=ENDTIME-STATIME
      TDIAT=TDIAT+CTIME
      IF (NDIACHG.EQ.0) GO TO 120
      WRITE (MOUT,370) ITFLO,CTIME,NDIACHG
  120 IF (NDIACHG.GT.0.OR.NPUMCHK.GT.0) GO TO 70
      IF (NLEQ.EQ.0) GO TO 140
      IF (IEX.EQ.1) GO TO 230
      IF (ILP.NE.0) GO TO 250
      IF (IEX.EQ.1) GO TO 220
C
C***** COMPARE CURRENT TO PREVIOUS SOLUTION
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C
      DO 130 I=1,NQ
         IF (Z.GT.ZLAST) ALFA(I)=.6*ALFA(I)
  130 CONTINUE
      ZLAST=Z
  140 IF (Z.GT.ZFLOOP) GO TO 220
C
C***** SAVE IMPROVED SOLUTION
C
      CALL SECOND (STATIME)
      ZFLOOP=Z
      ITFLOOP=ITFLO
  150 CONTINUE
      IF (UNIT,11) 150,160,260
  160 REWIND 11
      BUFFER OUT (11,0) (D(1,1),D(45,3))
  170 CONTINUE
      IF (UNIT,12) 170,180,260
  180 REWIND 12
      BUFFER OUT (12,0) (PIZ(1),X(325))
C
C***** SAVE OPTIMAL SOLUTION FOR RESTART
C
  190 REWIND 8
      DO 200 I=1,NS
         WRITE (8,380) (Q(I,L),L=1,NQ)
  200 CONTINUE
      WRITE (8,390) ((IDN(I),IDY(I),MIND(I),MAXD(I)),I=1,NS)
      DO 210 I=1,NPUMP
         IF (NPUMP.GT.0) WRITE (8,380) PUCOEF(I)
  210 CONTINUE
      CALL SECOND (ENDTIME)
      CTIME=ENDTIME-STATIME
  220 ITFLO=ITFLO+1
      IF (IPUMP.EQ.1) GO TO 260
      IF (ITFLO.EQ.1.AND.IFLODIS.EQ.3) GO TO 260
      IF (IFLODIS.EQ.3) GO TO 240
      IF (ITFLO.GT.MXFLOIT) GO TO 240
      IF (NLEQ.EQ.0) GO TO 260
      IF (NDIACHG.EQ.0.AND.NPUMCHK.EQ.0.AND.IGRAD.EG.1) GO TO 260
  230 IF (IEX.EQ.1.AND.NDIACHG.EQ.0) READ (MIN,420,END=270) (DQ(J),J=1,N
     1LEQ)
C
C***** COMPUTE  GRADIENT FLOW VECTOR
C
      CALL SECOND (STATIME)
      CALL FGRAD
      CALL SECOND (ENDTIME)
      CTIME=ENDTIME-STATIME
      TGRAT=TGRAT+CTIME
C
C***** PERFORM LOOP FLOW CHANGES
C
      CALL SECOND (STATIME)
      CALL FLOCHG
      CALL SECOND (ENDTIME)
      CTIME=ENDTIME-STATIME
      TFLOT=TFLOT+CTIME
      WRITE (MOUT,410) NFLOCHG
  240 IF (IFLODIS.LT.2.OR.IFLOSEL.EQ.1.OR.IFLODIS.EQ.4) GO TO 250
      CALL SECOND (STATIME)
      CALL FLOSEL
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      CALL SECOND (ENDTIME)                                             WAT00372
      CTIME=ENDTIME-STATIME                                             WAT00373
      TFLOS=TFLOS+CTIME                                                 WAT00374
      WRITE (MOUT,420) CTIME                                            WAT00375
C                                                                       WAT00376
C***** CHANGE LOOP FLOWS                                                WAT00377
C                                                                       WAT00378
      WRITE (MOUT,430)                                                  WAT00379
      WRITE (MOUT,320)                                                  WAT00380
      WRITE (MOUT,440) (DQ(L),L=1,NLEG)                                 WAT00381
      CALL FLOCHG                                                       WAT00382
  250 IF (IFLODIS.EQ.3) GO TO 190                                       WAT00383
      GO TO 70                                                          WAT00384
C                                                                       WAT00385
C***** PREPARE REPORT                                                   WAT00386
C                                                                       WAT00387
  260 CALL SECOND (ETIME)                                               WAT00388
      TNETT=ETIME-STIME                                                 WAT00389
      CALL REPORT                                                       WAT00390
  270 CONTINUE                                                          WAT00391
      STOP                                                              WAT00392
C                                                                       WAT00393
  280 FORMAT (8F10.0)                                                   WAT00394
  290 FORMAT (///,22X,27H INITIAL FLOW DISTRIBUTION ,/,34H  LINK   LOAD1WAT00395
     1 - LOAD2   LOAD3   LOAD4   LOAD5   LOAD6,33H   LOAD7   LOAD8   LOAWAT00396
     2D9   LOAD10)                                                      WAT00397
  300 FORMAT (1X,I5,10F8.1)                                             WAT00398
  310 FORMAT (//,40X,19H FLOW ITERATION NO.,I3)                         WAT00399
  320 FORMAT (/,47H LOAD      LOOP      LOOP      LOOP      LOOP     ,58HLOWAT00400
     1OP      LOOP      LOOP      LOOP      LOOP      LOOP      LOOP,4HLOOP,/,WAT00401
     259H  NO.      NO.1      NO.2      NO.3      NO.4      NO.5,63H      NO.6WAT00402
     3      NO.7      NO.8      NO.9      NO.10     NO.11     NO.12)    WAT00403
  330 FORMAT (45H CHANGES FROM INITIAL LOOP FLOW DISTRIBUTION )         WAT00404
  340 FORMAT (3H $$,I1,14(2X,F7.1))                                     WAT00405
  350 FORMAT (43H LP COMPUTATION TIME FOR FLOW ITERATION NO.,I3,1H=,F8.4WAT00406
     1)                                                                 WAT00407
  360 FORMAT (19H FLOW ITERATION NO.,I3,25H PUMCHK COMPUTATION TIME=,F8.WAT00408
     14,/,32H NO. OF PUMP COEFFICIENT CHANGES,I3)                       WAT00409
  370 FORMAT (48H DIAMCHG COMPUTATION TIME FOR FLOW ITERATION NO.,I3,2H WAT00410
     1=,F8.4,/,34H NO. OF LINKS CHANGING DIAMETERS =,I3)                WAT00411
  380 FORMAT (1X,5F12.5)                                                WAT00412
  390 FORMAT (1X,30I4)                                                  WAT00413
  400 FORMAT (2F5.0)                                                    WAT00414
  410 FORMAT (29H NO. OF LOOPS CHANGING FLOW =,I3)                      WAT00415
  420 FORMAT (35H FLOSEL BALANCING COMPUTATION TIME=,F8.4)              WAT00416
  430 FORMAT (19H FLOSEL $$  CHANGES)                                   WAT00417
  440 FORMAT (4H $$ ,18F7.1)                                            WAT00418
C                                                                       WAT00419
      END                                                               WAT00420
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      SUBROUTINE DIAMCHK
      COMMON /BUF1/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /AMAT/ AMAT(110,275)
      COMMON /LINK/ AL(45),EXCAVF(45),HW(45),ICLASS(45),LINCOL(45),NDIAM
     1(45),TAB(30,1),IDN(45),IDX(45)
      COMMON /MIND/ MIND(45)
      COMMON /MAXD/ MAXD(45)
      COMMON /BASIC/ IBV(325),IPIV(125)
      COMMON /BUF12/ PIZ(125),HF(45,3),X(325)
      COMMON /PATH2/ PPTR(75),NLOAD(75)
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NG,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /MATRIX/ NMROWS,NMCOLS,NMSLACK,NDVAPS,NBUROW,MXLPIT
      COMMON /DIAMV/ NPDIAM,DPSPACE,IDMIN,IDMAX
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      COMMON /PRICE/ PIPACRF,PIPEM,STOACRF
      COMMON /PREQ/ NHEQ,NSEQ,NLEQ,NPEQ
      COMMON /NTIME/ NDIACHG,NPUMCHK,NFLOCHG,NROWPIV
      COMMON /MOUT/ MOUT,MIN
      COMMON /FLOV/ IFLOOP,ITFLOOP,ITFLO
      COMMON /OPTION/ IFLODIS,MAXWMIN,MCRASH,MINCOST
      COMMON /Z/ Z
      DIMENSION DOLD(5)
      REAL LMAX,LMIN
      INTEGER DMIN,DMAX,PPTR
      GRADI(AG,AD,AHW)=10.471*((AG/AHW)**1.852/(AD)**4.87)
      NDIACHG=0
      DO 140 I=1,NS
         IF (AL(I).LT.1.E-2) GO TO 140
         IF (IDN(I).EQ.IDX(I)) GO TO 140
         LMAX=1.E-7
         LMIN=1.E6
         NUM1=LINCOL(I)
         NUM2=NUM1+NDIAM(I)-1
C
C***** FIND THE LINK DIAMETERS WITH THE LONGEST(LMAX/IMAX) AND
C       AND SHORTEST (LMIN/IMIN)NONZERO PIPE LENGTHS
C
         DO 20 J=NUM1,NUM2
            IF (X(J).LT.LMAX) GO TO 10
            LMAX=X(J)
            IMAX=J-LINCOL(I)+1
   10       IF (X(J).GT.LMIN.OR.X(J).LT.1.E-7) GO TO 20
            LMIN=X(J)
            IMIN=J-LINCOL(I)+1
   20    CONTINUE
         DMIN=INT(D(I,IMIN))
         DMAX=INT(D(I,IMAX))
C
C     $100 FORMAT(* IMIN=*,I3,* IMAX=*,I3)
C     $     WRITE(MOUT,100) IMIN,IMAX
C***** CASE I LONGEST&SHORTEST LENGTH PIPES HAVE UNEQUAL DIAMETERS
C
         IF (DMIN.NE.DMAX) GO TO 140
C
C***** CASE II SINGLE DIAMETER AT IDMIN OR IDMAX
C
         IF (DMAX.EQ.IDMIN.OR.DMAX.EQ.IDMAX.OR.DMAX.EQ.MIND(I).OR.DMAX.E
     1   Q.MAXD(I)) GO TO 140
C
C***** CASE III SINGLE DIAMETER NOT AT IDN(I) OR INX(I)
```

```
C
      IF (DMAX.NE.IDN(I).AND.DMAX.NE.IDX(I)) GO TO 140
C
C***** CASE III SINGLE DIAMETER EQUAL TO IDN(I) OR IDX(I)
C
      IF (DMAX.EQ.IDX(I).AND.MAXWMIN.EQ.1.AND.Z.GT.1.E8.AND.ZFLOOP.LT
     1.1.E8) GO TO 140
      DO 30 K=1,NDIAM(I)
         DOLD(K)=D(I,K)
   30 CONTINUE
      ND=NDIAM(I)
      DD=DPSPACE*FLOAT(NDIAM(I))
      NCHG=0
      DO 60 K=1,NDIAM(I)
         DDD=D(I,K)+DD
         IF (DMAX.EQ.IDX(I)) GO TO 40
         DDD=D(I,K)-DD
         IF (INT(DDD).GE.IDMIN.AND.INT(DDD).GE.IDN(I)-(NPDIAM-2)*INT(
     1   DPSPACE).AND.INT(DDD).GE.MIND(I)) GO TO 50
         GO TO 60
   40    IF (INT(DDD).LE.IDMAX.AND.INT(DDD).LE.IDX(I)+(NPDIAM-2)*INT(
     1   DPSPACE).AND.INT(DDD).LE.MAXD(I)) GO TO 50
         GO TO 60
   50    NCHG=NCHG+1
         D(I,K)=DDD
   60 CONTINUE
      IF (DMAX.NE.IDX(I)) NCHG=-NCHG
C
C    $    WRITE(MOUT,105)I,IDN(I),IDX(I),((K,DOLD(K)),K=1,ND)
C    $105 FORMAT(* LINK*,I3,* IDN=*,I3,*IDX=*,I3,5(*(DOLD(*,I3,*)=*,F5.
C
      IDN(I)=IDN(I)+NCHG*INT(DPSPACE*DD/ABS(DD))
      IDX(I)=IDX(I)+NCHG*INT(DPSPACE*DD/ABS(DD))
C
C    $    WRITE(MOUT,110)I,IDN(I),IDX(I),((K,D(I,K)),K=1,ND)
C    $110 FORMAT(* LINK*,I3,* IDN=*,I3,* IDX=*,I3,5(*DNEW(*,I3,*)=*,F5.
C
      NDIACHG=NDIACHG+1
      DO 110 II=1,NPSD
         IQ=NLOAD(II)
         DO 70 J=IABS(PPTR(II))+1,IABS(PPTR(II))+NO(IABS(PPTR(II)))
            L=IABS(ND(J))
            IF (L.EQ.I) GO TO 80
   70    CONTINUE
         GO TO 110
   80    SN=FLOAT(L/ND(J))
         IF (ABS(Q(I,IQ)).GT.1.E-2) SN=FLOAT(L/ND(J))*Q(I,IQ)/ABS(Q(I
     1   ,IQ))
         NUM1=LINCOL(I)
         NUM2=LINCOL(I)+NDIAM(I)-1
         III=0
         DO 100 NUM=NUM1,NUM2
            III=III+1
            IF (ABS(DOLD(III)-D(I,III)).LT.1.E-7) GO TO 100
            IF (IBV(NUM).GT.0) IPIV(IBV(NUM))=1
            GROLD=GRAD1(ABS(Q(I,IQ)),DOLD(III),HW(I))*PSCALE
            GRNEW=GRAD1(ABS(Q(I,IQ)),D(I,III),HW(I))*PSCALE
            DEL=(GRNEW-GROLD)*SN
            IART=NOVARS+NMSLACK+II
            DO 90 IROW=1,NMROWS
               AMAT(IROW,NUM)=AMAT(IROW,NUM)+AMAT(IROW,IART)*DEL
   90       CONTINUE
```

```
100       CONTINUE
110     CONTINUE
        IART=NOVARS+NMSLACK+NBURO+
        III=0
        DO 130 NUM=NUM1,NUM2
          III=III+1
          IF (ABS(DOLD(III)-D(I,III)).LT.1.E-7) GO TO 130
          ID=INT(D(I,III))
          BCOLD=PIPACRF*(TAB(INT(DOLD(III)),1)+EXCAVF(I))+PIPEM*DOLD(I
     1    II)/5280.
          BCNEW=PIPACRF*(TAB(ID,1)+EXCAVF(I))+PIPEM*FLOAT(ID)/5280.
          DEL=BCNEW-BCOLD
          DO 120 IROW=1,NMROWS
            AMAT(IROW,NUM)=AMAT(IROW,NUM)+AMAT(IROW,IART)*DEL
120       CONTINUE
130     CONTINUE
140   CONTINUE
      IF (NDIACHG.GT. ) ILPFORM=1
      RETURN
C
      END
```

```
      SUBROUTINE FGRAD
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /INTER/ EQPTR(75),LCOM(325)
      COMMON /EQ/ IHEQ(3),ISEQ(3),ILEQ(3),NQHEQ(3),NQLEQ(3),NQSEQ(3)
      COMMON /PATH2/ PPTR(75),NLOAD(75)
      COMMON /LINK/ AL(45),EXCAVF(45),HW(45),ICLASS(45),LINCOL(45),NDIAM
     1(45),TAB(30,1),IDN(45),IDX(45)
      COMMON /BUF12/ PIZ(125),HF(45,3),X(325)
      COMMON /FLOA/ DQ(45),QD(45),ALFA(3)
      COMMON /ZPEN/ ZPEN(3)
      COMMON /GRAD/ INTER,ICG,IBFGS,GZMCOST,GZMPER,ALPHA,IALP,ICRIT
      COMMON /Z/ Z
      COMMON /MOUT/ MOUT,MIN
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      COMMON /OPTION/ IFLODIS,MAXMMIN,MCRASH,MINCOST
      COMMON /PREQ/ NHEQ,NSEQ,NLEQ,NPEQ
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NQ,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /FLOW/ ZFLOOP,ITFLOOP,ITFLO
      COMMON /IEX/ IEX
      COMMON /NNORM/ NNORM
      DIMENSION GZX(45), GMX(3), GZ(45), DBDQ(45), GZL(45), DOLD(45), GZ
     1OLD(45), Y(45)
      INTEGER PEQ,PPTR,EQPTR
      GRAD1(AQ,AD,AC)=10.471*((AQ/AC)**1.852/(AD)**4.87)
C
***** COMPUTATION OF HEAD LOSS FLOW RATIOS
C
      IGRAD=0
      DO 40 I=1,NS
         II=LINCOL(I)-1
         DO 10 J=1,NQ
            HF(I,J)=0.
 10      CONTINUE
         DO 30 J=1,NDIAM(I)
            II=II+1
            DO 20 L=1,NQ
               HH=GRAD1(ABS(Q(I,L)),D(I,J),HW(I))*X(II)
               HF(I,L)=HF(I,L)+HH
 20         CONTINUE
 30      CONTINUE
 40   CONTINUE
      IF (NLEQ.EQ.0.OR.IEX.EQ.1) GO TO 210
      IF (ITFLO.GT.1.AND.ICG.EQ.1) GO TO 60
      DO 50 I=1,NLEQ
         DOLD(I)=0.
         GZOLD(I)=0.
         Y(I)=0.
 50   CONTINUE
      ZLAST=Z
 60   LOOP=
      GMAX= .
      DO 140 L=1,NQ
         IF (NQLEQ(L).EQ.0) GO TO 140
         IC=0
         IF (ICRIT.EQ.1.AND.ABS(ZPEN(L)).LT.1.E10.AND.L.GT.1) IC=1
         GMX(L)=0.
C
C*****COMPUTE GRADIENTS AND FLOW CHANGES IN LOOPS
C
         DO 130 LEQ=ILEQ(L),ILEQ(L)+NQLEQ(L)-1
```

```
      LOOP=LOOP+1
      DQ(LOOP)=0.
      GZ(LOOP)=0.
      GZL(LOOP)=0.
      GZX(LOOP)=0.
      DBDQ(LOOP)=0.
      IF (MAXMMIN.EQ.1.AND.L.LE.NNORM.AND.ICRIT.EQ.1) GO TO 130
      IF (INTER.EQ.0) GO TO 100
C
C     $     WRITE(MOUT,200)L,LOOP
C     $200 FORMAT(20X,* LOADING NO. *,I2,* LOOP NO.*,I3,
C     $   1 /,*  PEQ   LINK     Q    HF       SN      DQDBX     DUAL      GZX
C***** COMPUTE GRADIENT INTERACTION COMPONENT
C
      IF (EQPTR(LOOP).EQ.0) GO TO 100
      K=EQPTR(LOOP)+1
      DO 90 I=1,LCOM(EQPTR(LOOP))
         PEQ=LCOM(K)
         IF (PEQ.LT.0) GO TO 90
         IF (IC.EQ.1.AND.PEQ.GT.NHEQ) GO TO 80
         IF (ABS(PIZ(PEQ)).LT.1.E-30) GO TO 80
         DO 70 J=1,LCOM(K+1)
            KK=IABS(LCOM(K+J+1))
            IF (ABS(Q(KK,L)).LT.1.E-7) GO TO 70
            DBDQX=HF(KK,L)/ABS(Q(KK,L))
            GZX(LOOP)=DBDQX*PIZ(PEQ)
C
C     $     WRITE(MOUT,205)PEQ,KK,Q(KK,L),HF(KK,L),SN,DBDQX,PIZ(PEQ),GZX(
C     $205 FORMAT(2I5,2X,2F7.2,F5.1,3G12.4)
C
   70    CONTINUE
   80    K=K+LCOM(K+1)+2
   90 CONTINUE
      IF (IC.EQ.1) GO TO 120
C
C***** LOOP GRADIENT COMPONENT
C
  100 DO 110 J=IABS(PPTR(LEQ))+1,IABS(PPTR(LEQ))+NO(IABS(PPTR(LEQ)
     1))
         KK=IABS(NO(J))
         IF (ABS(Q(KK,L)).LT.1.E-7) GO TO 110
         DBDQ(LOOP)=DBDQ(LOOP)+HF(KK,L)/ABS(Q(KK,L))
  110 CONTINUE
      GZL(LOOP)=ABS(DBDQ(LOOP))*PIZ(LEQ)
  120 GZ(LOOP)=GZX(LOOP)+GZL(LOOP)
      IF (ABS(GZ(LOOP)).GT.GMX(L)) GMX(L)=ABS(GZ(LOOP))
  130 CONTINUE
      IF (ABS(GMX(L)).GT.GMAX) GMAX=ABS(GMX(L))
  140 CONTINUE
      NICG=0
      K1=1
      K2=1
      K3=NLEG
C
C***** CHECK FOR RESTART OF CONJUGATE GRADIENT
C
      IF (ICG.EQ.0.OR.ITFLO.EQ.2) NICG=1
      IF (ZLAST.GT.1.E9.AND.Z.LT.1.E9) NICG=1
      IF (ZLAST.LT.1.E9.AND.Z.GT.1.E9) NICG=1
      ZLAST=Z
      DO 190 L=1,K1
         IF (NICG.EQ.1) GO TO 170
```

```
C
C***** COMPUTE CONJUGATE GRADIENT
C
      BETAN=0.
      BETAD=0.
      DO 150 K=K2,K3
         Y(K)=GZ(K)-GZOLD(K)
         GZOLD(K)=GZ(K)
         BETAN=BETAN+Y(K)*GZ(K)
         BETAD=BETAD+Y(K)*DOLD(K)
  150 CONTINUE
      BETAN=BETAN/BETAD
      GMAX=0.
      WRITE (MOUT,230)
      DO 160 K=K2,K3
         GZ(K)=GZ(K)-BETAN*DOLD(K)
         IF (ABS(GZ(K)).GT.GMAX) GMAX=ABS(GZ(K))
         WRITE (MOUT,240) K,L,BETAN,DOLD(K),GZ(K)
  160 CONTINUE
C
C***** COMPUTE FLOW CHANGE
C
  170 WRITE (MOUT,250) L,K1,K2,K3,ILEQ(L)
      DO 180 K=K2,K3
         DQ(K)=ALFA(L)*GZ(K)/GMAX
         DOLD(K)=GZ(K)
         GZOLD(K)=FLOAT(NICG)*GZ(K)+(1.-FLOAT(NICG))*GZOLD(K)
  180 CONTINUE
  195 CONTINUE
      WRITE (MOUT,220)
      DO 200 LOOP=1,NLEQ
         WRITE (MOUT,260) LOOP,PIZ(LOOP+NHEQ+NSEG),DBDQ(LOOP),GZX(LOOP),
     1   GZL(LOOP),GZ(LOOP),DQ(LOOP)
  200 CONTINUE
  210 CONTINUE
      RETURN
C
  220 FORMAT (//5X,95HINTERMEDIATE RESULTS FOR COMPUTING GRADIENTS AND F
     1LOW CHANGES IN LOOPS FOR NEXT MAJOR ITERATION./3X,27HLOOP DUAL
     2     DB(LOOP)  ,48H  G(INTER)     G(LOOP)      GRAD        FLOW CHANGE
     3)
  230 FORMAT (37H LOOP LOAD      BETA      DOLD      DNEW)
  240 FORMAT (2I5,3G12.4)
  250 FORMAT (3H L=,I3,4H K1=,I3,4H K2=,I3,4H K3=,I3,9H ILEQ(L)=,I3)
  260 FORMAT (I5,5G12.4,F10.2)
C
      END
```

```
      SUBROUTINE FLOCHG
      TRACE STATEMENT NUMBERS
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /AMAT/ AMAT(110,275)
      COMMON /EQ/ IHEQ(3),ISEQ(3),ILEQ(3),NQHEQ(3),NQLEQ(3),NQSEQ(3)
      COMMON /PATH2/ PPTR(75),NLOAD(75)
      COMMON /LINK/ AL(45),EXCAVF(45),HW(45),ICLASS(45),LINCOL(45),NDIAM
     1(45),TAB(30,1),IDN(45),IDX(45)
      COMMON /BASIC/ IBV(325),IPIV(125)
      COMMON /FLOA/ DQ(45),QD(45),ALFA(3)
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NG,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      COMMON /NTIME/ NDIACHG,NPUMCHK,NFLOCHG,NROWPIV
      COMMON /PPEQ/ NHEQ,NSEQ,NLEQ,NPEQ
      COMMON /MOUT/ MOUT,MIN
      COMMON /MATRIX/ NMROWS,NMCOLS,NMSLACK,NOVARS,NHUROW,MXLPIT
      COMMON /QRATIO/ QRATIO
      COMMON /Z/ Z
      INTEGER PPTR
      GRAD1(AQ,AD,AC)=10.471*((AQ/AC)**1.852/(AD)**4.87)
      NFLOCHG=0
      LEQNO=NPEQ-NLEQ+1
   10 DQMIN=QRATIO*ALFA(NLOAD(LEQNO))
      IF (IFLOSEL.EQ.1.AND.ABS(DQ(LEQNO+NLEQ-NPEQ)).LT.DQMIN) GO TO 90
      NFLOCHG=NFLOCHG+1
      ILPFORM=2
      IQ=NLOAD(LEQNO)
C
C*****CHANGE FLOWS IN LOOPS, AND UPDATE THE MATRIX
C
      DO 80 J=IABS(PPTR(LEQNO))+1,IABS(PPTR(LEQNO))+NO(IABS(PPTR(LEQNO))
     1)
         L=IABS(NO(J))
         NUM1=LINCOL(L)
         NUM2=LINCOL(L)+NDIAM(L)-1
C
C***** FIND BASIC VARIABLES FOR LOOP LINKS
C
         DO 20 I=NUM1,NUM2
            IF (IBV(I).GT.0) IPIV(IBV(I))=1
   20    CONTINUE
         QOLD=Q(L,IQ)
         SN=FLOAT(NO(J)/IABS(NO(J)))
         Q(L,IQ)=Q(L,IQ)+DQ(LEQNO-NPEQ+NLEQ)*SN
C
C     $    WRITE(MOUT,95)QOLD,L,IQ,Q(L,IQ),SN,DQ(LEQNO-NPEQ+NLEQ)
C     $ 95 FORMAT(* QOLD=*,G15.4,*Q(*,I2,*,*,I2,*)=*,G15.4,*SN=*,F3.0,*D
C     $   1 G15.4)
C
         IF (QOLD*Q(L,IQ).LE.0.) WRITE (MOUT,100) L,IQ
         DO 70 II=IHEQ(IQ),ILEQ(IQ)+NQLEQ(IQ)-1
            IF (NLOAD(II).NE.IQ) GO TO 70
            DO 30 JJ=IABS(PPTR(II))+1,IABS(PPTR(II))+NO(IABS(PPTR(II)))
               IF (L.EQ.IABS(NO(JJ))) GO TO 40
   30       CONTINUE
            GO TO 70
   40       LINK=IABS(NO(JJ))
            SN1=FLOAT(NO(JJ)/LINK)
            SN2=SN1
            IF (ABS(Q(L,IQ)).GT.1.E-7) SN2=SN1*Q(L,IQ)/ABS(Q(L,IQ))
```

```
            IF (ABS(QOLD).GT.1.E-7) SN1=SN1*QOLD/ABS(QOLD)                  FL000062
            LA=0                                                            FL000063
            DO 60 NUM=NUM1,NUM2                                             FL000064
               LA=LA+1                                                      FL000065
               GROLD=GRAD1(ABS(QOLD),D(L,LA),HW(L))*SN1*PSCALE              FL000066
               GRNEW=GRAD1(ABS(Q(L,I3)),D(L,LA),HW(L))*SN2*PSCALE           FL000067
               DEL=GRNEW-GROLD                                              FL000068
               IART=NDVARS+NMSLACK+II                                       FL000069
C                                                                           FL000070
C***** UPDATE COEFFICIENT MATRIX                                            FL000071
C                                                                           FL000072
               DO 50 IR=1,NMROWS                                            FL000073
                  AMAT(IR,NUM)=AMAT(IR,NUM)+DEL*AMAT(IR,IART)               FL000074
   50          CONTINUE                                                     FL000075
   60       CONTINUE                                                        FL000076
   70    CONTINUE                                                           FL000077
   80 CONTINUE                                                              FL000078
      QQ(LEGNO-NPEQ+NLEQ)=QQ(LEGNO-NPEQ+NLEQ)+DQ(LEGNO-NPEQ+NLEQ)           FL000079
   90 CONTINUE                                                              FL000080
      DQ(LEGNO-NPEQ+NLEQ)=0.                                                FL000081
      LEGNO=LEGNO+1                                                         FL000082
      IF (LEGNO.LE.NPEQ) GO TO 10                                           FL000083
      IFLOSEL=0                                                             FL000084
      RETURN                                                                FL000085
C                                                                           FL000086
  100 FORMAT (/35X,23H FLOW DIRECTION OF LINK,I3,13H IN LOAD NO. ,I2,18HFL000087
     1 CHANGED DIRECTION)                                                   FL000088
C                                                                           FL000089
      END                                                                   FL000090
```

```
      SUBROUTINE FLOSEL
      TRACE SUBSCRIPTS
      TRACE STATEMENT NUMBERS
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /EQ/ IHEQ(3),ISEQ(3),ILEQ(3),NQHEQ(3),NQLEQ(3),NQSEQ(3)
      COMMON /PATH2/ PPTR(75),NLOAD(75)
      COMMON /LINK/ AL(45),EXCAVF(45),HW(45),ICLASS(45),LINCOL(45),NDIAM
     1(45),TAB(30,1),IDN(45),IDX(45)
      COMMON /BUF12/ PIZ(125),HF(45,3),X(325)
      COMMON /FLOA/ DQ(45),CD(45),ALFA(43)
      COMMON /MOUT/ MOUT,MIN
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      COMMON /HARDY/ MXHCIT,HDEVMX
      COMMON /PREQ/ NHEQ,NSEQ,NLEQ,NPEQ
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NQ,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /NNORM/ NNORM
      COMMON /OPTION/ IFLODIS,MAXMIN,MCRASH,MINCOST
      DIMENSION Q1(45)
      INTEGER PPTR
      GRAD1(AQ,AD,AH)=10.471*(AQ/AH)**1.852/(AD**4.87)
      IFLOSEL=0
      DO 10 I=1,NLEQ
         DQ(I)=0.
   10 CONTINUE
      DO 100 J=1,NQ
         IF (NQLEQ(J).EQ.0) GO TO 100
         IF (IFLODIS.EQ.2.AND.J.GT.NNORM) GO TO 100
C
C***** STORE INITIAL FLOW DISTRIBUTION FOR LOADING
C
         DO 20 L=1,NS
            Q1(L)=Q(L,J)
   20    CONTINUE
C
C***** PERFORM HARDY-CROSS  NETWORK BALANCE
C
         DO 80 II=1,MXHCIT
            AMAX=-9999.
C
C*****  CALCULATE NEW HEAD LOSSES AND HEAD LOSS/FLOW RATIOS
C
            DO 40 I=1,NS
               HF(I,J)=0.
               DO 30 L=1,NDIAM(I)
                  HF(I,J)=HF(I,J)+GRAD1(ABS(Q1(I)),D(I,L),HW(I))*X(LINCO
     1            L(I)+L-1)
   30          CONTINUE
   40       CONTINUE
            DO 70 M=ILEQ(J),ILEQ(J)+NQLEQ(J)-1
               HDEV=0.
               DER1=0.
               DO 50 L=IABS(PPTR(M))+1,IABS(PPTR(M))+NO(IABS(PPTR(M)))
                  LINK=IABS(NO(L))
                  IF (ABS(Q1(LINK)).LT.1.E-7) GO TO 50
                  SN=FLOAT(NO(L)/LINK)*Q1(LINK)/ABS(Q1(LINK))
                  HDEV=HDEV+SN*HF(LINK,J)
                  DER1=DER1+1.852*HF(LINK,J)/ABS(Q1(LINK))
   50          CONTINUE
               FCHG=-HDEV/DER1
               DQ(M-NHEQ-NSEQ)=DQ(M-NHEQ-NSEQ)+FCHG
```

```
            LOOP=M-NSEQ-NHEQ                                              FL000062
C                                                                         FL000063
C     $     WRITE(MOUT,220)M,LOOP,FCHG,DQ(LOOP),HDEV                      FL000064
C     $220 FORMAT(* EQ. NO.*,I3,* LOOP NO.*,I3,* FCHG=*,F8.2,* CUM=*,F8.FL000065
C     $   1 * HDEV=*,F8.2)                                                FL000066
C***** CHANGE LINK FLOWS                                                  FL000067
C                                                                         FL000068
            DO 60 L=IABS(PPTR(M))+1,IABS(PPTR(M))+NO(IABS(PPTR(M)))      FL000069
               LINK=IABS(NO(L))                                           FL000070
               SN=FLOAT(NO(L)/LINK)                                       FL000071
               QI(LINK)=QI(LINK)+SN*FCHG                                  FL000072
   60       CONTINUE
            IF (ABS(HDEV).GT.AMAX) AMAX=ABS(HDEV)                         FL000074
   70     CONTINUE                                                        FL000075
          III=II                                                          FL000076
          IF (AMAX.LT.HDEVMX) GO TO 90                                    FL000077
   80   CONTINUE                                                          FL000078
   90   WRITE (MOUT,110) J,AMAX,III                                       FL000079
  100 CONTINUE                                                            FL000080
      RETURN                                                              FL000081
C                                                                         FL000082
  110 FORMAT (  2H$$,32H MAXIMUM HEAD DEVIATION FOR LOAD,I3,2H =,F8.4,6HFL000083
     1 WITH ,I3,11H ITERATIONS)                                           FL000084
C                                                                         FL000085
      END                                                                 FL000086
```

```
      SUBROUTINE HCOMP (LOAD)
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /LINK/ AL(45),EXCAVF(45),HW(45),ICLASS(45),LINCOL(45),NDIAM
     1(45),IAB(30,1),ION(45),IOX(45)
      COMMON /EQ/ IHEQ(3),ISEQ(3),ILEQ(3),NQHEQ(3),NQLEQ(3),NQSEQ(3)
      COMMON /PATH1/ NSTART(75),NFINISH(75)
      COMMON /PATH3/ HCOPR(60),ISTOR(60,3),IPN(60,3)
      COMMON /STORE/ STCOST(7),STMAX(7)
      COMMON /NODE1/ PR(28,3),ELV(28)
      COMMON /NODE2/ NPTR(28,3),NREF(28,3),SOURCE(4)
      COMMON /LOADCOL/ LOADCOL(4)
      COMMON /BUF12/ PI2(125),HF(45,3),X(325)
      COMMON /ZLOAD/ ZLOAD(3)
      COMMON /PUMPA/ HPMIN(5),HPMAX(5),HMIN(5,3),HMAX(5,3),LPUMP(5,3),LP
     1UCRIT(5),NOPUMP(3),PML(5),PUCOEF(5),PUMPHR(5,3),PVL(1)
      COMMON /MOUT/ MOUT,MIN
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NQ,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /OPTION/ IFLODIS,MAXWMIN,MCRASH,MINCOST
      COMMON /NRHSCHG/ NRHSCHG
      COMMON /NNORM/ NNORM
      COMMON /ILAX/ ILAX
      INTEGER PPTR,SOURCE
      DIMENSION HC(4), H(28), IHMAX(28), IHMIN(28)
      GRAD1(AQ,AD,AC)=10.471*((AQ/AC)**1.852/(AD)**4.87)
C
C****** COMPUTE LINK HEAD LOSSES
C
      DO 40 I=1,NS
         II=LINCOL(I)-1
         DO 10 J=1,NQ
            HF(I,J)=0.
   10    CONTINUE
         DO 30 J=1,NDIAM(I)
            II=II+1
            DO 20 L=1,NQ
               HH=GRAD1(ABS(Q(I,L)),D(I,J),HW(I))*X(II)
               HF(I,L)=HF(I,L)+HH
   20       CONTINUE
   30    CONTINUE
   40 CONTINUE
C
C***** COMPUTE SOURCE NODE ADJUSTMENTS FOR PUMPS/STORAGE
C
      NRHSCHG=0
      DO 100 J=1,NSOURCE
         DO 50 I=IHEQ(LOAD),IHEQ(LOAD)+NQHEQ(LOAD)-1
            IF (SOURCE(J).EQ.NSTART(I)) GO TO 60
   50    CONTINUE
         GO TO 100
   60    HC(J)=HCORR(I)
C
C***** ELEVATED STORAGE HEAD
C
         IF (ISTOR(I,1).EQ.0) GO TO 80
         DO 70 II=1,3
            IF (ISTOR(I,II).EQ.0) GO TO 70
            SN=1.0
            IF (STCOST(IABS(ISTOR(I,II))).LT.0.) SN=-1.0
            HC(J)=HC(J)+SN*X(IABS(ISTOR(I,II)))/PSCALE
   70    CONTINUE
```

```
   80    IF (NPUMP.EQ.0) GO TO 100
         IF (NOPUMP(LOAD).EQ.0) GO TO 100
C
C***** PUMP HEAD
C
         DO 90 L=1,3
            IF (IABS(IPN(I,L)).EQ.0) GO TO 90
            IF (LPUMP(IABS(IPN(I,L)),LOAD).EQ.0) GO TO 90
            HC(J)=HC(J)+X(LOADCOL(LOAD)+LPUMP(IABS(IPN(I,L)),LOAD)-1)/PS
     1      CALE
   90    CONTINUE
  100 CONTINUE
C
C***** COMPUTE NODAL HEADS FOR LOADING
C
      NLEAVE=0
      DO 140 I=1,NJ
         M=0
         DO 110 J=1,NSOURCE
            IF (NREF(I,LOAD).EQ.SOURCE(J)) M=J
  110    CONTINUE
         H(I)=99999.
         IF (NPTR(I,LOAD).EQ.0.OR.M.EQ.0) GO TO 140
         H(I)=ELV(NREF(I,LOAD))-ELV(I)+HC(M)
         N=IABS(NPTR(I,LOAD))
         L=0
         M=0
         DO 120 K=N+1,N+NO(N)
            IF (NO(K).GT.0) L=1
            IF (NO(K).LT.0) M=1
  120    CONTINUE
         SN=1.0
         DO 130 K=N+1,N+NO(N)
            LINK=IABS(NO(K))
            IF (L+M.EQ.1) SN=FLOAT(NO(K)/LINK)
            IF (ABS(Q(LINK,LOAD)).GT.1.E-2) SN=Q(LINK,LOAD)/ABS(Q(LINK,L
     1      OAD))*SN
            H(I)=H(I)-SN*HF(LINK,LOAD)
  130    CONTINUE
         IF (NPTR(I,LOAD).GT.0.OR.H(I).GE.0.) GO TO 140
         NLEAVE=NLEAVE+1
         IHMIN(NLEAVE)=I
  140 CONTINUE
      WRITE (MOUT,200) LOAD
      WRITE (MOUT,210) ((I,H(I)),I=1,NJ)
      IF (ILAX.EQ.1) GO TO 240
      GO TO 240
      IF (NLEAVE.EQ.0) GO TO 240
      IF (NLEAVE.EQ.1) GO TO 170
C
C***** ORDER VIOLATED NODAL HEADS
C
      DO 160 I=1,NLEAVE-1
         DO 150 J=I+1,NLEAVE
            IF (H(IHMIN(I)).LT.H(IHMIN(J))) GO TO 150
            K=IHMIN(I)
            IHMIN(I)=IHMIN(J)
            IHMIN(J)=K
  150    CONTINUE
  160 CONTINUE
C
C***** ORDER CURRENT HEAD CONSTRAINTS BY DECREASING SLACK
```

```
C
  170 J=0
      DO 180 I=IHEQ(LOAD),IHEQ(LOAD)+NQHEQ(LOAD)-1
         J=J+1
         IHMAX(J)=I
  180 CONTINUE
      IF (NQHEQ(LOAD).LT.2) GO TO 210
      DO 200 I=1,NQHEQ(LOAD)-1
         DO 190 J=I+1,NQHEQ(LOAD)
            IF (H(NFINISH(IHMAX(I))).GT.H(NFINISH(IHMAX(J)))) GO TO 190
            K=IHMAX(I)
            IHMAX(I)=IHMAX(J)
            IHMAX(J)=K
  190    CONTINUE
  200 CONTINUE
C
C***** EXCHANGE VIOLATED HEAD CONTRAINT FOR CONSTRAINT WITH MOST SLACK
C
  210 DO 230 K=1,NLEAVE
         I=IHMIN(K)
         DO 220 J=1,NQHEQ(LOAD)
            IF (IHMAX(J).EQ.0) GO TO 220
            IF (NREF(NFINISH(IHMAX(J)),LOAD).NE.NREF(I,LOAD)) GO TO 220
            HI=ELV(NREF(I,LOAD))-ELV(I)-PR(I,LOAD)
            HJ=ELV(NREF(NFINISH(IHMAX(J)),LOAD))-ELV(NFINISH(IHMAX(J)))-
     1      PR(NFINISH(IHMAX(J)),LOAD)
            IF (HI+HJ.LT.0.) GO TO 220
            CALL TRADE (IHMAX(J),I,LOAD)
            IHMAX(J)=0
            GO TO 230
  220    CONTINUE
  230 CONTINUE
  240 CONTINUE
      RETURN
C
  250 FORMAT (/40X,  2H$$,19H HEADS FOR LOADING ,I2)
  260 FORMAT (   2H$$,5(3H H(,I2,3H)=,F10.2))
C
      END
```

```
      SUBROUTINE LP
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /AMAT/ AMAT(125,275)
      COMMON /EQ/ IHEQ(3),ISEQ(3),ILEQ(3),NQHEQ(3),NQLEQ(3),NQSEQ(3)
      COMMON /BCVEC/ B(125),C(325)
      COMMON /BASIC/ IBV(325),IPIV(125)
      COMMON /BUF12/ PIZ(125),HF(45,3),X(325)
      COMMON /LOADCOL/ LOADCOL(4)
      COMMON /ZLOAD/ ZLOAD(3)
      COMMON /PIPE/ PIPE(45)
      COMMON /PUMPA/ HPMIN(5),HPMAX(5),HMIN(5,3),HMAX(5,3),LPUMP(5,3),LP
     1UCRIT(3),NOPUMP(3),PML(5),PUCOEF(5),PUMPHR(5,3),PVL(1)
      COMMON /MOUT/ MOUT,MIN
      COMMON /FLOW/ ZFLOOP,ITFLOOP,ITFLO
      COMMON /NUMBER/ MXFLOIT,NJ,NJ,NG,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /Z/ Z
      COMMON /MATRIX/ NMROWS,NMCOLS,NMSLACK,NOVARS,NBUROW,MXLPIT
      COMMON /PREQ/ NHEQ,NSEQ,NLEQ,NPEQ
      COMMON /OPTION/ IFLODIS,MAXMIN,MCRASH,MINCOST
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      DIMENSION CBAR(275), IREJ(50)
      INTEGER PIPE
      ILP=0
      NIMBV=0
      Z=0.
      ZNP=0.
      NREJ=0
      NUMI=0
      IPOS=0
      DO 10 J=1,NMCOLS
         X(J)=0.
   10 CONTINUE
      DO 20 I=1,NMROWS
         IPIV(I)=0
   20 CONTINUE
   30 IF (NUMI.GE.MXLPIT) GO TO 160
      NUMI=NUMI+1
      OF=0.
      IF (IPOS.EQ.1) GO TO 50
C
C***** CHECK FOR FEASIBLE SOLUTION
C
      DO 40 I=1,NMROWS
         IB=IBC(I)
         IF (C(IB).GT.1.E9) GO TO 50
         OF=OF+B(I)*C(IB)
   40 CONTINUE
      IPOS=IPOS+1
      WRITE (MOUT,270) NUMI,OF
   50 AMIN=1.E15
      NBV=0
      DO 70 J=1,NMCOLS
         CBAR(J)=0.0
         IF (IBV(J).NE.0) GO TO 70
         DO 60 I=1,NMROWS
            IB=IBC(I)
            CBAR(J)=CBAR(J)+C(IB)*AMAT(I,J)
   60    CONTINUE
C
C***** FIND BASIC VARIABLE TO ENTER
```

```
C
         IF (C(J).GT.1.E9.AND.J.GT.NOVARS+NMSLACK+NPEQ) GO TO 70
C
C***** COMPUTE REDUCED COSTS
C -      -
         CBAR(J)=C(J)-CBAR(J)
         IF (CBAR(J).GT.AMIN) GO TO 70
         AMIN=CBAR(J)
         NBV=J
   70 CONTINUE
C
C***** CHECK FOR OPTIMALITY
C
      IF (AMIN.GT.-1.E-03) GO TO 160
      AMIN=10.E+15
C
C***** FIND BASIC VARIABLE TO LEAVE
C
      DO 80 I=1,NMROWS
         IF (AMAT(I,NBV).LE.0.) GO TO 80
         IF ((B(I)/AMAT(I,NBV)).GT.AMIN) GO TO 80
         AMIN=B(I)/AMAT(I,NBV)
         IROW=I
   80 CONTINUE
C
C***** CHECK FOR UNROUNDED SOLUTION
C
      IF (AMIN.GT.1.E14) GO TO 260
C
C***** CHECK FOR PIVOT LEVEL TOLERANCE
C
      IF (AMAT(IROW,NBV).GT.1.E-6) GO TO 90
      NREJ=NREJ+1
      IBV(NBV)=-1
      IREJ(NREJ)=NBV
      WRITE (MOUT,280) IROW,NBV,AMAT(IROW,NBV)
      GO TO 30
   90 IBV(IBC(IROW))=0
      PIV=AMAT(IROW,NBV)
      IF (NREJ.EQ.0) GO TO 110
      DO 100 J=1,NREJ
         IBV(IREJ(J))=0
  100 CONTINUE
      NREJ=0
  110 CONTINUE
C
C     $    WRITE(MOUT,205)IROW,IBC(IROW),NBV,PIV
C     $205 FORMAT(* ROW*,I3,* LEAVING VAR *,I3,* ENTERING VAR*,I3,
C     $   1 * PIV=*,G12.3)
C
      IBC(IROW)=NBV
      IBV(NBV)=IROW
      DO 130 I=1,NMROWS
         IF (I.EQ.IROW) GO TO 130
         B(I)=B(I)-B(IROW)*AMAT(I,NBV)/PIV
         DO 120 J=1,NMCOLS
            IF (J.EQ.NBV) GO TO 120
            AMAT(I,J)=AMAT(I,J)-AMAT(IROW,J)*AMAT(I,NBV)/PIV
  120    CONTINUE
  130 CONTINUE
      DO 140 I=1,NMROWS
         IF (I.EQ.IROW) GO TO 140
```

```
          AMAT(I,NBV)=0.0
  140 CONTINUE
      DO 150 J=1,NMCOLS
  150 AMAT(IROW,J)=AMAT(IROW,J)/PIV
      B(IROW)=B(IROW)/PIV
      GO TO 70
C
C*****END OF LINEAR PROGRAM
C*****CHECK FEASIBILITY OF THE SOLUTION
C
  160 IF (NUMI.GE.MXLPIT) WRITE (MOUT,360)
      DO 240 I=1,NMROWS
         K=IBC(I)
         Z=Z+B(I)*C(K)
         IF (ABS(C(K)).GT.1.E+8) GO TO 170
         IF (MINCOST.EQ.1.AND.C(K).GT.0.) ZLOAD(I)=B(I)
         IF (MAXWMIN.EQ.1.AND.C(K).LT.0.) ZLOAD(K-NDVARS+NQ)=B(I)/PSCALE
         ZNP=ZNP+B(I)*C(K)
         GO TO 240
C
C***** FIND LOADING ASSOCIATED WITH $$INFEASIBILITY
C
  170    IF (K.LE.NDVARS+NMSLACK+NPEQ) GO TO 180
         IF (K.GT.NMCOLS) GO TO 230
         LINK=K-NDVARS-NMSLACK-NPEQ
         WRITE (MOUT,290) K,B(I),PIPE(LINK)
         IFLOSEL=1
         GO TO 240
  180    DO 200 L=1,NQ
            IF (NQSEQ(L).EQ.0) GO TO 190
            IF (K.GE.LOADCOL(L)+NQPUMP(L).AND.K.LE.LOADCOL(L)+NQPUMP(L)+
     1      NQSEQ(L)-1) GO TO 210
            IF (K.GE.NDVARS+NMSLACK+ISEQ(L).AND.K.LE.NDVARS+NMSLACK+ISEQ
     1      (L)+NQSEQ(L)-1) GO TO 210
  190       IF (NQLEQ(L).EQ.0) GO TO 200
            IF (K.GE.LOADCOL(L)-NQPUMP(L)+NQSEQ(L).AND.K.LT.LOADCOL(L+1)
     1      ) GO TO 220
            IF (K.GE.NDVARS+NMSLACK+ILEQ(L).AND.K.LE.NDVARS+NMSLACK+ILEQ
     1      (L)+NQLEQ(L)-1) GO TO 220
  200    CONTINUE
  210    LSOURCE=K-LOADCOL(L)-NQPUMP(L)+1
         IF (K.GT.NDVARS) LSOURCE=K-NDVARS-NMSLACK-ISEQ(L)+1
         WRITE (MOUT,300) K,B(I),LSOURCE,L
         GO TO 240
  220    LOOP=K-LOADCOL(L)-NQPUMP(L)-NQSEQ(L)+1
         IF (K.GT.NDVARS) LOOP=K-NDVARS-NMSLACK-ILEQ(L)+1
         WRITE (MOUT,310) K,B(I),LOOP,L
         GO TO 240
  230    NIMBV=NIMBV+1
         ILP=2
         WRITE (MOUT,320) I,K
  240 X(K)=B(I)
      WRITE (MOUT,340) ((J,ZLOAD(J)),J=1,NQ)
      WRITE (MOUT,330) NUMI,ITFLO,Z,ZNP
      WRITE (MOUT,350) NIMBV
      DO 250 I=1,NMROWS
         J=NDVARS+NMSLACK+I
         PIZ(I)=C(J)-CBAR(J)
         IF (C(J).GT.1.E9.AND.J.GT.NDVARS+NMSLACK+NPEQ) PIZ(I)=CBAR(J)
         PIZ(I)=PSCALE*PIZ(I)
  250 CONTINUE
      RETURN
```

```
  260 WRITE (MOUT,370) NBV
      ILP=1
      RETURN
C
  270 FORMAT (///2X,32HFEASIBLE SOLUTION REACHED AFTER ,I5,17H MINOR ITE
     1RATIONS,3X,6H COST=,F12.2)
  280 FORMAT (25X,29H PIVOT ELEMENT REJECTED AMAT(,I3,1H,,I3,2H)=,G12.3)
  290 FORMAT (19H $$INFEASIBILITY X(,I3,2H)=,F10.2,6H LINK ,I3)
  300 FORMAT (21H $$$$INFEASIBILITY X(,I3,2H)=,F10.2,8H SOURCE ,I3,5H LO
     1AD,5H NO. ,I3)
  310 FORMAT (19H $$INFEASIBILITY X(,I3,2H)=,F10.2,6H LOOP ,I3,5H LOAD,5
     1H NO. ,I3)
  320 FORMAT (5X,11H$$CONS. NO.,I5,5X,17H  ART.  VAR.  NO.,I5)
  330 FORMAT (/,25H  SOLUTION REACHED AFTER ,I4,17H MINOR ITERATIONS,/,4
     1H $$ ,I3,12H (W/PENALTY),G12.4,14H (W/O PENALTY),G12.4)
  340 FORMAT (3H $$,5(3H Z(,I1,2H)=,F10.2))
  350 FORMAT (42H NO. OF IMAGINARY BASIC VARIABLES AFTER LP,I3)
  360 FORMAT (25X,66H     COMPUTATION STOPPED AS A RESULT OF TOO MANY
     1    ITERATIONS )
  370 FORMAT (//5X,26H SOLUTION IS UN LIMITED  VK=,I5)
C
      END
```

```
      SUBROUTINE LPFORM
      TRACE STATEMENT NUMBERS
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /AMAT/ AMAT(110,275)
      COMMON /BCVEC/ B(125),C(325)
      COMMON /BASIC/ IBV(325),IPIV(125)
      COMMON /RHSCHG/ HEQNO(10),DELRHS(10)
      COMMON /FLOW/ LFLOOP,ITFLOOP,ITFLO
      COMMON /MATRIX/ NMROWS,NMCOLS,NMSLACK,NOVARS,NBUROW,MXLPIT
      COMMON /MOUT/ MOUT,MIN
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      COMMON /NRHSCHG/ NRHSCHG
      COMMON /NTIME/ NDIACHG,NPUMCHK,NFLOCHG,NROWPIV
      INTEGER HEQNO
      NROWPIV=0
      ILPFORM=0
C
C*****RE-ORGANIZE THE MATRIX
C***** PIVOT TO PUT MATRIX IN STANDARD FORM
C
      DO 90 L=1,NMROWS
         IF (IPIV(L).EQ.0) GO TO 90
         IROW=L
         NROWPIV=NROWPIV+1
         ICOL=IBC(IROW)
         IF (IROW.EQ.NMROWS) GO TO 30
         IR=IROW
         AMAX=ABS(AMAT(IROW,ICOL))
         DO 10 LLL=IROW+1,NMROWS
            IF (ABS(AMAT(LLL,ICOL)).LE.AMAX) GO TO 10
            AMAX=ABS(AMAT(LLL,ICOL))
            IR=LLL
   10    CONTINUE
         IF (IR.EQ.IROW) GO TO 30
         DO 20 J=1,NMCOLS
            AM=AMAT(IROW,J)
            AMAT(IROW,J)=AMAT(IR,J)
            AMAT(IR,J)=AM
   20    CONTINUE
         BM=B(IROW)
         B(IROW)=B(IR)
         B(IR)=BM
   30    PIV=AMAT(IROW,ICOL)
C
C     $     WRITE(MOUT,100)IROW,ICOL,PIV
C     $     WRITE(MOUT,101)IR,AMAX
C     $101 FORMAT(' IR=',I3,' AMAX=',F8.4)
C     $100 FORMAT(/,' ROW ',I5,' COLUMN ',I5,' PIVOT= ',F10.5)
C
         IF (ABS(PIV).GT.1.E-6) GO TO 40
C
C***** CHECK FOR ZERO PIVOT TOLERANCE
C
         WRITE (MOUT,100) IROW,ICOL,PIV
         ILPFORM=1
         RETURN
C
C***** UPDATE RHS
C
   40    DO 60 I=1,NMROWS
            IF (I.EQ.IROW) GO TO 60
```

```
            B(I)=B(I)-B(IROW)*AMAT(I,ICOL)/PIV
C
C***** PERFORM ROW PIVOTS
C
            DO 50 J=1,NMCOLS
               IF (J.EQ.ICOL) GO TO 50
               AMAT(I,J)=AMAT(I,J)-AMAT(IROW,J)*AMAT(I,ICOL)/PIV
   50       CONTINUE
   60    CONTINUE
C
C***** ZERO ALL OTHER ELEMENTS IN UPDATED COLUMN
C
         DO 70 I=1,NMROWS
            IF (I.EQ.IROW) GO TO 70
            AMAT(I,ICOL)=0.0
   70    CONTINUE
C
C***** DIVIDE BASIC ROW BY PIVOT ELEMENT
C
         DO 80 J=1,NMCOLS
            AMAT(IROW,J)=AMAT(IROW,J)/PIV
   80    CONTINUE
         B(IROW)=B(IROW)/PIV
   90 CONTINUE
C
C***** CHANGE RHS FOR NEW PRESSURE CONSTRAINT
C
      IF (NRHSCHG.EQ.0) GO TO 120
      DO 110 J=1,NRHSCHG
         IART=NOVARS+NMSLACK+HEQNO(J)
         DO 100 I=1,NMROWS
            B(I)=B(I)+AMAT(I,IART)*DELRHS(J)
  100    CONTINUE
  110 CONTINUE
      NRHSCHG=0
  120 NIMBV=0
C
C     $    WRITE(MOUT,112)((I,B(I)),I=1,NMROWS)
C     $112 FORMAT(5(* B(*,I3,*)=*,G10.3))
C
      DO 140 I=1,NMROWS
         IF (B(I).GE.0.0) GO TO 140
         B(I)=-B(I)
         DO 130 J=1,NMCOLS
            AMAT(I,J)=-AMAT(I,J)
  130    CONTINUE
         NIMBV=NIMBV+1
         N=NMCOLS+NIMBV
C
C     $    WRITE(MOUT,111),I,IBC(I),N
C     $111 FORMAT(* ROW*,I3,* LEAVING VAR*,I3,* ENTERING VAR*,I3)
C
         IF (IBV(IBC(I)).GT.0) IBV(IBC(I))=0
         IBC(I)=NMCOLS+NIMBV
         IBV(NMCOLS+NIMBV)=I
         C(NMCOLS+NIMBV)=1.E10
  140 CONTINUE
      WRITE (MOUT,160) NIMBV
      RETURN
C
  150 FORMAT (25H NO PIVOT ELEMENT IN ROW ,I3,/,23H INTENDED PIVOT IN COL
     1L,I3,5H = ,G12.3)
```

```
 160 FORMAT (45H NO. OF IMAGINARY BASIC VARIABLES AFTER LPFORM,I3)      LPF00124
C                                                                        LPF00125
      END                                                                LPF00126
```

```
      SUBROUTINE MATGEN
      TRACE SUBSCRIPTS
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /AMAT/ AMAT(110,275)
      COMMON /BCVEC/ B(125),C(325)
      COMMON /INTER/ EQPTR(75),LCOM(325)
      COMMON /EQ/ IHEQ(3),ISEQ(3),ILEQ(3),NOHEQ(3),NOLEQ(3),NOSEQ(3)
      COMMON /PATH1/ NSTART(75),NFINISH(75)
      COMMON /PATH2/ PPTR(75),NLOAD(75)
      COMMON /PATH3/ HCOBR(60),ISTOR(60,3),IPN(60,3)
      COMMON /NODE1/ PR(28,3),ELV(28)
      COMMON /NODE2/ NPTR(28,3),NREF(28,3),SOURCE(4)
      COMMON /LINK/ AL(45),EXCAVF(45),HW(45),ICLASS(45),LINCOL(45),NDIAM
     1(45),TAR(20,1),IDN(45),IDX(45)
      COMMON /MIND/ MIND(45)
      COMMON /MAXD/ MAXD(45)
      COMMON /PIPE/ PIPE(45)
      COMMON /STORE/ STCOST(7),STMAX(7)
      COMMON /LOADCOL/ LOADCOL(4)
      COMMON /PUMPA/ HPMIN(5),HPMAX(5),HMIN(5,3),HMAX(5,3),LPUMP(5,3),LP
     1CRIT(5),NOPUMP(3),PML(5),PUCOEF(5),PUMPHR(5,3),PVL(1)
      COMMON /PPUMP/ PPUMP(5)
      COMMON /QPUMP/ QPUMP(5,3)
      COMMON /PUMPF/ PUMPF(5)
      COMMON /GRAD/ INTER,ICG,IBFGS,GZMCOST,GZMPER,ALPHA,IALP,ICRIT
      COMMON /PREQ/ NHEQ,NSEQ,NLEQ,NPEQ
      COMMON /PUMPV/ PUMPEFF,POWCOST,PUMPM,PCOIFF,WATDEN,PUMACRF,TIPCOST
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NQ,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /DIAMV/ NPDIAM,DPSPACE,IDMIN,IDMAX
      COMMON /OPTION/ IFLODIS,MAXWMIN,MCRASH,MINCOST
      COMMON /MOUT/ MOUT,MIN
      COMMON /IMATGEN/ IMATGEN
      COMMON /MATRIX/ NMROWS,NMCOLS,NMSLACK,NOVARS,NBUROW,MXLPIT
      COMMON /PRICE/ PIPACRF,PIPEM,STOACRF
      COMMON /HARDY/ MXHCIT,HDEVMX
      COMMON /IEX/ IEX
      COMMON /ILAX/ ILAX
      COMMON /QRATIO/ QRATIO
      COMMON /NNORM/ NNORM
      COMMON /IPUMP/ IPUMP
      DIMENSION LOMVPTR(5), A(1,1), WL(3), LREF(75), HSTART(5), PCON(5,3
     1), LPCON(5,3)
      INTEGER PML,PVL,PPTR,SOURCE,EQPTR,PIPE,PCON
      REAL IRATE,LIMBAL
C
C*****FUNCTION FOR GRADIENT COMPUTATION
C
      GRAD1(AQ,AJ,AD)=10.471*((AQ/AC)**1.852)/((AD)**4.87)
C
C***** CAPITAL PUMP COST FUNCTION
C
      PUMCOST(AGP,AH)=15.14*(AGP**.453)*(AH**.642)
C
C*****INITIAL VALUE FOR PENALTY FACTOR
C
      IMATGEN=0
      PENFAC=1.010
      WATDEN=1.94
      NMSLACK=0
      NRELAX=0
```

```
      TIPCOST=0.
C
C***** READ AND WRITE PROBLEM PROBLEM TITLE
C
      READ (MIN,960) (C(I),I=1,40)
      WRITE (MOUT,970) (C(I),I=1,40)
C
C***** READ AND WRITE INITIAL DATA
C
      READ (MIN,980) MINCOST,MAXWMIN,IEX,IPUMP
      READ (MIN,980) MCRASH,IMAT,IFLODIS,ILAX
      READ (MIN,980) INTER,ICG,IBFGS,IALP,ICRIT
      IF (MINCOST.EQ.1) WRITE (MOUT,1060)
      IF (MAXWMIN.EQ.1) WRITE (MOUT,990)
      IF (ICG.EQ.1) WRITE (MOUT,1000)
      IF (ICG.EQ.0.AND.IBFGS.EQ.1) WRITE (MOUT,1010)
      IF (IBFGS.EQ.1) WRITE (MOUT,1020)
      IF (INTER.EQ.0) WRITE (MOUT,1030)
      IF (INTER.EQ.1) WRITE (MOUT,1040)
      IF (IALP.EQ.1) WRITE (MOUT,1050)
      READ (MIN,980) NS,NJ,IDMIN,IDMAX,NEXCAV,NG,NEMERG,NPUMP,NVL,NST,NC
     1LASS,NSOURCE
      READ (MIN,1070) NPDIAM,DPSPACE
      DO 10 I=1,NS
         EXCAVF(I)=0.
   10 CONTINUE
      DO 20 I=1,NJ
      DO 20 J=1,NQ
         NREF(I,J)=0
         NOPUMP(J)=0
         NPTR(I,J)=0
   20 CONTINUE
      DO 30 J=1,275
         C(J)=0.
      DO 30 I=1,110
         AMAT(I,J)=0.
         IPC(I)=0
   30 CONTINUE
      READ (MIN,1090) BMAX,IRATE,NYPIPE,SVPIPE,PIPEM
      NNORM=NG-NEMERG
      IF (MINCOST.EQ.1) GO TO 40
      READ (MIN,1400) (WL(I),I=NNORM+1,NG)
      WRITE (MOUT,1040) ((I,WL(I)),I=NNORM+1,NQ)
   40 READ (MIN,107 ) MXHCIT,HDEVMX,LIMBAL,SIMBAL
      IF (NCLASS.EQ. ) NCLASS=1
      WRITE (MOUT,1100) NS,NJ,IDMAX,IDMIN,NG,NEMERG,NNORM,NSOURCE,NEXCAV
     1,NPUMP,NVL,NST
      WRITE (MOUT,1120) BMAX,IRATE,NYPIPE,SVPIPE,PIPEM
      IF (NPUMP.EQ.0) GO TO 80
C
C*****INPUT/OUTPUT PUMP DATA
C
      READ (MIN,1070) NYPUMP,SVPUMP,PUMPEFF,POWCOST,PUMPM,PCDIFF
      WRITE (MOUT,1130)
      WRITE (MOUT,1140) NYPUMP,SVPUMP,PUMPEFF,POWCOST,PUMPM,PCDIFF
C
C***** READ IN DATA FOR EACH PUMP
C
      WRITE (MOUT,1170)
      DO 50 I=1,NPUMP
         READ (MIN,1150) K,PML(K),HPMIN(K),HPMAX(K),LPUCRIT(K),PPUMP(K),
     1   HSTART(K),PUMPF(K)
```

```
      IF (NQ.GT.1) READ (MIN,980) ((PCON(I,J),LPCON(I,J)),J=1,NQ)
      IF (PPUMP(I).LT.1.0) PPUMP(I)=1.
      READ (MIN,1160) ((LPUMP(I,J),QPUMP(I,J)),J=1,NQ)
      READ (MIN,1250) (PUMPHR(I,J),J=1,NQ)
      WRITE (MOUT,1180) K,PML(K),HPMIN(K),HPMAX(K),LPUCRIT(K),HSTART(
     1 K)
   50 CONTINUE
C
C***** COMPUTE LOADING CONDITONS DATA
C
      DO 70 J=1,NQ
         DO 60 I=1,NPUMP
            WRITE (MOUT,1200) J,I,LPUMP(I,J),QPUMP(I,J),PUMPHR(I,J)
            IF (LPUMP(I,J).EQ.0) GO TO 60
            IF (PPUMP(I).GT.1.) WRITE (MOUT,1190) PPUMP(I)
            NQPUMP(J)=NQPUMP(J)+1
   60    CONTINUE
   70 CONTINUE
      PUMACRF=((IRATE*(1.+IRATE)**NYPUMP)/((1.+IRATE)**NYPUMP-1.))*(1.-S
     1VPUMP)+IRATE*SVPUMP
C
C*****INITIAL DATA FOR LOOPED NETWORK
C
   80 READ (MIN,1210) PSCALE,ALPHA,DQMAX,QRATIO,GZMCOST,GZMPER,MXFLOIT,M
     1XLPIT
      WRITE (MOUT,1220) MXFLOIT,MXLPIT
      WRITE (MOUT,1230) ALPHA,DQMAX,QRATIO
C
C***** ADDITIONAL EXCAVATION COST
C
      IF (NEXCAV.EQ.0) GO TO 100
      DO 90 KL=1,NEXCAV
   90 READ (MIN,1240) LL,EXCAVF(LL)
C
C***** VALVE LOCATIONS
C
  100 IF (NVL.GT.0) READ (MIN,980) (PVL(I),I=1,NVL)
C
C***** ANNUAL CAPITAL RECOVERY FACTOR COMPUTATION
C
      STOACRF=(IRATE*(1.+IRATE)**NYPIPE)/((1.+IRATE)**NYPIPE-1.)
      PIPACRF=STOACRF*(1.-SVPIPE)+IRATE*SVPIPE
      IF (NST.EQ.0) GO TO 120
C
C*****COST FOR ADDITIONAL STORAGE ELEVATION
C
      READ (MIN,1250) ((STCOST(I),STMAX(I)),I=1,NST)
      WRITE (MOUT,1260)
      DO 110 I=1,NST
  110 WRITE (MOUT,1280) I,STCOST(I),STMAX(I)
  120 CONTINUE
      WRITE (MOUT,1270)
C
C***** PIPE COST (BY CLASS)
C
      DO 130 I=1,IDMAX
      DO 130 J=1,NCLASS
         TAB(I,J)=1.01*(I**1.29)
         WRITE (MOUT,1240) I,TAB(I,J)
  130 CONTINUE
      WRITE (MOUT,1290)
C
```

```
C*****NODE DATA
C
      READ (MIN,980) (SOURCE(I),I=1,NSOURCE)
      DO 140 I=1,NJ
         READ (MIN,1240) K,ELV(K),(Q(K,L),L=1,NQ)
         READ (MIN,1300) (PR(K,L),L=1,NQ)
         WRITE (MOUT,1310) K,ELV(K),(PR(K,L),L=1,NQ)
  140 CONTINUE
      WRITE (MOUT,1330)
      DO 150 I=1,NJ
         WRITE (MOUT,1340) I,(Q(I,L),L=1,NQ)
  150 CONTINUE
      IF (IFLODIS.NE.1) GO TO 200
C
C***** CALCULATE INITIAL TREE FLOW DISTRIBUTION
C
      DO 160 I=1,NJ-1
      DO 160 J=1,NJ-1
         A(I,J)=0.
  160 CONTINUE
      N=NJ-1
      K=0
C
C***** INDEX RHS AND SOLUTION VECTOR
C
      DO 170 I=1,NS
         K=K+1
         IDN(K)=I
  170 CONTINUE
C
C***** READ IN COEFFICIENT MATRIX
C
      READ (MIN,1350) ((I,A(I,J),L,A(L,J)),J=1,NJ-1)
      DO 180 I=1,NJ-1
         WRITE (MOUT,1350) ((I,J,A(I,J)),J=1,NJ-1)
  180 CONTINUE
C
C***** CONVERT BACK TO PRIMARY LINK NO.
C
      DO 190 K=1,NJ-1
      DO 190 J=1,NQ
         Q(IDN(K),J)=Q(K,J)
         Q(NJ,J)=0.
  190 CONTINUE
  200 WRITE (MOUT,1120)
      DO 210 II=1,NS
         NDIAM(II)=NPDIAM
         I=II
C
C*****SECTION DATA
C
         READ (MIN,1340) PIPE(I),AL(I),HW(I),IDN(I),IDX(I),ICLASS(I)
         IF (IFLODIS.NE.1) READ (MIN,1300) (Q(I,L),L=1,NQ)
         IF (ICLASS(I).EQ.0) ICLASS(I)=1
  210 CONTINUE
      IF (MCRASH.EQ.0) GO TO 230
      DO 220 I=1,NS
         READ (9,1370) (Q(I,L),L=1,NQ)
  220 CONTINUE
      READ (9,1100) ((IDN(L),IDX(L),MIND(L),MAXD(L)),L=1,NS)
  230 DO 240 I=1,NS
         IF (MCRASH.EQ.0) MIND(I)=IDMIN
```

```
      IF (MCRASH.EQ.0) MAXD(I)=IDMAX
      IF (IDN(I).LT.0) MIND(I)=IABS(IDN(I))
      IF (IDX(I).LT.0) MAXD(I)=IABS(IDX(I))
      IDN(I)=IABS(IDN(I))
      IDX(I)=IABS(IDX(I))
      ND=0
C
C*****SELECTION OF ADMISSIBLE DIAMETERS FOR EACH PIPE
C
      IF (IDX(I).EQ.0) IDX(I)=IDMAX
C
C***** FIXED DIAMETER ON LINK I
C
      IF (IDX(I).NE.IDN(I)) GO TO 240
      ND=1
      NDIAM(I)=1
      D(I,1)=IDN(I)
      GO TO 260
  240 CONTINUE
      DO 250 J=1,NPDIAM
         D(I,J)=FLOAT(IDN(I))+DPSPACE*(FLOAT(J)-1.0)
  250 CONTINUE
      IDX(I)=INT(D(I,NDIAM(I)))
  260 CONTINUE
      DO 270 I=1,NS
      LPEF(PIPE(I))=I
  270 CONTINUE
C
C***** WRITE SECTION DATA AND SELECTED DIAMETERS FOR EACH PIPE
C
      DO 280 I=1,NS
      ND=NDIAM(I)
      WRITE (MOUT,1390) I,PIPE(I),AL(I),HW(I),IDN(I),IDX(I),ICLASS(I)
     1 ,(D(I,J),J=1,ND)
  280 CONTINUE
      WRITE (MOUT,1400)
      DO 290 I=1,NS
      WRITE (MOUT,1410) I,PIPE(I),(Q(I,L),L=1,NQ)
  290 CONTINUE
C
C***** TYPES OF PRESSURE CONSTRAINTS
C
      NPEQ=0
      NSEQ=0
      NLEQ=0
      NHEQ=0
      DO 300 L=1,NQ
      READ (MIN,480) NQHEQ(L),NQSEQ(L),NQLEQ(L)
      NHEQ=NHEQ+NQHEQ(L)
      IF (L.LE.NNORM) NNHEQ=NHEQ
      NSEQ=NSEQ+NQSEQ(L)
      NLEQ=NLEQ+NQLEQ(L)
      LOADCOL(1)=NST+1
      NCOL=NQPUMP(L)+NVL+NQSEQ(L)+NQLEQ(L)
      LOADCOL(L+1)=LOADCOL(L)+NCOL
      LOMVPTR(L)=LOADCOL(L)+NQPUMP(L)+NVL
C
C     $    WRITE(MOUT,825)L,LOADCOL(L)
C     $825 FORMAT(/* LOADCOL(*,I2,*)=*,I2)
C
  300 CONTINUE
      NPEQ=NHEQ+NSEQ+NLEQ
```

```
      DO 310 I=1,NHEQ+NSEQ
      DO 310 K=1,3
         IPN(I,K)=0
         ISTOR(I,K)=0
  310 CONTINUE
      IHEQ(1)=1
      IF (NQ.EQ.1) GO TO 330
      DO 320 L=2,NQ
         IHEQ(L)=IHEQ(L-1)+NQHEQ(L-1)
  320 CONTINUE
  330 IF (NSEQ.EQ.0) GO TO 350
      ISEQ(1)=NHEQ+1
      IF (NQ.EQ.1) GO TO 350
      DO 340 L=2,NQ
         ISEQ(L)=ISEQ(L-1)+NQSEQ(L-1)
  340 CONTINUE
  350 IF (NLEQ.EQ.0) GO TO 380
      ILEQ(1)=NHEQ+NSEQ+1
      IF (NQ.EQ.1) GO TO 380
      DO 360 L=2,NQ
         ILEQ(L)=ILEQ(L-1)+NQLEQ(L-1)
  360 CONTINUE
      DO 370 L=1,NQ
         IF (NQHEQ(L).EQ.0) IHEQ(L)=0
         IF (NQSEQ(L).EQ.0) ISEQ(L)=0
         IF (NQLEQ(L).EQ.0) ILEQ(L)=0
  370 CONTINUE
  380 WRITE (MOUT,990) NPEQ,NHEQ,NSEQ,NLEQ
C
C*****COMPUTE SIZE OF THE COEFFICIENT MATRIX
C
      LINCOL(1)=LOADCOL(NQ+1)
      DO 390 I=1,NS
         LINCOL(I+1)=LINCOL(I)+NDIAM(I)
  390 CONTINUE
      NDVARS=LINCOL(NS)+NDIAM(NS)
      IF (MAXWMIN.EQ.1) NDVARS=NDVARS+NEMERG
      NBUROW=NPEQ+1+NQ
      IF (NST.GT.0) NSTROW=NBUROW+1
      IF (NPUMP.EQ.0) GO TO 430
C
C***** COMPUTE MIN AND MAX HEAD AND INITIAL PUMP COSTS
C
      DO 410 I=1,NPUMP
         PML(I)=LREF(PML(I))
         DO 400 J=1,NQ
            HMIN(I,J)=0.
            HMAX(I,J)=9999.
            HMIN(I,J)=550.*HPMIN(I)*PUMPF(I)*PPUMP(I)/(WATDEN*Q(PML(I),J
     I      )+QPUMP(I,J))
            IF (HPMAX(I).GT.9000.) GO TO 400
            HMAX(I,J)=550.*HPMAX(I)*PUMPF(I)*PPUMP(I)/(WATDEN*Q(PML(I),J
     I      )+QPUMP(I,J))
  400    CONTINUE
         IF (HPMIN(I).LT.1.) GO TO 410
         K=LPUCRIT(I)
         PUICOST=PUMCOST(Q(PML(I),K)+QPUMP(I,K)/PPUMP(I),HMIN(I,K))*PUMA
     I   CRF*PPUMP(I)
         PMCOST=PPUMP(I)*PUMPM*HPMIN(I)*QPUMP(I,K)
         ECOST=PPUMP(I)*.746*HPMIN(I)*POWCOST*PUMPHR(I,K)*QPUMP(I,K)
         WRITE (MOUT,1430) I,K,PUICOST,ECOST,PMCOST
         TIPCOST=TIPCOST+PUICOST+PMCOST+ECOST
```

```
  410 CONTINUE
C
C***** COMPUTE PUMPCOST COEFFICIENTS FOR BUDGET ROW
C
      DO 420 I=1,NPUMP
         J=LPUCRIT(I)
         IF (LPUMP(I,J).EQ.0) GO TO 420
         KK=LOADCOL(J)+LPUMP(I,J)-1
C
C***** COMPUTE CAPITAL,MAINTENANCE,AND ENERGY COEFFICIENTS
C
         PUCOEF(I)=PPUMP(I)*PUMCOST(Q(PML(I),J)*QPUMP(I,J),HSTART(I))*PU
     1   MACPF/HSTART(I)
         IF (MCRASH.EQ.1) READ (R,1370) PUCOEF(I)
         PMCOST=PPUMP(I)*PUMPM*WATDEN*Q(PML(I),J)*QPUMP(I,J)/(550.*PUMPF
     1   (I))
         HP=PMCOST/PUMPM
         ECOST=.746*POWCOST*PUMPHR(I,J)*HP
         AMAT(NBURO*,KK)=PUCOEF(I)+PMCOST+ECOST
         WRITE (MOUT,1440) I,PUCOEF(I),PMCOST,ECOST
  420 CONTINUE
  430 WRITE (MOUT,1450)
      IF (NHED.GT.0) WRITE (MOUT,1490)
      I=0
      LPTR=1
      IDUP=1
      NO(1)=1
  440 IF (IDUP.EQ.0) LPTR=LPTR+NO(LPTR)+1
      READ (MIN,980) ITYP,IDUP,NSTAR,NFINIS,NLOA,IPM,ISS
      IF (ITYP.EQ.99999) GO TO 660
      IF (IABS(ITYP).EQ.1) NREF(NFINIS,NLOA)=NSTAR
      IF (IDUP.NE.0) GO TO 480
C
C***** NON-DUPLICATEHEAD PATH CONSTRAINT
C
      READ (MIN,980) NO(LPTR)
      N=LPTR+NO(LPTR)
      READ (MIN,980) (NO(K),K=LPTR+1,N)
      DO 450 K=LPTR+1,N
         NO(K)=LREF(IABS(NO(K)))*NO(K)/IABS(NO(K))
  450 CONTINUE
      IF (IABS(ITYP).EQ.1) NPTR(NFINIS,NLOA)=ITYP*LPTR
      IF (IABS(ITYP).NE.1) GO TO 470
      NPTR(NFINIS,NLOA)=ITYP*LPTR
      IF (ITYP.LT.0.AND.IPM.GT.0) READ (MIN,980)
      IF (ITYP.LT.0.AND.ISS.GT.0) READ (MIN,980)
      IF (MAXMIN.EQ.1.AND.NLOA.GT.NNORM) GO TO 470
      HEIGHT=ELV(NSTAR)-ELV(NFINIS)-PR(NFINIS,NLOA)
      IF (HEIGHT.GE.0.) GO TO 470
      DO 460 K=LPTR+1,N
         NO(K)=-NO(K)
  460 CONTINUE
  470 IF (ITYP.EQ.-1) GO TO 440
      N1=LPTR+1
      N2=LPTR+NO(LPTR)
      GO TO 490
C
C***** REFERENCE SAME HEAD PATH AS IN LOADING IDUP
C
  480 IF (IABS(ITYP).NE.1) GO TO 490
      NPTR(NFINIS,NLOA)=ITYP*IABS(NPTR(NFINIS,IDUP))
      IF (ITYP.LT.0.AND.IPM.GT.0) READ (MIN,980)
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```
      IF (ITYP.LT.0.AND.ISS.GT.0) READ (MIN,980)
      IF (ITYP.EQ.-1) GO TO 440
      N1=IABS(NPTR(NFINIS,NLOA))+1
      N2=N1+NO(IABS(NPTR(NFINIS,NLOA)))-1
C
C***** ENFORCED PATH CONSTRAINTS
C
  490 I=I+1
      IF (IDUP.EQ.0) PPTR(I)=LPTR*ITYP/IABS(ITYP)
      IF (IDUP.EQ.0) GO TO 500
      IF (ITYP.EQ.1) PPTR(I)=NPTR(NFINIS,NLOA)
      IF (ITYP.NE.1) PPTR(I)=IABS(PPTR(NHEQ+(IABS(ITYP)-2)*NSEQ+IDUP))*I
     1TYP/IABS(ITYP)
      N1=IABS(PPTR(I))+1
      N2=IABS(PPTR(I))+NO(IABS(PPTR(I)))
  500 IF (IABS(ITYP).EQ.3) GO TO 510
      NSTART(I)=NSTAR
      NFINISH(I)=NFINIS
  510 NLOAD(I)=NLOA
      IF (I.EQ.(NHEQ+NSEQ+1)) WRITE (MOUT,1470)
      IF (I.EQ.(NHEQ+1).AND.NSEQ.GT.0) WRITE (MOUT,1480)
C
C*****CONSTRAINT DATA, INCLUDING THE SEQUENCE OF PIPES
C
      IQI=NLOAD(I)
      WRITE (MOUT,1490) NSTART(I),NFINISH(I),IQI,(NO(J),J=N1,N2)
      B(I)=0.
      IF (IABS(ITYP).EQ.3) GO TO 550
      HCORR(I)=0.
      IF (IPM.LE.0) GO TO 530
C
C***** READ THE PUMPS AND REAL VALVES IN THE CONSTRAINT
C
      READ (MIN,980) (IPN(I,J),J=1,IPM)
      WRITE (MOUT,1500) (IPN(I,J),J=1,IPM)
      DO 520 J=1,IPM
         K=IABS(IPN(I,J))
         ISN=IPN(I,J)/K
         HCORR(I)=HCORR(I)-FLOAT(ISN)*HMIN(K,IQI)
  520 CONTINUE
  530 CONTINUE
      IF (ISS.EQ.0) GO TO 540
C
C*****STORAGE APPEARING IN THE CONSTRAINT
C
      READ (MIN,980) (ISTOR(I,J),J=1,ISS)
      WRITE (MOUT,1510) (ISTOR(I,J),J=1,ISS)
  540 CONTINUE
C
C*****COMPUTATION OF THE R.H.S. OF THE CONSTRAINTS
C
      B(I)=ELV(NSTART(I))-ELV(NFINISH(I))+HCORR(I)-PR(NFINISH(I),IQI)
      IF (MAXMIN.EQ.1.AND.NLOAD(I).GT.NNORM) B(I)=B(I)+PR(NFINISH(I),IQ
     1I)
  550 DO 560 J=N1,N2
         L=IABS(NO(J))
         SN=FLOAT(NO(J)/L)
         IF (ABS(Q(L,IQI)).GT.1.E-7) SN=SN*Q(L,IQI)/ABS(Q(L,IQI))
         K1=LINCOL(L)
         K2=K1+NDIAM(L)-1
         LL=0
C
```

```
C*****PIPE VARIABLES
C
      DO 560 K=K1,K2
         LL=LL+1
         AMAT(I,K)=GRAD1(ABS(Q(L,IQI)),D(L,LL),HW(L))*SN
  560 CONTINUE
      IF (IPM.EQ.0.AND.ITYP.EQ.1) GO TO 590
      IF (IPM.EQ.0.AND.IABS(ITYP).GT.1) GO TO 580
C
C*****PUMPS AND VALVES ELEMENTS
C
      DO 570 J=1,IPM
         K=IPN(I,J)
         KK=IABS(K)
         IF (LPUMP(KK,IQI).EQ.0) GO TO 570
         KKK=LOADCOL(IQI)+LPUMP(KK,IQI)-1
         AMAT(I,KKK)=K/KK
  570 CONTINUE
C
C***** DUMMY VALVES
C
  580 IF (ITYP.EQ.1) GO TO 590
      KKK=LDMVPTR(IQI)
      AMAT(I,KKK)=-1.0
      C(KKK)=PENFAC
      IF (ITYP.LT.0) C(KKK)=0.
      LDMVPTR(IQI)=LDMVPTR(IQI)+1
  590 IF (ISS.EQ.0) GO TO 610
C
C*****STORAGE ELEMENTS
C
      DO 600 J=1,ISS
         K=ISTOR(I,J)
         KK=IABS(K)
         AMAT(I,KK)=+K/KK
         AMAT(NSUROW,KK)=STOACRF*STCOST(J)
  600 CONTINUE
C
C***** CHECKING NEGATIVE B(I)FOR PUMPS/STORAGES/HEAD GAINS
C
  610 IF (B(I).GE.0.) GO TO 650
      DO 620 J=1,NOVARS
  620 IF (AMAT(I,J).LT.0.) GO TO 630
      WRITE (MOUT,1000) NSTART(I),NFINISH(I),B(I),I
      IMATS N=1
  630 DO 640 J=1,LINCOL(I)-1
  640 AMAT(I,J)=-AMAT(I,J)
      B(I)=-B(I)
      IF (ITYP.EQ.2) GO TO 440
      IRC(I)=1
      NMSLACK=NMSLACK+1
      AMAT(I,NOVARS+NMSLACK)=-1.0
C
C***** ADD POSITIVE SLACK VALUES FOR RELAXED LOOP/SOURCE EQUATIONS
C
  630 IF (PPTR(I).GT.0) GO TO 440
      NRELAX=NRELAX+1
      AMAT(I,NOVARS+NMSLACK+NRELAX)=1.0
      GO TO 440
  650 WRITE (MOUT,1030) ((J,NO(J)),J=1,LPTR)
      DO 670 I=1,NJ
      DO 670 J=1,NQ
```

```
         IF (NREF(I,J).EQ.0) NREF(I,J)=NREF(I,1)
  670 CONTINUE
      DO 680 J=1,NQ
         WRITE (MOUT,1540) ((I,J,NPTR(I,J)),I=1,NJ)
  680 CONTINUE
      WRITE (MOUT,1550) ((I,PPTR(I)),I=1,NPEQ)
      IF (INTER.EQ.0) GO TO 740
C
C***** COMPUTE INTERACTION BETWEEN LOOP AND PRESSURE EQUATIONS
C
      CALL SECOND (STATIME)
      LPTR=1
      K1=2
      DO 730 J=1,NQ
         DO 720 I=ILEQ(J),ILEQ(J)+NQLEQ(J)-1
            EQPTR(I-NHEQ-NQEQ)=0
            NCOM=0
            DO 710 K=1,NQHEQ(J)+NJ+NQLEQ(J)
               IF (K.LE.NQHEQ(J)) N1=IABS(PPTR(IHEQ(J)+K-1))
               IF (K.GT.NQHEQ(J).AND.K.LE.NQHEQ(J)+NJ.AND.NPTR(K-NQHEQ(J
     1         ),J).LE.0) GO TO 710
               IF (K.GT.NQHEQ(J).AND.K.LE.NQHEQ(J)+NJ) N1=IABS(NPTR(K-NQ
     1         HEQ(J),J))
               IF (K.GT.NQHEQ(J)+NJ) N1=IABS(PPTR(ILEQ(J)+(K-NQHEQ(J)-NJ
     1         )-1))
               IF (K.GT.NQHEQ(J)+NJ.AND.I.EQ.ILEQ(J)+K-NQHEQ(J)-NJ-1) GO
     1          TO 710
               NLINK=0
               DO 700 L=IABS(PPTR(I))+1,IABS(PPTR(I))+NO(IABS(PPTR(I)))
                  DO 690 M=N1+1,N1+NO(N1)
                     IF (IABS(NO(L)).NE.IABS(NO(M))) GO TO 690
                     NLINK=NLINK+1
                     LCOM(K1+NLINK+1)=(NO(L)/NO(M))*IABS(NO(L))
  690             CONTINUE
  700          CONTINUE
               IF (NLINK.EQ.0) GO TO 710
               NCOM=NCOM+1
               LCOM(K1)=K+IHEQ(J)-1
               IF (K.GT.NQHEQ(J).AND.K.LE.NQHEQ(J)+NJ) LCOM(K1)=-(K-NQHE
     1         Q(J))
               IF (K.GT.NQHEQ(J)+NJ) LCOM(K1)=K-NQHEQ(J)-NJ+ILEQ(J)-1
               LCOM(K1+1)=NLINK
               K1=K1+NLINK+2
  710       CONTINUE
            IF (NCOM.EQ.0) GO TO 720
            EQPTR(I-NHEQ-NSEQ)=LPTR
            LCOM(LPTR)=NCOM
            LPTR=K1
            K1=LPTR+1
  720    CONTINUE
  730 CONTINUE
      CALL SECOND (ENDTIME)
      STATIME=ENDTIME-STATIME
      WRITE (MOUT,1560) STATIME
C
C     $     WRITE(MOUT,884)((I,EQPTR(I)),I=1,NLEQ)
C     $884 FORMAT(10(*EQPTR(*,I2,*)=*,I3))
C     $     WRITE(MOUT,886)((I,LCOM(I)),I=1,LPTR)
C     $886 FORMAT(10(*LCOM(*,I3,*)=*,I3))
C
  740 NPCON=0
      IF (NPUMP.EQ.0) GO TO 790
```

```
C
C***** COMPUTE PUMP CONSTRAINT COEFFICIENTS AND RHS ELEMENT
C
      NPCON=NBUROW+NST
      DO 770 J=1,NQ
         IF (NQPUMP(J).EQ.0) GO TO 770
         DO 760 I=1,NPUMP
C
C***** CHECK FOR NEED FOR CONSTRAINT
C
            IF (LPUMP(I,J).EQ.0) GO TO 760
            IF (NQ.EQ.1.AND.HPMAX(I).GT.9000.) GO TO 760
            IF (LPUCRIT(I).NE.J) GO TO 750
            IF (HPMAX(I).GT.9000.) GO TO 750
C
C***** HP UPPER BOUND ON CRITICAL LOADING
C
            NPCON=NPCON+1
            K=LOADCOL(J)+LPUMP(I,J)-1
            AMAT(NPCON,K)=WATDEN*G(PML(I),J)*GPUMP(I,J)*PPUMP(I)/(550.*P
     1      UMPE(I))
            B(NPCON)=HPMAX(I)-HPMIN(I)
  750       IF (PCON(I,J).EQ.0) GO TO 760
C
C***** LOGICAL HEAD UPPER BOUND CONSTRAINT
C
            NPCON=NPCON+1
            K1=LOADCOL(J)+LPUMP(I,J)-1
            K2=LOADCOL(LPCON(I,J))+LPUMP(PCON(I,J),LPCON(I,J))-1
            AMAT(NPCON,K1)=1.0
            AMAT(NPCON,K2)=-1.0
            B(NPCON)=0.
  760    CONTINUE
  770 CONTINUE
      NPCON=NPCON-NBUROW-NST
C
C***** COMPUTE RHS FOR MAXIMUM STORAGE HEIGHT
C
  780 IF (NST.EQ.0) GO TO 800
      DO 790 J=1,NST
         B(NSTROW+J-1)=STMAX(J)
         AMAT(NSTROW+J-1,J)=1.0
  790 CONTINUE
C
C***** ADD MAXIMUM IMBALANCE CONSTRAINTS FOR RELAXED LOOP EQUATIONS
C
  800 IF (NRELAX.EQ.0) GO TO 830
      KK=NPEQ+NS+NST+NPCON+1
      DO 820 I=NMEQ+1,NPLG
         IF (PPTR(I).GT.0) GO TO 820
         KK=KK+1
         DO 810 J=1,NQ
            IF (I.GE.ILEQ(J).AND.I.LE.ILEQ(J)+NGLEQ(J)-1) L=J
            IF (I.GE.ISEQ(J).AND.I.LE.ISEQ(J)+NQSEQ(J)-1) L=J
  810    CONTINUE
         K=LOADCOL(L)+NQPUMP(L)+NQSEQ(L)+I-ILEQ(L)
         IF (I.LE.NMEQ+NSEQ) K=LOADCOL(L)+NQPUMP(L)+I-ISEQ(L)
         AMAT(KK,K)=1.0
         AMAT(KK,NDVARS+NMSLACK+KK)=1.0
         B(KK)=LIMBAL
         IF (I.LE.NMEQ+NSEQ) B(KK)=SIMBAL
  820 CONTINUE
```

```
C
C***** COMPUTE SIZE OF COEFFICIENT MATRIX
C
  830 NMROWS=NPEQ+NS+NST+NPCON+1+NRELAX
      NMCOLS=NOVARS+NMSLACK+NMROWS+NRELAX
      NPSLACK=NHEQ-NMSLACK+NPCON+NST+1+NRELAX
      NART=NMSLACK+NS+NLEQ+NSEQ-NRELAX
      WRITE (MOUT,1570) NOVARS,NPSLACK,NMSLACK,NART,NHEQ,NSEQ,NLEQ,NS,NSM
     1T,NPCON,NRELAX,NMROWS,NMCOLS
C
C*****SECTION LENGTH  CONSTRAINTS
C
      I1=NPEQ
      DO 850 I=1,NS
         IC=ICLASS(I)
         I1=I1+1
         J1=LINCOL(I)
         J2=LINCOL(I)+NDIAM(I)-1
         L=0
         DO 840 J=J1,J2
            L=L+1
            AMAT(I1,J)=1.
            ID=D(I,L)
            AMAT(NBUROW,J)=PIPACRF*(TAB(ID,IC)+EXCAVF(I))+PIPEM*FLOAT(ID
     1      )/5280.
  840    CONTINUE
         B(I1)=AL(I)
  850 CONTINUE
      NMSLACK=NMSLACK+NRELAX
C
C*****BUILDING THE COEFFICIENT MATRIX
C
      DO 860 I=1,NMROWS
         J=NOVARS+NMSLACK+I
         IF (IBC(I).EQ.1) C(J)=PENFAC
         IBC(I)=J
         AMAT(I,J)=1.
         IF (I.GT.NHEQ.AND.I.LT.NBUROW) C(J)=PENFAC
         IF (I.GT.NPEQ) GO TO 860
  860 CONTINUE
      C(NOVARS)=1.0
      B(NBUROW)=0.
      AMAT(NBUROW,LINCOL(NS)+NDIAM(NS))=-1.0
      IF (MAX.MIN.NE.1) GO TO 890
C
C***** COMPUTE OBJECTIVE FUNCTION COEFFICIENTS&RELATED MATRIX ELEMENTS
C
      DO 870 J=NNORM+1,NQ
         C(NOVARS-NQ+J)=-WL(J)/PSCALE
  870 CONTINUE
      C(LINCOL(NS)+NDIAM(NS))=PENFAC
      B(NBUROW)=BMAX-TIPCOST
      DO 880 I=1,NHEQ
      DO 880 J=NNORM+1,NQ
         IF (NLOAD(I).EQ.J) AMAT(I,NOVARS-NQ+J)=1.0
  880 CONTINUE
C
C***** PRESSURE EQUATION SCALING
C
  890 DO 910 I=1,NPEQ
         B(I)=PSCALE*B(I)
      DO 900 J=LINCOL(1),LINCOL(NS)+NDIAM(NS)-1
```

```
         AMAT(I,J)=PSCALE*AMAT(I,J)
  900 CONTINUE
C
C***** STORAGE AND PUMP SCALING
C
      IF (NST.EQ.0.AND.NPCON.EQ.0) GO TO 920
      DO 910 I=NBUROW+1,NMROWS
         B(I)=PSCALE*B(I)
  910 CONTINUE
  920 DO 930 J=1,LINCOL(1)-1
         AMAT(NBUROW,J)=AMAT(NBUROW,J)/PSCALE
  930 CONTINUE
      IF (IMAT.EQ.0) GO TO 950
      WRITE (MOUT,1590) ((I,B(I)),I=1,NMROWS)
      WRITE (MOUT,1590) ((J,C(J)),J=1,NMCOLS)
      DO 940 I=1,NMROWS
      DO 940 J=1,NMCOLS
         IF (ABS(AMAT(I,J)).GT.1.E-7) WRITE (MOUT,1500) I,J,AMAT(I,J)
  940 CONTINUE
      CALL EXIT
  950 CONTINUE
      RETURN
C
  960 FORMAT (20A4,/20A4)
  970 FORMAT (1X,1H1,15(/),15X,20A4,2(/),10X,20A4,/5X,60(1H=)///)
  980 FORMAT (16I5)
  990 FORMAT (49H MAXIMIZE WEIGHTED SUM OF MINIMUM HEAD NODES OVER,32H E
     1MERGENCY LOADINGS SUBJECT TO MAXIMUM BUDGET LEVELS)
 1000 FORMAT (54H CONJUGATE GRADIENT USED IN COMPUTING DIRECTION VECTOR)
 1010 FORMAT (53H NEGATIVE GRADIENT USED IN COMPUTING DIRECTION VECTOR)
 1020 FORMAT (47H BFGS METHOD USED IN COMPUTING DIRECTION VECTOR)
 1030 FORMAT (29H NO INTERACTION BETWEEN PATHS)
 1040 FORMAT (47H INTERACTION BETWEEN PATHS COMPUTED IN GRADIENT)
 1050 FORMAT (51H SIGN IF LOOP TERMS IN GRADIENT COMPUTATION IGNORED)
 1060 FORMAT (//,75H MINIMIZE EQUIVALENT UNIFORM ANNUAL COST OF
     1      DISTRIBUTION SYSTEM ,/,92H SUBJECT TO MINIMUM PERFORMANCE
     2       LEVELS AT SELECTED NODES ON EACH LOADING CONDITION )
 1070 FORMAT (I5,10-5.0)
 1080 FORMAT (2(9H LOAD NO.,I2,27H OBJECTIVE FUNCTION WEIGHT=,F6.3))
 1090 FORMAT (F10.0,F5.0,I5,2F5.0)
 1100 FORMAT (1X,30I4)
 1110 FORMAT (1X,/25X,14H GENERAL  DATA,/26X,13(1H=),/5X,43H NUMBER  OF
     1 SECTIONS                         ,I4,/5X,60(1H-)/5X,43H NUMBER  OF
     2NODES                             ,I4,/5X,60(1H-)/5X,43H GREATEST  DIA
     3METER  ALLOWED (INCH)             ,I4,/5X,60(1H-)/5X,43H SMALLEST DIAME
     4TER ALLOWED    (INCH)            ,I4,/5X,60(1H-)/5X,43H NUMBER  OF  DIF
     5FERENT  FLOW  DISTRIBUTIONS,I4,/5X,60(1H-)/5X,41H NUMBER OF EMERGE
     6NCY LOADING CONDITIONS  ,I4,/5X,60(1H-)/5X,43H NUMBER OF NORMAL LO
     7ADING CONDITIONS        ,I4,/5X,60(1H-)/5X,20H NO. OF SOURCE NODES,
     8I4,5X,60(1H-)/5X,35H NO. OF LINKS W/HIGH EXCAVATION COST,3X,I5/5X,
     960(1H-)/5X,43H NUMBER  OF  PUMPS                            ,I4,/5X,6
     *0(1H-)/5X,43H NUMBER  OF  VALVES                            ,I4,/5X,60
     *(1H-)/5X,43H NUMBER  OF  STORAGES                           ,I4,/5X,)
 1120 FORMAT (/5X,39H ANNUAL TOTAL BUDGET                         ,F10.0,/5X,6
     10(1H-)/5X,40H INTEREST RATE                                ,F5.2,/5X,60(
     21H-)/5X,40H PIPE LIFE IN YEARS                           ,I4,/5X,60(1H-)
     3/5X,40H PIPE SALVAGE VALUE RATIO                ,F4.2/5X,60(1H-)/5X
     4,34H PIPELINE MAINTENANCE COST($/IN/MILE/YR,F5.1)
 1130 FORMAT (/20X,11H PUMPS DATA,/25X,12(1H=))
 1140 FORMAT (/,60(1H-)/5X,28H PUMP LIFE IN YEARS              ,I5,/5X,60(1H-
     1)/5X,40H PUMP SALVAGE VALUE RATIO                ,F4.2/5X,60(1H-)/5
     2X,33H PUMP-MOTOR COMBINED EFFICIENCY  ,F5.2,/5X,60(1H-)/5X,34H ELE
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    3CTRICITY COST($/KW-HR)          ,F5.2,/5X,60(1H-)/5X,36H PUMP MAINTEMAT00806
    *NANCE COST($/HP/YR)      ,F5.1,/5X,60(1H-)/5X,39H ALLOWABLE EST/ACTMAT00807
    5UAL COST % DIFFERENCE,F5.2,/5X)                                    MAT00808
1150 FORMAT (2I5,2F5.0,I5,3F5.0)                                        MAT00809
1160 FORMAT (10(I5,F5.0))                                               MAT00810
1170 FORMAT (48H PUMP NO.  LOCATION  HPMIN  HPMAX  C/LOAD HSTART)       MAT00811
1180 FORMAT (2(4X,I5),2(4X,F5.0),4X,I3,F4.0)                            MAT00812
1190 FORMAT (19H PUMP COMPOSED OF ,F2.0,18H PUMPS IN PARALLEL)          MAT00813
1200 FORMAT (9H LOAD NO.,I2,9H PUMP NO.,I2,14H LOAD PUMP NO.,I2,7H GPUMMAT00814
    1P ,F7.4,17H OPERATING HOURS ,F8.2)                                 MAT00815
1210 FORMAT (6F5.0,2I5)                                                 MAT00816
1220 FORMAT (42H MAXIMUM NO. OF FLOW ITERATIONS/NETWORK    ,I3,/,42H MAXMAT00817
    1IMUM NO. OF LP ITERATIONS/FLOW          ,I3)                       MAT00818
1230 FORMAT (30H INITIAL STEP SIZE(GPM)       =,F5.1,/,30H MINIMUM ALLOWMAT00819
    1ABLE STEPSIZE  =,F5.1,/,30H RATIO FOR MINIMUM FLO CHANGE=,F5.1)   MAT00820
1240 FORMAT (I5,7F10.0)                                                 MAT00821
1250 FORMAT (8F10.0)                                                    MAT00822
1260 FORMAT (//3X,49H ADDITIONAL STORAGE ELEVATION COST(PER UNIT ELEV),MAT00823
    1/3X,35H STORAGE               COST     MAX HEIGHT)                 MAT00824
1270 FORMAT (1X,////10X,10HPIPES COST,/10X,10(1H=),/,46H  DIAMETER      MAT00825
    1  UNIT COST(ACCORDING TO CLASS))                                   MAT00826
1280 FORMAT (I5,5X,6F10.1)                                              MAT00827
1290 FORMAT (////22X,13H NODES  DATA,/22X,11(1H=),/,36H                 MAT00828
    1        MIN/MAX          ,/,35H  NODE          ELEVATIONPRESSURE      ,/,34MAT00829
    2H                     FT.    ALLOWED      ,/,60H               LOAD1MAT00830
    3  LOAD2   LOAD3   LOAD4   LOAD5   LOAD6,29H  LOAD7   LOAD8   LOAD9   LOADMAT00831
    410)                                                                MAT00832
1300 FORMAT (15X,6F10.0)                                                MAT00833
1310 FORMAT (1X,I5,5X,F7.1,10(2X,F5.1))                                 MAT00834
1320 FORMAT (////22X,15H SECTIONS  DATA ,/22X,14(1H=),///22X,19HRANGE   MAT00835
    1OF             ,/2X,41HLINK LINK  LENGTH     -C-  ALLOWABLE CLASS,10X,MAT00836
    2RHSELECTED,3X,40HNO. NO.    (FT.)              DIAMETERS       ,10X,9HDIMAT00837
    3AMETERS,/22X,31H(INCHES)                  (INCHES))                MAT00838
1330 FORMAT (///,22X,17HCONSUMPTION DATA ,/,6H  NODE,22X,5H(GPM),/,10X,MAT00839
    148H   LOAD1   LOAD2   LOAD3   LOAD4   LOAD5   LOAD6,33H   LOAD7    MAT00840
    2LOAD8    LOAD9    LOAD10)                                          MAT00841
1340 FORMAT (1X,I5,4X,10F8.2)                                           MAT00842
1350 FORMAT (8(3H A(,I3,1H,,I3,2H)=,F5.0))                              MAT00843
1360 FORMAT (4(I5,F5.0,I5,F5.0))                                        MAT00844
1370 FORMAT (1X,5F10.5)                                                 MAT00845
1380 FORMAT (I5,2F10.0,3I5)                                             MAT00846
1390 FORMAT (1X,2I5,F7.1,F4.1,2I4,I6,3X,6F5.0)                          MAT00847
1400 FORMAT (///,22X,27H INITIAL FLOW DISTRIBUTION ,/,60H  LINK  LINK   MAT00848
    1 LOAD1    LOAD2    LOAD3    LOAD4    LOAD5    LOAD6,33H    LOAD7    LOAD8MAT00849
    2    LOAD9    LOAD10)                                               MAT00850
1410 FORMAT (1X,2I5,10F  .1)                                            MAT00851
1420 FORMAT (16F5.0)                                                    MAT00852
1430 FORMAT (20X,31H INITIAL TOTAL COST OF PUMP NO.,I2,29H CRITICAL     MAT00853
    1       LOADING NO.,I2,/,10H CAPITAL $,F8.2,9H ENERGY $,F8.2,14H MAINMAT00854
    2TENANCE $,F8.2)                                                    MAT00855
1440 FORMAT (9H PUMP NO.,I2,13H COST COEFFICIENT=,F8.2,18H MAINTENANCE MAT00856
    1COST=,F9.2,13H ENERGY COST=,F8.2)                                  MAT00857
1450 FORMAT (//5X,50HSTRINGS OF PIPES FOR PRESSURE OR LOOP  CONSTRAINTSMAT00858
    1,/,5X,50(1H-),/3X,45HFROM TO   LOAD         NUMBER ORDER OF SECTIONSMAT00859
    2,19X,15HNUMBER ORDER OF:,//2X,49HNODE NODE  NUM.        CONNECTED BMAT00860
    3ETWEEN THE NODES,14X,26HPUMPES/VALVES       STORAGES)              MAT00861
1460 FORMAT (1X,3I5,7X,9I4,/22X,9I4)                                    MAT00862
1470 FORMAT (1H ,14HLOOP       EQ,S.)                                   MAT00863
1480 FORMAT (1H ,13HSOURCES EQ,S.)                                      MAT00864
1490 FORMAT (1H ,14HPRESSURE EQ,S.)                                     MAT00865
1500 FORMAT (1H+,60X,4I4)                                               MAT00866
1510 FORMAT (1H+,80X,2I4)                                               MAT00867
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 1520 FORMAT (15H HEAD AT SOURCE,I4,24H LOWER THAN HEAD AT NODE,I4,3H BYMAT00868
     1,F6.2,32HNO HEAD GAINS ON PATH CONSTRAINT,I4,/,12H EXIT CALLED)   MAT00869
 1530 FORMAT (10(4H NO(,I3,2H)=,I3))                                    MAT00870
 1540 FORMAT (10(4HNPT(,I2,1H,,I1,2H)=,I3))                             MAT00871
 1550 FORMAT (10(5HPPIR(,I3,2H)=,I3))                                   MAT00872
 1560 FORMAT (50H COMPUTATION TIME FOR COMPUTING INTERACTION ARRAYS,F8.4MAT00873
     1)                                                                 MAT00874
 1570 FORMAT (1X,////10X,18H CONSTRAINTS  DATA,/10X,17(1H=),/5X,41HNUMBEMAT00875
     1R OF DECISION  VARIABLES                ,I5,/5X,38HNO. OF POSITIVE SLAMAT00876
     2CK VARIABLES          ,I5,/5X,38HNO. OF NEGATIVE SLACK VARIABLES     MAT00877
     3  ,I5,/5X,38HNO. OF ARTIFICIAL VARIABLES            ,I5,/5X,31HNUMBMAT00878
     4ER OF PRESSURE EQ.S.          ,I5,/5X,31HNUMBER OF EQ.S. BETWEEN SOURMAT00879
     5CES,I5,/5X,31HNUMBER OF LOOP EQ.S.              ,I5,/5X,31HNUMBER OF MAT00880
     6LENGTH CONSTRAINTS    ,I5,/5X,32HNUMBER OF STORAGE HEIGHT CONST'S,IMAT00881
     75,/5X,31HNUMBER OF PUMP CAPACITY CONST'S,I5,/5X,32HNUMBER OF LOOP MAT00882
     8SLACK CONSTRAINTS,I5,/5X,29HSIZE OF COEF. MATRIX:     ROWS,I5,9X,7HMAT00883
     9COLUMNS,I5)                                                       MAT00884
 1580 FORMAT (8(3H B(,I3,2H)=,G8.2))                                    MAT00885
 1590 FORMAT (8(3H C(,I3,2H)=,G8.2))                                    MAT00886
 1600 FORMAT (3H A(,I3,1H,,I3,2H)=,F10.4)                               MAT00887
C                                                                       MAT00888
      END                                                               MAT00889
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      SUBROUTINE PUMCHK
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /AMAT/ AMAT(110,275)
      COMMON /LOADCOL/ LOADCOL(4)
      COMMON /BASIC/ IBV(325),IPIV(125)
      COMMON /BUF12/ PI2(125),HF(45,3),X(325)
      COMMON /PUMPA/ HPMIN(5),HPMAX(5),HMIN(5,3),HMAX(5,3),LPUMP(5,3),LP
     1UCRIT(5),NJPUMP(3),PML(5),PUCOEF(5),PUMPHR(5,3),PVL(1)
      COMMON /PPUMP/ PPUMP(5)
      COMMON /QPUMP/ QPUMP(5,3)
      COMMON /PUMPF/ PUMPF(5)
      COMMON /PUMPV/ PUMPEFF,POWCOST,PUMPM,PCDIFF,WATDEN,PUMACRF,TIPCOST
      COMMON /MATRIX/ NMROWS,NMCOLS,NMSLACK,NDVARS,NBUROW,MXLPIT
      COMMON /NTIMES/ NDIACHG,NPUMCHK,NFLOCHG,NROWPIV
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NQ,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /MOUT/ MOUT,MIN
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      INTEGER PML
      NPUMCHK=0
      DO 20 I=1,NPUMP
         J=LPUCRIT(I)
         IF (LPUMP(I,J).EQ.0) GO TO 20
         K=LOADCOL(J)+LPUMP(I,J)-1
         ESTCOST=PUCOEF(I)*(X(K)/PSCALE)
         PUICOST=16.14*PUMACRF*(((Q(PML(I),J)*QPUMP(I,J))**.453)*((X(K)/
     1   PSCALE)+HMIN(I,J))**.642)-((Q(PML(I),J)*QPUMP(I,J)/PPUMP(I))**.
     2   453)*(HMIN(I,J)**.642)
         PMCOST=PUMPM*WATDEN*Q(PML(I),J)*QPUMP(I,J)*(X(K)/PSCALE)*PPUMP(
     1   I)/(550.*PUMPF(I))
         HP=PMCOST/PUMPM
         ECOST=.746*PUMPHR(I,J)*POWCOST*HP
         ACOST=PUICOST*PPUMP(I)
         WRITE (MOUT,30) I,ESTCOST,ACOST,PUICOST,PMCOST,ECOST,HP
         IF (ACOST.LT.1.E-2) GO TO 20
         IF (ABS(ESTCOST-ACOST)/ACOST.LT.PCDIFF) GO TO 20
C
C*****  ADJUST BUDGET ROW COEFFICIENTS
C
         OLD=PUCOEF(I)
         IF (X(K).GT.1.E-7) PUCOEF(I)=ACOST/(X(K)/PSCALE)
         WRITE (MOUT,40) I,OLD,PUCOEF(I)
         OLD=(PUCOEF(I)-OLD)/PSCALE
         IARY=NDVARS+NMSLACK+NBUROW
         IF (IBV(K).GT.0) IPIV(IBV(K))=1
         DO 10 KK=1,NMROWS
            AMAT(KK,K)=AMAT(KK,K)+AMAT(KK,IARY)*OLD
   10    CONTINUE
         NPUMCHK=NPUMCHK+1
         ILPFORM=2
   20 CONTINUE
      RETURN
C
   30 FORMAT (9H PUMP NO.,I2,9H EST COST,F8.2,10H ACT COST=,F8.2,14H CAP
     1ITAL COST=,F8.2,12H MAINT COST=,F8.2,13H ENERGY COST=,F8.2,4H HP=,
     2F8.2)
   40 FORMAT (10H PUMP NO. ,I2,18H OLD COEFFICIENT= ,F9.2,18H NEW COEFFI
     1CIENT =,F9.2)
C
      END
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      SUBROUTINE REPORT
      COMMON /BUF11/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /BCVEC/ B(125),C(325)
      COMMON /EQ/ IHEQ(3),ISEQ(3),ILEQ(3),NQHEQ(3),NQLEQ(3),NQSEQ(3)
      COMMON /PATH1/ NSTART(75),NFINISH(75)
      COMMON /PATH2/ PPTR(75),NLOAD(75)
      COMMON /NODE1/ PR(28,3),ELV(28)
      COMMON /NODE2/ NPTR(28,3),NREF(28,3),SOURCE(4)
      COMMON /LINK/ AL(45),EXCAVF(45),HW(45),ICLASS(45),LINCOL(45),NDIAM
     1(45),FAH(30,1),ION(45),IOX(45)
      COMMON /PIPE/ PIPE(45)
      COMMON /STORE/ STCOST(7),STMAX(7)
      COMMON /LOADCOL/ LOADCOL(4)
      COMMON /BUF12/ PIZ(125),HF(45,3),X(325)
      COMMON /FLOA/ DG(45),QD(45),ALFA(3)
      COMMON /ZLOAD/ ZLOAD(5)
      COMMON /ZPEN/ ZPEN(3)
      COMMON /PUMPA/ HPMIN(5),HPMAX(5),HMIN(5,3),HMAX(5,3),LPUMP(5,3),LP
     1UCRIT(5),NGPUMP(3),PML(5),PUCOEF(5),PUMPHR(5,3),PVL(1)
      COMMON /QPUMP/ QPUMP(5,3)
      COMMON /PUMPF/ PUMPF(5)
      COMMON /PPUMP/ PPUMP(5)
      COMMON /MATRIX/ NMROWS,NMCOLS,NMSLACK,NOVARS,NBUROW,MXLPIT
      COMMON /PREQ/ NHEQ,NSEQ,NLEQ,NPEQ
      COMMON /PUMPV/ PUMPEFF,POWCOST,PUMPM,PCDIFF,WATDEN,PUMACRF,TIPCOST
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NQ,NVL,NPUMP,NST,NCLASS,NSOURCE,PSCA
     1LE
      COMMON /MOUT/ MOUT,MIN
      COMMON /IMATGEN/ IMATGEN
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      COMMON /CTIME/ TMATT,TNETT,TFLOS,TLPT,TLPFT,TPUMT,TGRAT,TDIAT,TSAV
     1T,TFLOT
      COMMON /FLOV/ ZFLOOP,ITFLOOP,ITFLO
      COMMON /PRICE/ PIPACRF,PIPEM,STOACRF
      INTEGER PPTR,PIPE,PVL,PML
      DIMENSION AAL(5), DOP(5), LDMVPTR(10)
      IF (ILP.EQ.0.AND.ILPFORM.NE.1.AND.IMATGEN.EQ.0) GO TO 10
      WRITE (MOUT,240) ((I,B(I)),I=1,NMROWS)
      WRITE (MOUT,250) ((J,C(J)),J=1,NMCOLS)
      IF (IMATGEN.EQ.0) GO TO 10
      RETURN
   10 WRITE (MOUT,260) TMATT,TNETT,TFLOS,TLPT,TLPFT,TPUMT,TGRAT,TDIAT,TF
     1LOT,TSAVT
   20 CONTINUE
      IF (UNIT,11) 20,30,230
   30 REWIND 11
      BUFFER IN (11,0) (D(1,1),Q(45,3))
   40 CONTINUE
      IF (UNIT,12) 40,50,230
   50 REWIND 12
      BUFFER IN (12,0) (PIZ(1),X(325))
      WRITE (MOUT,270) ITFLOOP
      CALL FLOSEL
      CALL FLOCHG
      DO 60 J=1,NQ
         CALL HCOMP (J)
   60 CONTINUE
      WRITE (MOUT,280)
      II=LINCOL(1)-1
      TOTAL=0.0
      TOTPIM=0.0
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      TOTPIC=0.
      DO 100 I=1,NS
         DO 70 J=1,3
            DOP(J)=0.
            AAL(J)=0.
   70    CONTINUE
         KL=0
         K=ICLASS(I)
         DO 80 J=1,NQ
            LOMVPTR(J)=LOADCOL(J)+NQPUMP(J)+NVL+NQSEQ(J)-1
   80    CONTINUE
         DO 90 J=1,NDIAM(I)
            ID=INT(D(I,J))
            II=II+1
C
C***** BREAK OUT PIPE CAPITAL AND OPERATING COSTS
C
            PICOST=PIPACRF*(TAB(ID,K)+EXCAVE(I))*X(II)
            PIMCOST=PIPEM*FLOAT(ID)*X(II)/5280.
            IF (X(II).LT.1.E-5) GO TO 90
            KL=KL+1
            AAL(KL)=X(II)
            DOP(KL)=D(I,J)
            TOTAL=TOTAL+PICOST+PIMCOST
            TOTPIC=TOTPIC+PICOST
            TOTPIM=TOTPIM+PIMCOST
   90    CONTINUE
C
C*****PRINT OUT SECTION DATA - INCLUDING LENGTH OF SELECTED SEGMENTS
C
         WRITE (MOUT,290) PIPE(I),AL(I),((DOP(J),AAL(J)),J=1,3)
  100 CONTINUE
      WRITE (MOUT,300)
      DO 110 I=1,NS
         WRITE (MOUT,310) PIPE(I),(Q(I,L),L=1,NQ)
  110 CONTINUE
      WRITE (MOUT,320)
      DO 120 I=1,NS
         WRITE (MOUT,310) PIPE(I),(HF(I,L),L=1,NQ)
  120 CONTINUE
      WRITE (MOUT,330) TOTAL
      WRITE (MOUT,340) TOTPIC
      WRITE (MOUT,350) TOTPIM
      K=0
C
C*****PRINT COST FOR ADDITIONAL STORAGE ELEVATION
C
      SCOST=0.
      TPUCOST=0.
      IF (NST.EQ.0) GO TO 140
      DO 130 I=1,NST
         X(I)=X(I)/PSCALE
         SCOST=SCOST+STCOST(I)*X(I)*STOACRF
  130 CONTINUE
      WRITE (MOUT,360) SCOST
  140 IF (NPUMP.EQ.0) GO TO 160
      WRITE (MOUT,370)
      DO 150 I=1,NPUMP
         J=LPUCRIT(I)
         IF (LPUMP(I,J).EQ.0) GO TO 150
         K=LOADCOL(J)+LPUMP(I,J)-1
         PUCOST=16.14*PPUMP(I)*PUMACRF*((((Q(PML(I),J)*QPUMP(I,J))**.45
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     1    3)*((X(K)/PSCALE)*HMIN(I,J))**.642)-((Q(PML(I),J)*QPUMP(I,J))**
     2    .453)*(HMIN(I,J)**.642))
          PMCOST=PUMPM*WATDEN*Q(PML(I),J)*(X(K)/PSCALE)*QPUMP(I,J)*PPUMP(
     1    I)/(550.*PUMPE(I))
          HP=PMCOST/PUMPM
          ECOST=HP*POWCOST*PUMPHR(I,J)*.745
          PTCOST=(PUICOST+PMCOST+ECOST)
          TPUCOST=TPUCOST+PTCOST
          WRITE (MOUT,380) I,PTCOST,PUICOST,PMCOST,ECOST,HP
  150 CONTINUE
C
C*****PRINT OUT PENALTY COST (FOR THE DUMMY VARIABLES)
C
  160 TOTAL=TOTAL+SCOST+TPUCOST
      WRITE (MOUT,390) TOTAL
C
C*****COMPUTE AND PRINT RESULTS FOR NODES
C
      WRITE (MOUT,400)
      IF (NHEQ.GT.0) WRITE (MOUT,410)
      IF (NPUMP.EQ.0) GO TO 190
C
C*****PUMP OPERATION DATA
C
      WRITE (MOUT,420)
      WRITE (MOUT,430)
      DO 180 J=1,NQ
         DO 170 I=1,NPUMP
            Q(I,J)=HMIN(I,J)
            IF (LPUMP(I,J).EQ.0) GO TO 170
            Q(I,J)=Q(I,J)+X(LOADCOL(J)+LPUMP(I,J)-1)/PSCALE
  170    CONTINUE
         WRITE (MOUT,440) J,(Q(K,J),K=1,NPUMP)
  180 CONTINUE
  190 IF (NVL.EQ.0) GO TO 210
C
C*****VALVE OPERATION DATA
C
      WRITE (MOUT,450)
      WRITE (MOUT,460)
      DO 200 I=1,NQ
         K1=LOADCOL(I)+NOPUMP(I)
         K2=LOADCOL(I)+NOPUMP(I)+NVL-1
         WRITE (MOUT,440) I,(X(J),J=K1,K2)
  200 CONTINUE
  210 IF (NLEQ.EQ.0.AND.NSEQ.EQ.0) GO TO 220
C
C*****DUMMY VARIABLES - OPERATIONAL STATUS
C
      WRITE (MOUT,470)
      WRITE (MOUT,480)
  220 CONTINUE
      IF (NST.LE.0) GO TO 230
C
C*****ADDITIONAL STORAGE ELEVATION
C
      WRITE (MOUT,490) (I,I=1,NST)
      WRITE (MOUT,500) (X(J),J=1,NST)
  230 CONTINUE
      RETURN
C
  240 FORMAT (8(3H B(,I3,2H)=,G9.2))
```

```
250 FORMAT (8(3H C(,I3,2H)=,G8.2))                                     REP00186
260 FORMAT (///,36H COMPUTATION TIME TOTALS(IN SECONDS),/,19H SUBROUTIREP00187
   1NE MATGEN ,F8.4,/,19H SUBROUTINE NETOP  ,F8.4,/,19H SUBROUTINE FLOREP00188
   2SEL ,F8.4,/,19H SUBROUTINE LP     ,F8.4,/,19H SUBROUTINE LPFORM ,FREP00189
   38.4,/,19H SUBROUTINE PUMCHK ,F8.4,/,19H SUBROUTINE GRAD   ,F8.4,/,REP00190
   419H SUBROUTINE DIAMCHG,F8.4,/,19H SUBROUTINE FLOCHG ,F8.4,/,19H SUREP00191
   5BROUTINE SAVEOPT,F8.4)                                             REP00192
270 FORMAT (20X,29H $$OPTIMAL FLOW ITERATION NO.,I3)                   REP00193
280 FORMAT (///10X,20H $$OPTIMAL DIAMETERS,/10X,39(1H-)//2X,35H$$SEC LREP00194
   1ENGTH            DIAM1  LENGTH1,36H DIAM2  LENGTH2    DIAM3     LENGTH3REP00195
   2  ,/2X,32H $$NO    FT.              IN        FT,34H         IN   FT.    REP00196
   3  IN.       FT. ,X,/,5H-----,10(9H---------+))                     REP00197
290 FORMAT (1X,2H$$,I3,4X,10(F8.2))                                    REP00198
300 FORMAT (///22X,32H $$LINK FLOW DISTRIBUTION (GPM) ,/,54H $$LINK  LREP00199
   1OAD1   LOAD2   LOAD3   LOAD4   LOAD5   LOAD6,33H   LOAD7   LOAD8  REP00200
   2 LOAD9   LOAD10)                                                   REP00201
310 FORMAT (1X,2H$$,I3,8(F11.3),2H$$)                                  REP00202
320 FORMAT (///22X,21H LINK HEAD LOSS (FT) ,/,,97H LINK   LOAD1   LOADREP00203
   12   LOAD3   LOAD4   LOAD5   LOAD6                  LOAD7   LOAD8   LOAREP00204
   2D9   LOAD10)                                                       REP00205
330 FORMAT (/10X,40H $$TOTAL EQUIVALENT ANNUAL PIPELINE COST,F13.0)   REP00206
340 FORMAT (/2X,42H $$EQUIVALENT ANNUAL PIPELINE CAPITAL COST,F13.0)  REP00207
350 FORMAT (/2X,42H $$ANNUAL PIPELINE O&M COST                 ,F13.0)  REP00208
360 FORMAT (//10X,40H $$TOTAL EQUIVALENT ANNUAL STORAGE COST ,F13.0)  REP00209
370 FORMAT (50H $$PUMP NO.   TOTAL    CAPITAL    MAINT    ENERGY  HP)  REP00210
380 FORMAT (3H $$,I2,5F10.2)                                           REP00211
390 FORMAT (20X,46H $$TOTAL EQUIVALENT ANNUAL NETWORK(NO PENALTY),F12.REP00212
   10)   .                                                             REP00213
400 FORMAT (1X,////15X,15H      NODES DATA ,/20X,10(1H=),/7X,13H   NODEREP00214
   1   NO.,4X,8HFRICTION,8X,7HMIN/MAX,7X,19HEXISTING        DUAL/25X,6HLREP00215
   2OSSES,4X,16HPRESSURE ALLOWED,6X,20HPRESSURE      ACTIVITY,/)       REP00216
410 FORMAT (1H ,14HPRESSURE EQ.S.)                                     REP00217
420 FORMAT (//13X,20HPUMPS  ACTIVITY (FT),/13X,15(1H-))                REP00218
430 FORMAT (/,50H LOAD      PUMP      PUMP      PUMP      PUMP      PUMP,59REP00219
   1H  NO.       NO.1      NO.2      NO.3      NO.4      NO.5      NO.6)    REP00220
440 FORMAT (/,1X,2H$$,I3,6(3X,F6.1),2H$$)                              REP00221
450 FORMAT (//13X,20HREAL VALVE  ACTIVITY,/13X,16(1H-))                REP00222
460 FORMAT (/,50H LOAD      VALVE      VALVE      VALVE      VALVE      ,55REP00223
   1HVALVE      VALVE      VALVE      VALVE      VALVE      VALVE,/,61H  NOREP00224
   2.       NO.1      NO.2      NO.3      NO.4      NO.5        ,44HNO.6 REP00225
   3     NO.7      NO.8      NO.9      NO.10)                          REP00226
470 FORMAT (//13X,21HDUMMY VALVE  ACTIVITY,/13X,25(1H-))               REP00227
480 FORMAT (/,49H LOAD      SOURCE      SOURCE      SOURCE      SOURCE,11HREP00228
   1      SOURCE,/,41H  NO.       NO.1      NO.2      NO.3      NO.4,,9H     REP00229
   2NO.5)                                                              REP00230
490 FORMAT (/5X,35H ADDITIONAL STORAGE ELEVATIONS (FT),/5X,33(1H-),/,1REP00231
   12H STORAGE NO.,5X,10I6)                                            REP00232
500 FORMAT (/,18H $$ADDED ELEVATION,10F6.1)                            REP00233
C                                                                      REP00234
      END                                                              REP00235
```

```
      SUBROUTINE TRADE (LVEQ,NENTER,LOAD)
      COMMON /BUF1I/ D(45,4),IBC(125),NO(325),Q(45,3)
      COMMON /AMAT/ AMAT(100,275)
      COMMON /BCVEC/ B(125),C(325)
      COMMON /INTER/ EQPTR(75),LCOM(325)
      COMMON /EQ/ IHEQ(3),ISEQ(3),ILEQ(3),NQHEQ(3),NQLEQ(3),NQSEQ(3)
      COMMON /PATH1/ NSTART(75),NFINISH(75)
      COMMON /PATH2/ PPTR(75),NLOAD(75)
      COMMON /NODE1/ PR(28,3),ELV(28)
      COMMON /NODE2/ NPTR(28,3),NREF(28,3),SOURCE(4)
      COMMON /LINK/ AL(45),EXCAVF(45),HW(45),ICLASS(45),LINCOL(45),NDI
     1(45),TAB(30,1),IDN(45),IDX(45)
      COMMON /BASIC/ IBV(325),IPIV(125)
      COMMON /FLOW/ DQ(45),QD(45),ALFA(3)
      COMMON /RHSCHG/ HEQNO(10),DELRHS(10)
      COMMON /NUMBER/ MXFLOIT,NS,NJ,NG,NVL,NPUMP,NST,NCLASS,NSOURCE,PS
     1LE
      COMMON /MATRIX/ NMROWS,NMCOLS,NMSLACK,NOVAPS,NHUROW,MXLPIT
      COMMON /STATUS/ ILPFORM,IGRAD,IFLOSEL,ILP
      COMMON /GRAD/ INTER,ICG,IREGS,GZMCOST,GZMPER,ALPHA,IALP,ICRIT
      COMMON /MOUT/ MOUT,MIN
      COMMON /PREQ/ NHEQ,NSEQ,NLEQ,NPEQ
      COMMON /NRHSCHG/ NRHSCHG
      INTEGER EQPTR,PPTR,HEQNO
      WRITE (MOUT,70) LOAD,NFINISH(LVEQ),NENTER,LVEQ
      ILPFORM=2
**    ** CHANGE COEFFICIENT MATRIX
      K1=PPTR(LVEQ)+1
      K2=PPTR(LVEQ)+NO(PPTR(LVEQ))
      DO 40 J=1,2
         IF (J.EQ.2) K1=IABS(NPTR(NENTER,LOAD))+1
         IF (J.EQ.2) K2=IABS(NPTR(NENTER,LOAD))+NO(IABS(NPTR(NENTER,LO
     1   )))
 1
         DO 30 I=K1,K2
            LINK=IABS(NO(I))
            SN=FLOAT(LINK/NO(I))*G(LINK,LOAD)/ABS(G(LINK,LOAD))
            NUM1=LINCOL(LINK)
            NUM2=NUM1+NDIAM(LINK)-1
            II=0
            DO 20 NUM=NUM1,NUM2
               II=II+1
               IF (IBV(NUM).GT.0) IPIV(IBV(NUM))=1
               DEL=FLOAT((-1)**J)*SN*10.471*((G(LINK,LOAD)/HW(LINK))**
     1                 852)/(D(LINK,II)**4.87)
 1
               IART=NOVAPS+NMSLACK+LVEQ
               DO 10 III=1,NMROWS
                  AMAT(III,NUM)=AMAT(III,NUM)+AMAT(III,IART)*DEL
 10            CONTINUE
 20         CONTINUE
 30      CONTINUE
 40   CONTINUE
**    ** CHANGE RIGHT HAND SIDE
      NRHSCHG=NRHSCHG+1
2T
      TRADEOL=TRAUTODS40C
      DELRHS(NRHSCHG)=ELV(NREF(NENTER,LOAD))-ELV(NENTER)-PR(NENTER,LOA
     1-(ELV(NSTART(LVEQ))-ELV(NFINISH(LVEQ))-PR(NFINISH(LVEQ),LOAD))
      HEQNO(NRHSCHG)=LVEQ
**    ** CHANGE INTERACTION ARRAYS
```

```
      IF (INTER.EQ.1.OR.NQLEQ(LOAD).EQ.0) GO TO 80                      TRA00062
      DO 60 I=ILEQ(LOAD),ILEQ(LOAD)+NQLEQ(LOAD)-1                       TRA00063
         LOOP=I-NHEQ-NSEQ                                               TRA00064
         IF (EQPTR(LOOP).EQ.0) GO TO 60                                 TRA00065
         K=EQPTR(LOOP)+1                                                TRA00066
         DO 50 J=1,LCOM(K-1)                                            TRA00067
            IF (LCOM(K).EQ.-NENTER) LCOM(K)=LVEQ                        TRA00068
            IF (LCOM(K).EQ.LVEQ) LCOM(K)=-NFINISH(LVEQ)                 TRA00069
            K=K+LCOM(K+1)+2                                             TRA00070
   50    CONTINUE                                                       TRA00071
   60 CONTINUE                                                          TRA00072
      NPTR(NFINISH(LVEQ),LOAD)=-NPTR(NFINISH(LVEQ),LOAD)                TRA00073
      NPTR(NENTER,LOAD)=-NPTR(NENTER,LOAD)                              TRA00074
      NFINISH(LVEQ)=NENTER                                              TRA00075
      PPTR(LVEQ)=NPTR(NENTER,LOAD)                                      TRA00076
      RETURN                                                            TRA00077
    5 FORMAT (  9H LOAD NO.,I2,  6H  NODE,I2, 13H LEAVING NODE,I2,   9HTRA00078
     1ENTERING, 17H  IN EQUATION NO.,I3)                                TRA00079
C                                                                       TRA00080
      END                                                               TRA00081
```

GLOSSARY

This glossary defines the symbols used in this paper including where applicable the units of measurement. The section or Appendix where the symbol is introduced is given in parenthesis following the definition.

A_k--the cross-sectional area of link k (square inches) (3.3.4.1)

a_i--the constants used to define sets on the real line (5.4.3)

α--the maximum step length in the detailed design solution algorithm (GPM) (5.5.2.7)

α^k--the maximum step length at iteration k (GPM) (5.5.2.7)

α_{min}--the step length below which the detailed design solution algorithm terminates (GPM) (5.5.2.8)

B--the linear program basis matrix (5.5.2.6)

BMAX--the maximum budget level (dollars) (5.3.2.6.4)

b_i--the external flow at node i (GPM) (1.1.4)

$\hat{C}_B$--the cost vector of the linear program basic variable (5.5.2.6)

C_E--the cost of electricity per kilowatt-hr (dollars) (5.3.2.6.3.2.1)

C_k--the total capital cost of link k (dollars) (3.3.5.1)

CCP($\hat{Q}$)--the optimal objective value of the complementary convex program with flow distribution $\hat{Q}$ (5.5.2.1)

CFR--the capital recovery factor (5.3.2.6.2)

CL_{kj}--the total equivalent uniform annual cost per foot for installing a segment of diameter $j \varepsilon S_k$ (dollars/foot) (3.2.2.1)

c_k--the total estimated cost of installing redundant link k in the system at minimum diameter (dollars) (4.4.1.1)

c_{kj}--the total estimated cost of installing candidate diameter redundant link $j \varepsilon S_k$ (dollars) (4.4.2.1)

$\bar{c}_j$--the reduced cost of the j th linear programming variable (3.5.2)

D--the link diameter (inches) (1.1.3)

D_k--the diameter of link k (inches) (1.1.4)

D_k^*--the optimal link diameter for link k (inches) (Appendix C)

D_{kj}--the jth candidate diameter for link k where $j \in S_k$ (inches) (3.3.2.1)

DNODE--the set of demand nodes (3.2.2.1)

d_i--the minimum total redundant link capacity required to cover the failure of primary link i (GPM) (4.4.2.1)

δ-method--one of the two principal methods of separable programming (3.3.5.2)

ΔD_k--the change in diameter for link k (inches) (4.4.4)

E--the general symbol for energy (ft-lb or kw-hr) (1.1.2)

EL--the vertical distance (elevation) above a fixed datum plane (feet) (1.1.2)

EL_i--the elevation at node i (feet) (1.1.2)

$EQCAP_i$--the average excess primary link flow capacity available from the alternate source in case of failure of primary link i (GPM) (4.4.4)

EUAC--the equivalent uniform annual costs (5.3.2.6.2)

e'--the thickness of the pipe wall (inches) (1.1.3)

e_{ik}, e_{ikj}--the discrete valued constants used in defining the constraint matrix for the set (Problem P6) and flow (Problem P7) models (4.4.1 and 4.4.2)

ε_1, ε_2--the constants used as stopping criteria for the Hardy Cross balancing method (1.2.1)

ΔENERGY--the estimate of the external energy which must be added to the system to attain minimum normal nodal pressure levels (feet) (3.4)

F--the feasible region of the MAXWMIN problem (Problem P12) (5.3.3.2)

f'--the dimensionless friction factor in the Darcy-Weisbach rational friction loss formula (1.1.3)

$f_k(\ )$, $f_{i\ell}(\ )$, $\overline{f}_\ell(\ )$--general arbitrarily defined real valued functions

G_i--the gradient for loop i (5.5.2.7)

$GMAX^k$--the largest absolute value of G_i at iteration k (5.5.2.7)

GPM--the abbreviation for gallons per minute (1.1.3)

GRAPH--an undirected graph (3.3.1)

g'--the gravitational constant (ft/sec^2) (1.1.2)

$g(\ )$, $g_{ik}(\ )$, $\bar{g}(\ )$--general arbitrary real valued functions (3.3.5.2)

γ--the specific weight of a fluid (lb/ft^3) (1.1.2)

H_i--the head at node i (feet) (3.2.2.1)

$H_i(\ell)$--the head at node i under loading condition ℓ (feet) (5.3.2.1)

ΔH_i--the change in head at node i during application of the nodal form of the Hardy Cross method (feet) (1.2.1)

ΔHF--the frictional head loss on a link (feet) (1.1.2)

ΔHF_k--the frictional head loss on link k (feet) (1.1.4)

ΔHF_k^*--the optimal frictional head loss on link k (feet) (Appendix C)

$\Delta HF_k(\ell)$--the frictional head loss on link k during loading ℓ (feet) (5.3.2.1)

$HMIN_i$--the minimum head at node i (feet) (3.2.2.1)

$HMIN_i(\ell)$--the minimum head at node i under loading ℓ (feet) (5.3.2.1)

HP_k--the horsepower of pump k (horsepower) (5.3.2.6.3.2.1)

$HPMAX_k$--the maximum horsepower of pump k (horsepower) (5.3.2.5)

$HPMIN_k$--the minimum horsepower of pump k (horsepower) (5.3.2.5)

HW--the dimensionless Hazen-Williams roughness coefficient (1.1.3)

HW_k--the Hazen-Williams roughness coefficient for link k (1.1.4)

$h_i(\hat{x})$--a general nonlinear function (1.2.1)

I--the interest rate on funds (5.3.2.6.2)

inf--the infimum of a function (5.4.3)

J_k--the hydraulic gradient for link k, i.e., head loss per unit length of pipe (3.3.4.1)

J^*_{kj}--the optimal hydraulic gradient on the j th segment of link k (Appendix C)

$J^{*}_{kj\ell}$--the optimal hydraulic gradient on the j th segment of link k on loading ℓ (Appendix C)

$\bar{J}$--a uniform hydraulic gradient (3.3.4.1)

JAC^{k}--the Jacobian matrix at iteration k of the Newton-Rhapson method (1.2.2)

K--the general multiplicative constant in the empirical frictional head loss equation (1.1.3.)

K_k--the constant multiplier for frictional head loss in link k (1.1.4)

K_{kj}--the constant multiplier for frictional head loss on segment $j \varepsilon S_k$ on link k (5.3.2.1)

$\bar{K}_k$--constant multiplier used in development of nonlinear minimum cost flow model (3.3.5.1)

L--the link length (feet) (1.1.3)

L_k--the length of link k (feet) (1.1.4)

LC_i--the set of loops which have links in common with loop i (5.5.2.7)

LE--the set of emergency loading conditions (5.3.3.2)

LINK--the set of links in the distribution system (3.3.1)

LN--the set of normal loading conditions (5.3.3.4)

$LOOP_i$--the set of links in loop i (1.1.4)

$LOOP_i(\ell)$--the set of links in loop i under loading conditions ℓ (5.3.4)

LP_{ij}--the length of the j th path from the source to node i in the shortest path tree model (feet) (3.3.4.1)

ℓ_{c_k}--the critical loading condition for pump k (5.3.2.6.1.2)

ℓ_1, ℓ_2--the dimensionless constants used in defining the capital pump cost function (3.3.5.1)

ℓ_3--a dimensionless constant used in development of the nonlinear minimum cost flow model (3.3.5.1)

ℓ_4, ℓ_5, ℓ_6--the dimensionless constants used in defining the capital pump cost function (5.3.2.6.1.2)

λ-method--the method of separable programming used to solve the nonlinear minimum cost flow model (3.3.5.2 and Appendix B)

λ'--the expected number of link failures per foot of pipe per year (4.3.2)

λ''_{kj}--the weight used in the proof of THEOREM II (Appendix C)

M--the number of decision variables in the separable program (Appendix B)

M'(GRAPH)--the tree matrix used to count the number of spanning trees in a graph (3.3.1)

MAXFLOIT--the maximum number of flow iterations in the detailed design solution algorithm (5.5.2.8)

MAXIMB--the maximum head imbalance in the Hardy Cross method (Appendix A)

MAXMIN--the objective function to maximize the minimum nodal head over all emergency loading conditions (5.3.3.1)

MAXWMIN--the objective function to maximize a weighted sum of the minimum nodal heads over all emergency loading conditions. This term also refers to Problem P12. (5.3.3.1)

MAXWNODE--the objective function to maximize a weighted sum of nodal heads over all emergency loading conditions (5.3.3.1)

MINCOST--the objective function to minimize equivalent uniform annual costs. The term also refers to Problem P13 (5.4.2)

m--the dimensionless constant exponent for the diameter in the empirical frictional head loss equation (1.1.3)

m'_{ij}--the i,j element of M'(GRAPH) (3.3.1)

N--the number of equations in a system of equations (1.2.1)

NLINK--the number of links in the distribution system (1.1.4)

NLOAD--the number of loadings (Appendix C)

NLOOP--the number of independent loops in the distribution system (1.1.4)

NLOOP(ℓ)--the number of active loops under loading condition ℓ (5.3.4)

NNODE--the total number of nodes in the distribution system (1.1.4)

NODE--the set of nodes in the distribution system (3.3.1)

NP_i--the number of tree paths from the source to node i in the shortest path tree model (3.3.4.1)

$NPPUMP_k$--the number of identical parallel pumps composing pump k (5.3.2.6.1.2)

NPUMP--the number of pumps in the distribution system (3.2.2.1)

NSOURCE--the number of sources (5.3.2.2)

NST--the number of elevated storages in the distribution system (3.2.2.1)

NYEAR--the economic life of an item of capital equipment (years) (5.3.2.6.2)

η_k--the pump-motor efficiency of pump k (5.3.2.5)

n--the exponent of Q in the empirical head loss equation (1.1.3)

O_i--the set of links with flows leaving node i (1.1.4)

Ω--a closed, bounded set (3.3.5.1)

$PATH_{si}$--the set of links, pumps, and storages on the path from source node s to demand node i (3.2.2.1)

$PEN_{k\ell}$--the penalty coefficient used in the quadratic programming problem, Problem P18 (Appendix C)

$PHMIN_k$--the minimum head for pump k (feet) (5.3.2.5)

$PHMAX_k$--the maximum head for pump k (feet) (5.3.2.5)

PL--the set of links in the core tree (3.2.2.1)

$\overline{PL}$--the set of non-tree or candidate redundant links (3.2.2.1)

$\overline{PL}_k$--this term used to identify a specific subset of non-tree links (4.4.3)

PU $[XP_k({}^{\ell}c_k), QP_k({}^{\ell}c_k)]$--the total equivalent uniform annual capital and operating cost for pump k (dollars) (3.2.2.1)

π--the dimensionless constant which is the ratio of the circumference of a circle to its diameter (3.3.2.1)

$\hat{\pi} = (\pi_1, \ldots)$--the vector of dual variables (5.5.2.6)

p--the fluid pressure (lb/ft^2) (1.1.2)

p_i--the fluid pressure at point i (lb/ft^2) (1.1.2)

Q--the flow rate (GPM) (1.1.3)

Q_k--the flow rate on link k (GPM) (1.1.4)

$Q_k(\ell)$--the flow rate on link k on loading ℓ (GPM) (5.3.2.1)

$\hat{Q} = (Q_1, \ldots, Q_{NLINK})$--the link flow distribution vector (GPM) (5.5.2.1)

$\hat{Q}^k$--the link flow distribution vector at the k^{th} iteration of the detailed design solution algorithm (5.5.2.7)

Q_k^0--the initial estimate of flow on link k for the linear theory balancing method (1.2.3)

Q_k^*--the optimal flow on link k (GPM) (5.5.4)

Q_{k_i}--the expected flow on link k after failure of pirmary link i (GPM) (4.4.4)

$QMAX_k$--the flow capacity of link k (GPM) (3.3.4.1)

$\overline{Q}_k$--the average daily flow rate on link k (GPM) (4.3.1)

ΔQ_i--the flow change on loop i (GPM) (1.2.1)

$\Delta\hat{Q} = (\Delta Q_1, \ldots, \Delta Q_{NLOOP})$--the vector of loop flow changes (GPM) (5.5.2.1)

$\Delta\hat{Q}^k$--the vector of loop flow changes at the k^{th} iteration of the detailed design solution algorithm (GPM) (5.5.2.7)

$\Delta QMIN^k$--the minimum loop flow change at iteration k used in the detailed design solution algorithm (GPM) (5.5.2.7)

QP_k--the flow through pump k (GPM) (3.2.2.1)

$QP_k(\ell)$--the flow through pump k under loading ℓ (GPM) (5.3.2.5)

R--used to define a specific convex set (5.4.3)

Re--the dimensionless Reynolds number (1.1.3)

RMAX--the maximum resistance which a valve can provide (feet) (5.6.4.3.3)

r_i--the minimum number of redundant links required to cover the failure of primary link i (4.4.1.1)

S_k--the set of candidate diameters for link k (3.2.2.1)

$SHMAX_k$--the maximum height storage k may be elevated (feet) (5.3.2.4)

SNODE--the set of source nodes (3.2.2.1)

$SOURCE_j$--the j^{th} source (4.4.4)

STC_k--the equivalent uniform annual cost per foot for elevating storage k (3.2.2.1)

SSP_i--*the set of primary links on the source-to-source path from* the alternative source to primary link i (4.4.4)

SV--the salvage value ratio for an item of capital equipment (5.3.2.6.2)

T_i--the set of links with flows entering node i (1.1.4)

t_i--the expected repair time for repairing failure of primary link i (minutes) (4.3.1)

U--the load factor for computing the pump energy usage (5.3.2.6.3.2.1)

u_i--the expected unsatisfied demand resulting from each failure of primary link i (gallons) (4.3.1)

$\bar{u}_i$--the expected annual unsatisfied demand resulting from failure of primary link i (gallons) (4.3.2)

V--the velocity of water flow (ft/sec) (1.1.2)

V_k--the velocity of water flow on link k (ft/sec) (1.1.2)

w_ℓ--the weight assigned to emergency loading ℓ in the MAXWMIN problem (5.3.3.2)

X--the general set of decision values in a mathematical programming problem (5.3.3.2)

XL_{kj}--the length of pipe of diameter $j \varepsilon S_k$ to install on link k (feet) (3.2.2.1)

XP--the head lift provided by a pump (feet) (1.1.2)

XP_k--the head lift provided by pump k (feet) (3.2.2.1)

$XP_k(\ell)$--the head lift provided by pump k on loading ℓ (feet) (5.3.2.1)

XS_k--the height to elevate storage reservoir k (feet) (3.2.2.1)

XV_i^+, XV_i^---the resistance provided by valve i (feet) (5.5.2.6)

x--a general one dimensional real variable (5.4.3)

$\hat{x} = (x_1, \ldots)$--a general vector of real variables (1.2.1)

x_j--a single component of the vector $\hat{x}$ (1.2.1)

$\hat{x}^k$--the value of $\hat{x}$ at iteration k (1.2.1)

$\Delta\hat{x}^k$--the change in $\hat{x}$ at iteration k (1.2.1)

Δx_j--the change in $x_j \varepsilon \hat{x}$ (1.2.1)

Δx_j^k--the change in $x_j^k \varepsilon \hat{x}^k$ at iteration k (1.2.2)

y_i--a general 0-1 decision variable (3.2.2.1)

y_{ij}--a general 0-1 decision variable (3.3.4.1)

$\bar{y}_i$--a discrete valued variable (4.4.4)

z--the objective function value for a mathematical programming problem (3.2.2.1)

z*, z**--the optimal objective function value for a mathematical programming problem (3.2.2.2)

z_ℓ--the value of the minimum nodal head on emergency loading ℓ (5.3.3.2)

Δz--the change in objective function value (3.5.2)

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